

Board – CBSE

Class – 12

Topic – Application of derivatives

## 1. Rate of change

### LEVEL -I

1. A balloon, which always remains spherical, has a variable diameter  $\frac{3}{2}(2x + 1)$ . Find the rate of change of its volume with respect to  $x$ .
2. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long?
3. The radius of a circle is increasing at the rate of 0.7 cm/sec. what is the rate of increase of its circumference?

### LEVEL -II

4. Find the point on the curve  $y^2 = 8x$  for which the abscissa and ordinate change at the same rate?
5. A man 2-meter-high walks at a uniform speed of 6km /h away from a lamp post 6-meter high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.
6. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3cm/sec. find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm

### LEVEL III

7. A particle moves along the curve  $6y = x^3 + 2$ , Find the points on the curve at which  $y$  coordinate is changing 8 times as fast as the  $x$ -coordinate.
8. Water is leaking from a conical funnel at the rate of  $5 \text{ cm}^3/\text{sec}$ . If the radius of the base of the funnel is 10 cm and altitude is 20 cm, Find the rate at which water level is dropping when it is 5 cm from top.
9. From a cylinder drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 ml/sec. If the radius of the drum is 10cm and height 50cm, find the rate at which the level of the petrol is changing when petrol level is 20 cm

## 2. Increasing & decreasing functions

### LEVEL I

10. Show that  $f(x) = x^3 - 6x^2 + 18x + 5$  is an increasing function for all  $x \in \mathbb{R}$ .
11. Show that the function  $x^2 - x + 1$  is neither increasing nor decreasing on  $(0,1)$

12. Find the intervals in which the function  $f(x) = \sin x - \cos x$ ,  $0 < x < 2\pi$  is increasing or decreasing

### LEVEL II

13. Indicate the interval in which the function  $f(x) = \cos x$ ,  $0 \leq x \leq 2\pi$  is decreasing.
14. Show that the function  $f(x) = \frac{\sin x}{x}$  is strictly decreasing on  $(0, \pi/2)$
15. Find the intervals in which the function  $f(x) = \frac{\log x}{x}$  is increasing or decreasing.

### LEVEL III

16. Find the interval of monotonicity of the function  $f(x) = 2x^2 - \log x$ ,  $x \neq 0$
17. Prove that the function  $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$  is an increasing function of  $\theta$  in  $[0, \pi/2]$

[CBSE 2011]

### 3. Tangents & Normal

#### LEVEL-I

18. Find the equations of the normal to the curve  $3x^2 - y^2 = 8$  which are parallel to the line  $x + 3y = 4$ .
19. Find the point on the curve  $y = x^2$  where the slope of the tangent is equal to the x-coordinate of the point.
20. At what points on the circle  $x^2 + y^2 - 2x - 4y + 1 = 0$ , the tangent is parallel to x axis ?

#### LEVEL-II

21. Find the equation of the normal to the curve  $ay^2 = x^3$  at the point  $(am^2, am^3)$
22. For the curve  $y = 2x^2 + 3x + 18$ , find all the points at which the tangent passes through the origin.
23. Find the equation of the normal to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$
24. Show that the equation of tangent at  $(x_1, y_1)$  to the parabola  $yy_1 = 2a(x + x_1)$

[CBSE 2012 Comptt.]

#### LEVEL- III

25. Find the equation of the tangent line to the curve  $y = \sqrt{5x - 3} - 2$  which is parallel to the line  $4x - 2y + 3 = 0$