



**SpeedLabs**

**MATHS**

**CBSE 12<sup>th</sup>**

**TEEVRA EDUTECH PVT. LTD.**

# Application of Derivatives

## Exercise 6.4

1. Using differentials, find the approximate value of each of the following up to 3 places of decimal.

- (i)  $\sqrt{25.3}$       (ii)  $\sqrt{49.5}$       (iii)  $\sqrt{0.6}$       (iv)  $(0.009)^{\frac{1}{3}}$       (v)  $(0.999)^{\frac{1}{10}}$   
(vi)  $(15)^{\frac{1}{4}}$       (vii)  $(26)^{\frac{1}{3}}$       (viii)  $(255)^{\frac{1}{4}}$       (ix)  $(82)^{\frac{1}{4}}$       (x)  $(401)^{\frac{1}{2}}$   
(xi)  $(0.0037)^{\frac{1}{2}}$       (xii)  $(26.57)^{\frac{1}{3}}$       (xiii)  $(81.5)^{\frac{1}{4}}$       (xiv)  $(3.968)^{\frac{3}{2}}$       (xv)  $(32.15)^{\frac{1}{5}}$

Sol. (i)  $\sqrt{25.3}$

Consider  $y = \sqrt{x}$ . Let  $x = 25$  and  $\Delta x = 0.3$ . Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$$

$$\Rightarrow \sqrt{25.3} = \Delta y + 5$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (0.3) \quad \left[ \text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2\sqrt{25}} (0.3) = 0.03$$

Hence, the approximate value of  $\sqrt{25.3}$  is  $0.03 + 5 = 5.03$ .

(ii)  $\sqrt{49.5}$

Consider  $y = \sqrt{x}$ . Let  $x = 49$  and  $\Delta x = 0.5$ . Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{49.5} - \sqrt{49} = \sqrt{49.5} - 7$$

$$\Rightarrow \sqrt{49.5} = 7 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (0.5) \quad \left[ \text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2\sqrt{49}} (0.5) = \frac{1}{14} (0.5) = 0.035$$

Hence, the approximate value of  $\sqrt{49.5}$  is  $7 + 0.035 = 7.035$

(iii)  $\sqrt{0.6}$

Consider  $y = \sqrt{x}$ . Let  $x = 1$  and  $\Delta x = -0.4$ .

Then

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{0.6} - 1$$

$$\Rightarrow \sqrt{0.6} = 1 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (\Delta x) \quad \left[ \text{as } y = \sqrt{x} \right]$$

$$= \frac{1}{2}(-0.4) = -0.2$$

Hence, the approximate value of  $\sqrt{0.6}$  is  $1 + (-0.2) = 1 - 0.2 = 0.8$ .

(iv)  $(0.009)^{\frac{1}{3}}$

Consider  $y = x^{\frac{1}{3}}$ . Let  $x = 0.008$  and  $\Delta x = 0.001$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - (0.008)^{\frac{1}{3}} = (0.009)^{\frac{1}{3}} - 0.2$$

$$\Rightarrow (0.009)^{\frac{1}{3}} = 0.2 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{3(x)^{\frac{2}{3}}} (\Delta x) \quad \left[ \text{as } y = x^{\frac{1}{3}} \right]$$

$$\frac{1}{3 \times 0.04} (0.001) = \frac{0.001}{0.12} = 0.008$$

Hence, the approximate value of  $(0.009)^{\frac{1}{3}}$  is  $0.2 + 0.008 = 0.208$ .

(v)  $(0.999)^{\frac{1}{10}}$

Consider  $y = x^{\frac{1}{10}}$ . Let  $x = 1$  and  $\Delta x = -0.001$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{10}} - (x)^{\frac{1}{10}} = (0.999)^{\frac{1}{10}} - 1$$

$$\Rightarrow (0.999)^{\frac{1}{10}} = 1 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{10(x)^9} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{10}} \right]$$

$$\frac{1}{10}(-0.001) = -0.0001$$

Hence, the approximate value of  $(0.999)^{\frac{1}{10}}$  is  $1 + (-0.0001) = 0.9999$ .

(vi)  $(15)^{\frac{1}{4}}$

Consider  $y = x^{\frac{1}{4}}$ . Let  $x = 16$  and  $\Delta x = -1$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{4}} - x^{\frac{1}{4}} = (15)^{\frac{1}{4}} - (16)^{\frac{1}{4}} = (15)^{\frac{1}{4}} - 2$$

$$\Rightarrow (15)^{\frac{1}{4}} = 2 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{4(x)^{\frac{3}{4}}} (\Delta x) \quad \left[ \text{as } y = x^{\frac{1}{4}} \right]$$

$$= \frac{1}{4(16)^{\frac{3}{4}}} (-1) = \frac{-1}{4 \times 8} = \frac{-1}{32} = -0.03125$$

Hence, the approximate value of  $(15)^{\frac{1}{4}}$  is  $2 + (-0.03125) = 1.96875$ .

