



SpeedLabs

MATHS

CBSE 8th

TEEVRA EDUTECH PVT. LTD.

Algebraic Expressions and Identities

Exercise 9.5

Q.1 Use a suitable identity to get each of the following products.

(i) $(x + 3)(x + 3)$

(ii) $(2y + 5)(2y + 5)$

(iii) $(2a - 7)(2a - 7)$

(iv) $(3a - \frac{1}{2})(3a - \frac{1}{2})$

(v) $(1.1m - 0.4)(1.1m + 0.4)$

(vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$

(viii) $(-a + c)(-a + c)$

(IX) $(\frac{x}{2} + \frac{3y}{4})(\frac{x}{2} + \frac{3y}{4})$

(x) $(7a - 9b)(7a - 9b)$

Sol:

(i) $(x + 3)(x + 3) = (x + 3)^2$

$= (x)^2 + 2 \times x \times 3 + (3)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$= x^2 + 6x + 9$

(ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$= (2y)^2 + 2 \times 2y \times 5 + (5)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$= 4y^2 + 20y + 25$

(iii) $(2a - 7)(2a - 7) = (2a - 7)^2$

$= (2a)^2 - 2 \times 2a \times 7 + (7)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$= 4a^2 - 28a + 49$$

$$= 4a^2 - 28a + 49$$

$$(iv) \left(3a - \frac{1}{2}\right) \left(3a - \frac{1}{2}\right) = \left(3a - \frac{1}{2}\right)^2$$

$$= (3a)^2 - 2 \times 3a \times \frac{1}{2} + \left(\frac{1}{2}\right)^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$= 9a^2 - 3a + \frac{1}{4}$$

$$(v) (1.1m - 0.4)(1.1m + 0.4)$$

$$= (1.1m - 0.4)(1.1m + 0.4) = (1.1m)^2 - (0.4)^2$$

Using identity $(a - b)(a + b) = a^2 - b^2$

$$= 1.21m^2 - 0.16$$

$$(vi) (a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$$

$$= (b^2)^2 - (a^2)^2 \text{ Using identity } (a + b)(a - b) = a^2 - b^2$$

$$= b^4 - a^4$$

$$(vii) (6x - 7)(6x + 7)$$

$$= (6x)^2 - (7)^2 \text{ Using identity } (a - b)(a + b) = a^2 - b^2$$

$$= 36x^2 - 49$$

$$(viii) (-a + c)(-a + c) = (-a + c)^2$$

$$= (-a)^2 + 2(-a) \times c + (C)^2$$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= a^2 - 2ac + c^2$$

Aliter. $(-a + c)(-a + c) = (-a + c)^2 = (c - a)^2$

$$= (c)^2 - 2 \times c \times a + a^2 \text{ Using identity } (a - b)^2$$

$$= a^2 - 2ab + b^2$$

$$= c^2 - 2ac + a^2$$

$$(ix) \left(\frac{x}{2} + \frac{3y}{4}\right) \left(\frac{x}{2} + \frac{3y}{4}\right) = \left(\frac{x}{2} + \frac{3y}{4}\right)^2$$

$$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= \frac{x^2}{4} + \frac{3}{4}xy + \frac{9}{16}y^2$$

$$(x) (7a - 9b)(7a - 9b) = (7a - 9b)^2$$

$$= (7a)^2 - 2 \times 7a \times 9b + (9b)^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$= 49a^2 - 126ab + 81b^2.$$

Q.2 Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to the following products.

(i) $(x + 3)(x + 7)$

(ii) $(4x + 5)(4x + 1)$

(iii) $(4x - 5)(4x - 1)$

(iv) $(4x + 5)(4x - 1)$

(v) $(2x + 5y)(2x + 3y)$

(vi) $(2a^2 + 9)(2a^2 + 5)$

(vii) $(xyz - 4)(xyz - 2)$

Sol: (i) $(x + 3)(x + 7) = x^2 + (3 + 7)x + 3 \times 7$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= x^2 + 10x + 21$$

(ii) $(4x + 5)(4x + 1)$

$$(4x + 5)(4x + 1) = (4x)^2 + (5 + 1) \times 4x + 5 \times 1$$

By using above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= 16x^2 + 6 \times 4x + 5$$

$$= 16x^2 + 24x + 5$$

$$(iii) (4x - 5)(4x - 1)$$

$$(4x - 5)(4x - 1) = (4x)^2 + (-5 - 1) \times 4x + (-5)(-1)$$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= 16x^2 + (-5 - 1) \times 4x + 5$$

$$= 16x^2 + (-6) \times 4x + 5$$

$$= 16x^2 - 24x + 5$$

$$(iv) (4x + 5)(4x - 1)$$

$$(4x + 5)(4x - 1) = (4x)^2 + [5 + (-1)] \times 4x + (5) \times (-1)$$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= 16x^2 + (5 - 1) \times 4x - 5$$

