



**SpeedLabs**

**MATHS**

**CBSE 12<sup>th</sup>**

**TEEVRA EDUTECH PVT. LTD.**

# Application of Derivatives

## Exercise 6.2

1. Show that the function given by  $f(x) = 3x + 17$  is strictly increasing on  $\mathbb{R}$ .

**Sol.** Let  $x_1$  and  $x_2$  be any two numbers in  $\mathbb{R}$ .

Then, we have:

$$x_1 < x_2 \Rightarrow 3x_1 < 3x_2 \Rightarrow 3x_1 + 17 < 3x_2 + 17 \Rightarrow f(x_1) < f(x_2)$$

Hence,  $f$  is strictly increasing on  $\mathbb{R}$ .

Alternate method:

$$f'(x) = 3 > 0, \text{ in every interval of } \mathbb{R}.$$

Thus, the function is strictly increasing on  $\mathbb{R}$ .

2. Show that the function given by  $f(x) = e^{2x}$  is strictly increasing on  $\mathbb{R}$ .

**Sol.** Let  $x_1$  and  $x_2$  be any two numbers in  $\mathbb{R}$ .

Then, we have:

$$x_1 < x_2 \Rightarrow 2x_1 < 2x_2 \Rightarrow e^{2x_1} < e^{2x_2} \Rightarrow f(x_1) < f(x_2)$$

Hence,  $f$  is strictly increasing on  $\mathbb{R}$ .

3. Show that the function given by  $f(x) = \sin x$  is

(a) strictly increasing in  $\left(0, \frac{\pi}{2}\right)$

(b) strictly decreasing in  $\left(\frac{\pi}{2}, \pi\right)$

(c) neither increasing nor decreasing in  $(0, \pi)$

**Sol.** The given function is  $f(x) = \sin x$ .

$$\therefore f'(x) = \cos x$$

(a) Since for each  $x \in \left(0, \frac{\pi}{2}\right)$ ,  $\cos x > 0$ , we have  $f'(x) > 0$ .

Hence,  $f$  is strictly increasing in  $\left(0, \frac{\pi}{2}\right)$

(b) Since for each  $x \in \left(\frac{\pi}{2}, \pi\right)$ ,  $\cos x < 0$ , we have  $f'(x) < 0$ .

Hence,  $f$  is strictly decreasing in  $\left(\frac{\pi}{2}, \pi\right)$

(c) From the results obtained in (a) and (b), it is clear that  $f$  is neither increasing nor decreasing in  $(0, \pi)$ .

4. Find the intervals in which the function  $f$  given by  $f(x) = 2x^2 - 3x$  is

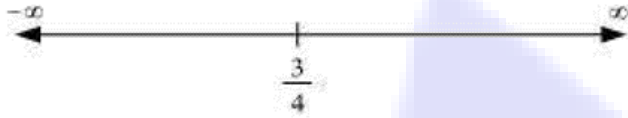
(a) strictly increasing (b) strictly decreasing

**Sol.** The given function is  $f(x) = 2x^2 - 3x$

$$f'(x) = 4x - 3$$

$$\therefore f'(x) = 0 \Rightarrow x = \frac{3}{4}$$

Now, the point  $\frac{3}{4}$  divides the real line into two disjoint intervals i. e.,  $(-\infty, \frac{3}{4})$  and  $(\frac{3}{4}, \infty)$ .



In interval  $(-\infty, \frac{3}{4})$ ,  $f'(x) = 4x - 3 < 0$ .

Hence, the given function (f) is strictly decreasing in interval  $(-\infty, \frac{3}{4})$ .

In interval  $(\frac{3}{4}, \infty)$ ,  $f'(x) = 4x - 3 > 0$ .

Hence, the given function (f) is strictly increasing in interval  $(\frac{3}{4}, \infty)$ .