$$= 16x^2 + 4 \times 4x - 5 = 16x^2 + 16x - 5$$

$$(v) (2x + 5y)(2x + 3y)$$

$$(2x + 5y)(2x + 3y) = (2x)^2 + (5y + 3y) \times 2x + 5y \times 3y$$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + a \times b$

$$= 4x^2 + 8y \times 2x + 15y^2$$

$$= 4x^2 + 16xy + 15y^2$$

$$(vi) (2a^2 + 9)(2a^2 + 5)$$

$$(2a^2 + 9)(2a^2 + 5) = (2a^2)^2 + (9 + 5) \times 2a^2 + 9 \times 5$$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= 4a^4 + 14 \times 2a^2 + 45 = 4a^4 + 28a^2 + 45$$

$$(vii) (xyz - 4)(xyz - 2)$$

$$(xyz - 4)(xyz - 2) = (xyz)^2 + (-4 - 2) \times xyz + (-4) \times (-2)$$

By using the above identity $(x + a)(x + b) = x^2 + (a + b)x + ab$

$$= x^2y^2z^2 + (-4 - 2) \times xyz + 8$$

$$= x^2y^2z^2 + (-6)xyz + 8$$

$$= x^2y^2z^2 - 6xyz + 8.$$

Q.3 Find the following squares by using the identities.

(i) $(b - 7)^2$

(ii) $(xy + 3z)^2$

(iii) $(6x^2 - 5y)^2$

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$

(v) $(0.4p - 0.5q)^2$

(vi) $(2xy + 5y)^2$

Sol: (i) $(b - 7)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}(b - 7)^2 &= (b)^2 - 2 \times b \times 7 + (7)^2 \\ &= b^2 - 14b + 49.\end{aligned}$$

(ii) $(xy + 3z)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}(xy + 3z)^2 &= (xy)^2 + 2 \times xy \times 3z + (3z)^2 \\ &= x^2y^2 + 6xyz + 9z^2.\end{aligned}$$

(iii) $(6x^2 - 5y)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}(6x^2 - 5y)^2 &= (6x^2)^2 - 2 \times 6x^2 \times 5y + (5y)^2 \\ &= 36x^4 - 60x^2y + 25y^2.\end{aligned}$$

(iv) $\left(\frac{2}{3}m + \frac{3}{2}n\right)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned}\left(\frac{2}{3}m + \frac{3}{2}n\right)^2 &= \left(\frac{2}{3}m\right)^2 + 2 \times \frac{2}{3}m \times \frac{3}{2}n + \left(\frac{3}{2}n\right)^2 \\ &= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2.\end{aligned}$$

(v) $(0.4p - 0.5q)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(0.4p - 0.5q)^2 = (0.4p)^2 - 2 \times 0.4p \times 0.5q + (0.5q)^2 \\ = 0.16p^2 - 0.40pq + 0.25q^2.$$

(vi) $(2xy + 5y)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(2xy + 5y)^2 = (2xy)^2 + 2 \times 2xy \times 5y + (5y)^2 \\ = 4x^2y^2 + 20xy^2 + 25y^2$$

Q. 4 Simplify.

(i) $(a^2 - b^2)^2$

(ii) $(2x + 5)^2 - (2x - 5)^2$

(iii) $(7m - 8n)^2 + (7m + 8n)^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$

(vi) $(ab + bc)^2 - 2ab^2c$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$.

Sol: (i) $(a^2 - b^2)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(a^2 - b^2)^2 = (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2 \\ = a^4 - 2a^2b^2 + b^4$$

(ii) $(2x + 5)^2 - (2x - 5)^2$

Using identities $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$

$$(2x + 5)^2 - (2x - 5)^2 = (2x)^2 + 2 \times 2x \times 5 + (5)^2 - [(2x)^2 - 2 \times 2x \times 5 + (5)^2] \\ = 4x^2 + 20x + 25 - (4x^2 - 20x + 25) \\ = 4x^2 + 20x + 25 - 4x^2 + 20x - 25$$

= 40x (\because Like terms with equal coefficient having opposite sign will be cancelled.)

$$(iii)(7m - 8n)^2 + (7m + 8n)^2$$

Using identities $(a - b)^2 = a^2 - 2ab + b^2$

and $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned} & (7m - 8n)^2 + (7m + 8n)^2 \\ & = (7m)^2 - 2 \times 7m \times 8n + (8n)^2 + (7m)^2 + 2 \times 7m \times 8n + (8n)^2 \\ & = 49m^2 - 112mn + 64n^2 + 49m^2 + 112mn + 64n^2 \\ & = 98m^2 + 128n^2 \end{aligned}$$