5. Find the intervals in which the function f given by  $f(x) = 2x^3 - 3x^2 - 36x + 7$  is  
 (a) strictly increasing                      (b) strictly decreasing

**Sol.** The given function is  $f(x) = 2x^3 - 3x^2 - 36x + 7$ .

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x + 2)(x - 3)$$

$$\therefore f'(x) = 0 \Rightarrow x = -2, 3$$

The points  $x = -2$  and  $x = 3$  divide the real line into three disjoint intervals i.e.,  $(-\infty, -2)$ ,  $(-2, 3)$  and  $(3, \infty)$ .



In intervals  $(-\infty, -2)$  and  $(3, \infty)$ ,  $f'(x)$  is positive while in interval  $(-2, 3)$ ,  $f'(x)$  is negative.

Hence, the given function (f) is strictly increasing in intervals  $(-\infty, -2)$  and  $(3, \infty)$ , while function (f) is strictly decreasing in interval  $(-2, 3)$ .

6. Find the intervals in which the following functions are strictly increasing or decreasing:

- (a)  $x^2 + 2x - 5$                       (b)  $10 - 6x - 2x^2$   
 (c)  $-2x^3 - 9x^2 - 12x + 1$         (d)  $6 - 9x - x^2$   
 (e)  $(x + 1)^3 (x - 3)^3$

**Sol.** (a) We have,

$$F(x) = x^2 + 2x - 5$$

$$\therefore f'(x) = 2x + 2$$

Now,

$$f'(x) = 0 \Rightarrow x = -1$$

Point  $x = -1$  divides the real line into two disjoint intervals i.e.,  $(-\infty, -1)$  and  $(-1, \infty)$

In interval  $(-\infty, -1)$ ,  $f'(x) = 2x + 2 < 0$ .

$\therefore f$  is strictly increasing in interval  $(-1, \infty)$ .

Thus,  $f$  is strictly increasing for  $x > -1$ .

(b) We have,

$$f(x) = 10 - 6x - 2x^2$$

$$\therefore f'(x) = -4 - 4x$$

Now,

$$f'(x) = 0 \Rightarrow x = -\frac{3}{2}$$

The point  $x = -\frac{3}{2}$  divides the real line into two disjoint intervals

i. e.,  $(-\infty, -\frac{3}{2})$  and  $(-\frac{3}{2}, \infty)$

In interval  $(-\infty, -\frac{3}{2})$  i. e., when  $x > -\frac{3}{2}$ ,  $f'(x) = -6 - 4x < 0$ .

$\therefore f$  is strictly increasing for  $x > -\frac{3}{2}$ .

(c) We have,

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

$$\therefore f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x + 1)(x + 2)$$

Now,

$$f'(x) = 0 \Rightarrow x = -1 \text{ and } x = -2$$

Points  $x = -1$  and  $x = -2$  divide the real line into three disjoint intervals

i.e.,  $(-\infty, -2)$   $(-2, -1)$  and  $(-1, \infty)$

In intervals  $(-\infty, -2)$  and  $(-1, \infty)$  i. e., when  $x < -2$  and  $x > -1$ ,

$$f'(x) = -6(x + 1)(x + 2) < 0$$

$\therefore f$  is strictly decreasing for  $x < -2$  and  $x > -1$ .

Now, in interval  $(-2, -1)$  i.e., when  $-2 < x < -1$ ,  $f'(x) = -6(x + 1)(x + 2) > 0$

$f$  is strictly increasing for  $-2 < x < -1$ .

(d) We have,

$$f(x) = 6 - 9x - x^2$$

$$\therefore f'(x) = -9x - 2x$$

Now,  $f'$

$$f'(x) = 0 \text{ gives } x = -\frac{9}{2}$$

The point  $x = -\frac{9}{2}$  divides the real line into two disjoint intervals i.e.,  $(-\infty, -\frac{9}{2})$  and  $(-\frac{9}{2}, \infty)$ .

In interval  $(-\infty, -\frac{9}{2})$  i.e., for  $x < -\frac{9}{2}$ ,  $f'(x) = -9 - 2x > 0$ .

$\therefore$   $f$  is strictly increasing for  $x < -\frac{9}{2}$

In interval  $(-\frac{9}{2}, \infty)$  i.e., for  $x > -\frac{9}{2}$ ,  $f'(x) = -9 - 2x < 0$ .

$\therefore$   $f$  is strictly decreasing for  $x > -\frac{9}{2}$ .

(e) We have,

$$f(x) = (x + 1)^3 (x - 3)^3$$

$$f'(x) = 3(x + 1)^2 (x - 3)^3 + 3(x - 3)^2 (x + 1)^3$$

$$= 3(x + 1)^2 (x - 3)^2 [x - 3 + x + 1]$$

$$= 3(x + 1)^2 (x - 3)^2 (2x - 2)$$

$$= 6(x + 1)^2 (x - 3)^2 (x - 1)$$

Now,

$$f'(x) = 0 \Rightarrow x = -1, 3, 1$$

The points  $x = -1$ ,  $x = 1$ , and  $x = 3$  divide the real line into four disjoint intervals i.e.,  $(-\infty, -1)$ ,  $(-1, 1)$ ,  $(1, 3)$ , and  $(3, \infty)$ .

In intervals  $(-\infty, -1)$  and  $(-1, 1)$ ,  $f'(x) = 6(x + 1)^2 (x - 3)^2 (x - 1) < 0$ .

$\therefore$   $f$  is strictly decreasing in intervals  $(-\infty, -1)$  and  $(-1, 1)$ .

In intervals  $(1, 3)$  and  $(3, \infty)$ ,  $f'(x) = 6(x + 1)^2 (x - 3)^2 (x - 1) > 0$ .

$\therefore$   $f$  is strictly increasing in intervals  $(1, 3)$  and  $(3, \infty)$ .

7. Show that  $y = \log(1 + x) - \frac{2x}{2 + x}$ ,  $x > -1$ , is an increasing function of  $x$  throughout its domain.

**Sol.** We have,

$$y = \log(1 + x) - \frac{2x}{2 + x}$$

$$\frac{dy}{dx} = \frac{1}{1 + x} - \frac{(2 + x)(2) - 2x(1)}{(2 + x)^2} = \frac{1}{1 + x} - \frac{4}{(2 + x)^2} = \frac{x^2}{(2 + x)^2}$$

$$\text{Now, } \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{x^2}{(2 + x)^2} = 0$$

$$\Rightarrow x^2 = 0 \quad [(2 + x) \neq 0 \text{ as } x > -1]$$

$$\Rightarrow x = 0$$

Since  $x > -1$ , point  $x = 0$  divides the domain  $(-1, \infty)$  in two disjoint intervals i.e.,  $-1 < x < 0$  and  $x > 0$ .

When  $-1 < x < 0$ , we have:

$$x < 0 \Rightarrow x^2 > 0$$

$$x > -1 \Rightarrow (2+x) > 0 \Rightarrow (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Also, when  $x > 0$ :

$$x > 0 \Rightarrow x^2 > 0, (2+x)^2 > 0$$

$$\therefore y' = \frac{x^2}{(2+x)^2} > 0$$

Hence, function  $f$  is increasing throughout this domain.

8. Find the values of  $x$  for which  $y = [x(x-2)]^2$  is an increasing function.

Sol. We have,

$$y = [x(x-2)]^2 = [x^2 - 2x]^2$$

$$\therefore \frac{dy}{dx} = y' = 2(x^2 - 2x)(2x - 2) = 4x(x-2)(x-1)$$

$$\therefore \frac{dy}{dx} = 0 \Rightarrow x = 0, x = 2, x = 1.$$

The points  $x = 0$ ,  $x = 1$ , and  $x = 2$  divide the real line into four disjoint intervals i.e.,

$(-\infty, 0)$ ,  $(0, 1)$ ,  $(1, 2)$  and  $(2, \infty)$ .