(\because Having equal coefficients of like terms with opposite sign will be cancelled.)

$$(iv)(4m + 5n)^2 + (5m + 4n)^2$$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{aligned} & (4m + 5n)^2 + (5m + 4n)^2 \\ & = [(4m)^2 + 2 \times 4m \times 5n + (5n)^2] + [(5m)^2 + 2 \times 5m \times 4n + (4n)^2] \\ & = 16m^2 + 40mn + 25n^2 + 25m^2 + 40mn + 16n^2 \\ & = 41m^2 + 80mn + 41n^2 \end{aligned}$$

$$(v)(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned} & (2.5p - 1.5q)^2 - (1.5p - 2.5q)^2 = (2.5p)^2 - 2 \times 2.5p \times 1.5q \\ & + (1.5q)^2 - [(1.5p)^2 - 2 \times 1.5p \times 2.5q + (2.5q)^2] \\ & = 6.25p^2 - 7.50pq + 2.25q^2 - (2.25p^2 - 7.50pq + 6.25q^2) \\ & = 6.25p^2 - 7.50pq + 2.25q^2 - 2.25p^2 + 7.50pq - 6.25q^2 \end{aligned}$$

(\because Equal like variables with opposite signs Will be cancelled.)

$$= 4p^2 - 4q^2.$$

$$(vi)(ab + bc)^2 - 2ab^2c$$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(ab + bc)^2 - 2ab^2c = (ab)^2 + 2 \times ab \times bc + (bc)^2 - 2ab^2c$$

$$= a^2b^2 + 2ab^2c + b^2c^2 + 2ab^2c$$

$$= a^2b^2 + b^2c^2.$$

$$(vii)(m^2 - n^2m)^2 + 2m^3n^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$(m^2 - n^2m)^2 + 2m^3n^2 = (m^2)^2 - 2 \times m^2 \times n^2m + (n^2m)^2 + 2m^3n^2$$

$$= m^4 - 2m^3n^2 + n^4m^2 + 2m^3n^2$$

$$= m^4 + n^4m^2$$

Q.5 Show that.

$$(i)(3x + 7)^2 - 84x = (3x - 7)^2$$

$$(ii)(9p - 5q)^2 + 180pq = (9p + 5q)^2$$

$$(iii)\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$$

$$(iv)(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

$$(v)(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Sol: (i) $(3x + 7)^2 - 84x = (3x - 7)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$\text{L. H. S.} = (3x + 7)^2 - 84x$$

$$= (3x)^2 + 2 \times 3x \times 7 + (7)^2 - 84x$$

$$= 9x^2 + 42x + 49 - 84x$$

$$= 9x^2 - 42x + 49$$

$$= (3x)^2 - 2 \times 3x \times 7 + (7)^2 [\because a^2 - 2ab + b^2 = (a - b)^2]$$

$$= (3x - 7)^2$$

$$\text{L. H. S.} = \text{R. H. S.}$$

$$(ii)(9p - 5q)^2 + 180pq = (9p + 5q)^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\text{L. H. S.} = (9p - 5q)^2 + 180pq$$

$$\begin{aligned}
&= (9p)^2 - 2 \times 9p \times 5q + (5q)^2 + 180pq \\
&= 81p^2 - 90pq + 25q^2 + 180pq \\
&= 81p^2 + 90pq + 25q^2 \\
&= (9p)^2 + 2 \times 9p \times 5q + (5q)^2 \\
&= (9p + 5q)^2 [\because a^2 + 2ab + b^2 = (a + b)^2]
\end{aligned}$$

\therefore L. H. S. = R. H. S.

$$(iii) \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}
\text{L. H. S.} &= \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn \\
&= \left(\frac{4}{3}m\right)^2 - 2 \times \frac{4}{3}m \times \frac{3}{4}n + \left(\frac{3}{4}n\right)^2 + 2mn \\
&= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn \\
&= \frac{16}{9}m^2 + \frac{9}{16}n^2
\end{aligned}$$

\therefore L. H. S. = R. H. S.

$$(iv) (4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$$

Using identities $(a + b)^2 = a^2 + 2ab + b^2$

and $(a - b)^2 = a^2 - 2ab + b^2$

$$\begin{aligned}
\text{L. H. S.} &= (4pq + 3q)^2 - (4pq - 3q)^2 \\
&= (4pq)^2 + 2 \times 4pq \times 3q + (3q)^2 - [(4pq)^2 - 2 \times 4pq \times 3q + (3q)^2] \\
&= 16p^2q^2 + 24pq^2 + 9q^2 - [16p^2q^2 - 4pq^2 + 9q^2] \\
&= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 4pq^2 - 9q^2 \\
&= 48pq^2
\end{aligned}$$

\therefore L. H. S. = R. H. S.

$$(V) (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$$

Using identity $(a - b)(a + b) = (a^2 - b^2)$

$$\text{L.H.S.} = (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$$

$$= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2)$$

$$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$$

$$= 0$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Q.6 Using identities, evaluate.