In intervals  $(-\infty, 0)$ , and  $(1, 2)$   $\frac{dy}{dx} < 0$

$\therefore y$  is strictly decreasing in intervals  $(-\infty, 0)$  and  $(1, 2)$

, However, in intervals  $(0, 1)$  and  $(2, \infty)$ ,  $\frac{dy}{dx} > 0$

$\therefore y$  is strictly increasing in intervals  $(0, 1)$  and  $(2, \infty)$ .

$\therefore y$  is strictly increasing for  $0 < x < 1$  and  $x > 2$ .

9. Prove that  $y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta$  is an increasing function of  $\theta$  in  $\left[0, \frac{\pi}{2}\right]$

Sol. We have,

$$y = \frac{4 \sin \theta}{(2 + \cos \theta)} - \theta$$

$$\therefore \frac{dy}{dx} = \frac{(2 + \cos \theta)(4 \cos \theta) - 4 \sin \theta (-\sin \theta)}{(2 + \cos \theta)^2} - 1$$

$$\frac{8 \cos \theta + 4 \cos^2 \theta + 4 \sin^2 \theta}{(2 + \cos \theta)^2} - 1$$

$$\frac{8 \cos \theta + 4}{(2 + \cos \theta)^2} - 1$$

$$\text{Now, } \frac{dy}{dx} = 0.$$

$$\Rightarrow \frac{8 \cos \theta + 4}{(2 + \cos \theta)^2} = 1$$

$$\Rightarrow 8 \cos \theta + 4 = 4 + \cos^2 \theta + 4 \cos \theta$$

$$\Rightarrow \cos^2 \theta - 4 \cos \theta = 0$$

$$\Rightarrow \cos \theta (\cos \theta - 4) = 0$$

$$\Rightarrow \cos \theta = 0 \text{ or } \cos \theta = 4$$

Since  $\cos \theta \neq 4$ ,  $\cos \theta = 0$

$$\cos \theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Now,

$$\frac{dy}{dx} = \frac{8 \cos \theta + 4 - (4 + \cos^2 \theta + 4 \cos \theta)}{(2 + \cos \theta)^2} = \frac{4 \cos \theta - \cos^2 \theta}{(2 + \cos \theta)^2} = \frac{\cos \theta (4 - \cos \theta)}{(2 + \cos \theta)^2}$$

In interval  $(0, \frac{\pi}{2})$ , we have  $\cos \theta > 0$ . Also,  $4 > \cos \theta \Rightarrow 4 - \cos \theta > 0$ .

$\cos \theta (4 - \cos \theta) > 0$  and also  $(2 + \cos \theta)^2 > 0$

$$\Rightarrow \frac{\cos \theta (4 - \cos \theta)}{(2 + \cos \theta)^2} > 0$$

$$\Rightarrow \frac{dy}{dx} > 0$$

, Therefore,  $y$  is strictly increasing in interval  $(0, \frac{\pi}{2})$ .

Also, the given function is continuous at  $x = 0$  and  $x = \frac{\pi}{2}$ .

Hence,  $y$  is increasing in interval  $[0, \frac{\pi}{2}]$ .

**10.** Prove that the logarithmic function is strictly increasing on  $(0, \infty)$ .

**Sol.** The given function is  $f(x) = \log x$ .

$$\therefore f'(x) = \frac{1}{x}$$

It is clear that for  $x > 0$ ,  $f'(x) = \frac{1}{x} > 0$ .

Hence,  $f(x) = \log x$  is strictly increasing in interval  $(0, \infty)$ .

**11.** Prove that the function  $f$  given by  $f(x) = x^2 - x + 1$  is neither strictly increasing nor strictly decreasing on  $(-1, 1)$ .

**Sol.** The given function is  $f(x) = x^2 - x + 1$ .

$$\therefore f'(x) = 2x - 1$$

$$\text{Now, } f'(x) = 0 \Rightarrow x = \frac{1}{2}$$

The point  $\frac{1}{2}$  divides the interval  $(-1, 1)$  into two disjoint intervals i. e.,  $(-1, \frac{1}{2})$  and  $(\frac{1}{2}, 1)$ .

Now, in interval  $\left(-1, \frac{1}{2}\right)$ ,  $f'(x) = 2x - 1 < 0$ .

Therefore,  $f$  is strictly decreasing in interval  $\left(-1, \frac{1}{2}\right)$

However, in interval  $\left(\frac{1}{2}, 1\right)$ ,  $f'(x) = 2x - 1 > 0$

, Therefore,  $f$  is strictly increasing in interval  $\left(\frac{1}{2}, 1\right)$ ,

Hence,  $f$  is neither strictly increasing nor decreasing in interval  $(-1, 1)$ .

12. Which of the following functions are strictly decreasing on  $\left(0, \frac{\pi}{2}\right)$ .

(A)  $\cos x$       (B)  $\cos 2x$       (C)  $\cos 3x$       (D)  $\tan x$

Sol. (A) Let  $f_1(x) = \cos x$

$$f_1'(x) = -\sin x$$

In interval  $\left(0, \frac{\pi}{2}\right)$ ,  $f_1'(x) = -\sin x < 0$

$f_1(x) = \cos x$  is strictly decreasing in interval  $\left(0, \frac{\pi}{2}\right)$ .

(B)  $f_2(x) = \cos 2x$

$$\therefore f_2'(x) = -2\sin 2x$$

Now,  $0 < x < \frac{\pi}{2} \Rightarrow 0 < 2x < \pi \Rightarrow \sin 2x > 0 \Rightarrow -2\sin 2x < 0$ .

$$\therefore f_2'(x) = -2\sin 2x < 0 \text{ on } \left(0, \frac{\pi}{2}\right)$$

$f_2(x) = \cos 2x$  is strictly decreasing in interval  $\left(0, \frac{\pi}{2}\right)$ .

(C) Let  $f_3(x) = \cos 3x$

$$\therefore f_3'(x) = -3\sin 3x$$

Now,  $f_3'(x) = 0$

$$\Rightarrow \sin 3x = 0 \Rightarrow 3x = \pi, \text{ as } x \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow x = \frac{\pi}{3}$$

The point  $x = \frac{\pi}{3}$  divides the interval  $\left(0, \frac{\pi}{2}\right)$  into two disjoint intervals i.e.,  $\left(0, \frac{\pi}{3}\right)$  and  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ .

Now, in interval  $\left(0, \frac{\pi}{3}\right)$ ,  $f_3(x) = -3\sin 3x < 0$  [as  $0 < x < \frac{\pi}{3} \Rightarrow 0 < 3x < \pi$ ]

$\therefore f_3$  is strictly decreasing in interval  $\left(0, \frac{\pi}{3}\right)$

However, in interval  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ ,  $f_3(x) = -3\sin 3x > 0$  [as  $\frac{\pi}{3} < x < \frac{\pi}{2} \Rightarrow \pi < 3x < \frac{3\pi}{2}$ ]

$\therefore f_3$  is strictly increasing in interval  $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ .



, Hence,  $f_3$  is neither increasing nor decreasing in interval  $\left(0, \frac{\pi}{2}\right)$

(D) Let  $f_4(x) = \tan x$

$$f_4'(x) = \sec^2 x$$

In interval  $\left(0, \frac{\pi}{2}\right)$ ,  $f_4'(x) = \sec^2 x > 0$

$\therefore f_4$  is strictly increasing in interval  $\left(0, \frac{\pi}{2}\right)$

, Therefore, functions  $\cos x$  and  $\cos 2x$  are strictly decreasing in  $\left(0, \frac{\pi}{2}\right)$ .

Hence, the correct answers are A and B.

13. On which of the following intervals is the function  $f$  given by  $f(x) = x^{100} + \sin x - 1$  strictly decreasing?