(i) 71^2

(ii) 998^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(vi) 297×303

(vii) 78×82

(viii) 8.9^2

(ix) 1.05×9.5

Sol: (i) $71^2 = (70 + 1)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= (70)^2 + 2 \times 70 \times 1 + (1)^2$$

$$= 4900 + 140 + 1 = 5041$$

(ii) $99^2 = (100 - 1)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$= (100)^2 - 2 \times 100 \times 1 + (1)^2$$

$$= 10000 - 200 + 1 = 9801$$

(iii) $102^2 = (100 + 2)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= (100)^2 + 2 \times 100 \times 2 + (2)^2$$

$$= 10000 + 400 + 4 = 10404$$

(iv) $998^2 = (1000 - 2)^2$

Using identity $(a - b)^2 = a^2 - 2ab + b^2$

$$= (1000)^2 - 2 \times 1000 \times 2 + (2)^2$$

$$= 1000000 - 4000 + 4 = 996004$$

(v) $5.2^2 = (5 + 0.2)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= (5)^2 + 2 \times 5 \times 0.2 + (0.2)^2$$

$$= 25 + 2.0 + 0.04 = 27.04$$

(vi) $297 \times 303 = (300 - 3) \times (300 + 3)$

Using identity $(a + b)(a - b) = a^2 - b^2$

$$= (300)^2 - (3)^2 = 90000 - 9 = 89991$$

(vii) $78 \times 82 = (80 - 2)(80 + 2)$

Using identity $(a + b)(a - b) = a^2 - b^2$

$$= (80)^2 - (2)^2$$

$$= 6400 - 4 = 6396$$

(viii) $8.9^2 = (8 + 0.9)^2$

Using identity $(a + b)^2 = a^2 + 2ab + b^2$

$$= (8)^2 + 2 \times 8 \times 0.9 + (0.9)^2$$

$$= 64 + 14.4 + 0.81 = 79.21$$

(ix) $1.05 \times 9.5 = (10 + 0.5)(10 - 0.5)$

Using identity $(a + b)(a - b) = a^2 - b^2$

$$= (10)^2 - (0.5)^2 = 100 - 0.25$$

$$= 99.75$$

Q.7 Using $a^2 - b^2 = (a + b)(a - b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Sol: (i) $51^2 - 49^2$

By using above identity

$$= (51 + 49)(51 - 49)$$

$$= 100 \times 2 = 200$$

(ii) $(1.02)^2 - (0.98)^2$

By using above identity

$$= (1.02 + 0.98)(1.02 - 0.98)$$

$$= 2.00 \times 0.04 = 0.08$$

(iii) $153^2 - 147^2$

By using above identity

$$= (153 + 147)(153 - 147)$$

$$= 300 \times 6 = 1800$$

(iv) $12.1^2 - 7.9^2$

By using above identity

$$= (12.1 + 7.9)(12.1 - 7.9)$$

$$= 20.0 \times 4.2 = 84.0 = 84.$$

Q.8 Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find

(i) 103×104 (ii) 5.1×5.2 (iii) 103×98 (iv) 9.7×9.8

Sol: (i) $103 \times 104 = (100 + 3) \times (100 + 4)$

By using above identity

$$= (100)^2 + (3 + 4) \times 100 + 3 \times 4$$

$$= 10000 + 7 \times 100 + 12$$

$$= 10000 + 700 + 12 = 10712$$

$$(ii) 5.1 \times 5.2 = (5 + 0.1) \times (5 + 0.2)$$

By using above identity

$$= (5)^2 + (0.1 + 0.2) \times 5 + 0.1 \times 0.2$$

$$= 25 + 0.3 \times 5 + 0.02$$

$$= 25 + 1.5 + 0.02 = 26.52$$

$$(iii) 103 \times 98 = (100 + 3) \times (100 - 2)$$

By using above identity

$$= (100)^2 + [3 + (-2)] \times 100 + 3 \times (-2)$$

$$= 10000 + (3 - 2) \times 100 - 6$$

$$= 10000 + 1 \times 100 - 6$$

$$= 10000 + 100 - 6 = 10094$$

$$(iv) 9.7 \times 9.8 = (10 - 0.3) \times (10 - 0.2)$$

By using above identity

$$= (10)^2 + \{(-0.3) + (-0.2)\} \times 10 + (-0.3) \times (-0.2)$$

$$= 100 + \{-0.3 - 0.2\} \times 10 + 0.06$$

$$= 100 - (0.5) \times 10 + 0.06$$

$$= 100 - 5 + 0.06$$

$$= 95.06.$$