(A)  $(0, 1)$

(B)  $\left(\frac{\pi}{2}, 2\right)$

(C)  $\left(\frac{\pi}{2}, 2\right)$

(D) None of these

**Sol.** We have  $f(x) = x^{100} + \sin x - 1$

$$\therefore f'(x) = 100x^{99} + \cos x$$

In interval  $(0, 1)$ ,  $\cos x > 0$  and  $100x^{99} > 0$

$$\therefore f'(x) > 0$$

Thus, function  $f$  is strictly increasing in interval  $(0, 1)$ .

In interval  $\left(\frac{\pi}{2}, \pi\right)$ ,  $\cos x < 0$  and  $100x^{99} > 0$ . Also,  $100x^{99} > \cos x$ .

$$\therefore f'(x) > 0 \text{ in } \left(\frac{\pi}{2}, \pi\right)$$

Thus, function  $f$  is strictly increasing in interval  $\left(\frac{\pi}{2}, \pi\right)$ .

In interval  $\left(0, \frac{\pi}{2}\right)$ ,  $\cos x > 0$  and  $100x^{99} > 0$ .

$$\therefore 100x^{99} + \cos x > 0$$

$$\Rightarrow f'(x) > 0 \text{ on } \left(0, \frac{\pi}{2}\right)$$

$\therefore f$  is strictly increasing in interval  $\left(0, \frac{\pi}{2}\right)$ .

Hence, function  $f$  is strictly decreasing in none of the intervals.

The correct answer is D.

**14.** Find the least value of  $a$  such that the function  $f$  given  $f(x) = x^2 + ax + 1$  is strictly increasing on  $(1, 2)$ .

**Sol.** We have,

$$f(x) = x^2 + ax + 1$$

$$\therefore f'(x) = 2x + a$$

Now, function  $f$  will be increasing in  $(1, 2)$ , if  $f'(x) > 0$  in  $(1, 2)$

$$f'(x) > 0$$

$$\Rightarrow 2x + a > 0$$

$$\Rightarrow 2x > -a$$

$$\Rightarrow x > \frac{-a}{2}$$

Therefore, we have to find the least value of  $a$  such that

$$x > \frac{-a}{2}, \text{ when } x \in (1, 2)$$

$$\Rightarrow x > \frac{-a}{2} \text{ (when } 1 < x < 2)$$

Thus, the least value of  $a$  for  $f$  to be increasing on  $(1, 2)$  is given by,

$$\frac{-a}{2} = 1$$

$$\frac{-a}{2} = 1 \Rightarrow a = -2$$

Hence, the required value of  $a$  is  $-2$ .

**15.** Let  $I$  be any interval disjoint from  $(-1, 1)$ . Prove that the function  $f$  given by  $f(x) = x + \frac{1}{x}$  is strictly

increasing on  $I$

**Sol.** We have

$$f(x) = x + \frac{1}{x}$$

$$\therefore f'(x) = x - \frac{1}{x^2}$$

Now,

$$f'(x) = 0 \Rightarrow \frac{1}{x^2} = 1 \Rightarrow x = \pm 1$$

The points  $x = 1$  and  $x = -1$  divide the real line in three disjoint intervals i.e.,

$(-\infty, -1), (-1, 1)$  and  $(1, \infty)$

In interval  $(-1, 1)$ , it is observed that:

$$-1 < x < 1$$

$$\Rightarrow x^2 < 1$$

$$\Rightarrow 1 < \frac{1}{x^2}, x \neq 0$$

$$\Rightarrow 1 - \frac{1}{x^2} < 0, x \neq 0$$

$$\therefore f'(x) = 1 - \frac{1}{x^2} < 0 \text{ on } (-1, 1) \sim \{0\}$$

$\therefore f$  is strictly decreasing on  $(-1, 1) \sim \{0\}$

In intervals  $(-\infty, -1)$  and  $(1, \infty)$ , it is observed that:

$$x < -1 \text{ or } 1 < x$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow 1 > \frac{1}{x^2}$$

$$\Rightarrow 1 - \frac{1}{x^2} > 0$$

$$\therefore f'(x) = 1 - \frac{1}{x^2} > 0 \text{ on } (-\infty, -1) \text{ and } (1, \infty)$$

$\therefore f$  is strictly increasing on  $(-\infty, -1)$  and  $(1, \infty)$ .

Hence, function  $f$  is strictly increasing in interval I disjoint from  $(-1, 1)$ .

Hence, the given result is proved.

16. Prove that the function  $f$  given by  $f(x) = \log \sin x$  is strictly increasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly decreasing on  $\left(\frac{\pi}{2}, \pi\right)$ .

Sol. We have,

$$f(x) = \log \sin x$$

$$\therefore f'(x) = \frac{1}{\sin x} \cos x = \cot x$$

In interval  $\left(0, \frac{\pi}{2}\right)$ ,  $f'(x) = \cot x > 0$ .

$\therefore f$  is strictly increasing in  $\left(0, \frac{\pi}{2}\right)$ .

In interval  $\left(\frac{\pi}{2}, \pi\right)$ ,  $f'(x) = \cot x < 0$ .

$\therefore f$  is strictly decreasing in  $\left(\frac{\pi}{2}, \pi\right)$ .

17. Prove that the function  $f$  given by  $f(x) = \log \cos x$  is strictly decreasing on  $\left(0, \frac{\pi}{2}\right)$  and strictly increasing on  $\left(\frac{\pi}{2}, \pi\right)$ .

**Sol.** We have,

$$f(x) = \log \cos x$$

$$\therefore f'(x) = \frac{1}{\cos x} (-\sin x) = -\tan x$$

In interval  $\left(0, \frac{\pi}{2}\right)$ ,  $\tan x > 0 \Rightarrow -\tan x < 0$

$\therefore f'(x) < 0$  on  $\left(0, \frac{\pi}{2}\right)$

$\therefore f$  is strictly decreasing on  $\left(0, \frac{\pi}{2}\right)$

In interval  $\left(\frac{\pi}{2}, \pi\right)$ ,  $\tan x < 0 \Rightarrow -\tan x > 0$ .

$\therefore f$  is strictly increasing on  $\left(\frac{\pi}{2}, \pi\right)$

18. Prove that the function given by  $f(x) = x^3 - 3x^2 + 3x - 100$  is increasing in  $\mathbb{R}$ .

Sol. We have,

$$f(x) = x^3 - 3x^2 + 3x - 100$$

$$f'(x) = 3x^2 - 6x + 3$$

$$= 3(x^2 - 2x + 1)$$

$$= 3(x - 1)^2$$

For any  $x \in \mathbb{R}$ ,  $(x - 1)^2 > 0$ .

Thus,  $f'(x)$  is always positive in  $\mathbb{R}$ .

Hence, the given function ( $f$ ) is increasing in  $\mathbb{R}$ .

19. The interval in which  $y = x^2e^{-x}$  is increasing is

(A)  $(-\infty, \infty)$       (B)  $(-2, 0)$       (C)  $(2, \infty)$       (D)  $(0, 2)$

Sol. We have,

$$y = x^2e^{-x}$$

$$\therefore \frac{dy}{dx} = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2 - x)$$

$$\text{Now, } \frac{dy}{dx} = 0.$$

$$\Rightarrow x = 0 \text{ and } x = 2$$

The points  $x = 0$  and  $x = 2$  divide the real line into three disjoint intervals i.e.,  $(-\infty, 0)$ ,  $(0, 2)$  and  $(2, \infty)$ .

In intervals  $(-\infty, 0)$  and  $(2, \infty)$ ,  $f'(x) < 0$  as  $e^{-x}$  is always positive.

$\therefore f$  is decreasing on  $(-\infty, 0)$  and  $(2, \infty)$ .

In interval  $(0, 2)$ ,  $f'(x) > 0$

$\therefore f$  is strictly increasing on  $(0, 2)$ .

Hence,  $f$  is strictly increasing in interval  $(0, 2)$ .

The correct answer is D.