(vii)  $(26)^{\frac{1}{3}}$

Consider  $y = (x)^{\frac{1}{3}}$ . Let  $x = 27$  and  $\Delta x = -1$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26)^{\frac{1}{3}} - 3$$

$$\Rightarrow (26)^{\frac{1}{3}} = 3 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{3(x)^{\frac{2}{3}}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{3}} \right]$$

$$\frac{1}{3(27)^{\frac{2}{3}}}(-1) = \frac{-1}{27} = -0.0370$$

Hence, the approximate value of  $(26)^{\frac{1}{3}}$  is  $3 + (-0.0370) = 2.9629$ .

(viii)  $(255)^{\frac{1}{4}}$

Consider  $y = (x)^{\frac{1}{4}}$ . Let  $x = 256$  and  $\Delta x = -1$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}} = (255)^{\frac{1}{4}} - (256)^{\frac{1}{4}} = (255)^{\frac{1}{4}} - 4$$

$$\Rightarrow (255)^{\frac{1}{4}} = 4 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{4(x)^{\frac{3}{4}}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{4}} \right]$$

$$= \frac{1}{4(256)^{\frac{3}{4}}} (-1) = \frac{-1}{4 \times 4^3} = -0.0039$$

Hence, the approximate value of  $(255)^{\frac{1}{4}}$  is  $4 + (-0.0039) = 3.9961$

(ix)  $(82)^{\frac{1}{4}}$

Consider  $y = (x)^{\frac{1}{4}}$ . Let  $x = 81$  and  $\Delta x = 1$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (82)^{\frac{1}{4}} - 3$$

$$\Rightarrow (82)^{\frac{1}{4}} = \Delta y + 3$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{4(x)^{\frac{3}{4}}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{4}} \right]$$

$$= \frac{1}{4(81)^{\frac{3}{4}}} (1) = \frac{1}{4(3)^3} = \frac{1}{108} = 0.009$$

Hence, the approximate value of  $(82)^{\frac{1}{4}}$  is  $3 + 0.009 = 3.009$ .

$$(x) (401)^{\frac{1}{2}}$$

Consider  $y = (x)^{\frac{1}{2}}$ . Let  $x = 400$  and  $\Delta x = 1$ . Then,

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{401} - \sqrt{400} = \sqrt{401} - 20$$

$$\Rightarrow \sqrt{401} = 20 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (\Delta x) \quad \left[ \text{as } y = x^{\frac{1}{2}} \right]$$

$$= \frac{1}{2 \times 20} (1) = \frac{1}{40} = 0.025$$

Hence, the approximate value of  $\sqrt{401}$  is  $20 + 0.025 = 20.025$ .

$$(xi) (0.0037)^{\frac{1}{2}}$$

Consider  $y = (x)^{\frac{1}{2}}$ . Let  $x = 0.0036$  and  $\Delta x = 0.0001$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{2}} - (x)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - (0.0036)^{\frac{1}{2}} = (0.0037)^{\frac{1}{2}} - 0.06$$

$$\Rightarrow (0.0037)^{\frac{1}{2}} = 0.06 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{2\sqrt{x}} (\Delta x) \quad \left[ \text{as } y = x^{\frac{1}{2}} \right]$$

$$= \frac{1}{2 \times 0.06} (0.0001)$$

$$= \frac{0.0001}{0.12} = 0.00083$$

Thus, the approximate value of  $(0.0037)^{\frac{1}{2}}$  is  $0.06 + 0.00083 = 0.06083$ .

$$(xii) (26.57)^{\frac{1}{3}}$$

Consider  $y = (x)^{\frac{1}{3}}$ . Let  $x = 27$  and  $\Delta x = -0.43$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{3}} - (x)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - (27)^{\frac{1}{3}} = (26.57)^{\frac{1}{3}} - 3$$

$$\Rightarrow (26.57)^{\frac{1}{3}} = 3 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{3(x)^{\frac{2}{3}}} (\Delta x) \quad \left[ \text{as } y = x^{\frac{1}{3}} \right]$$

$$= \frac{1}{3(9)} (-0.43) = \frac{-0.43}{27} = -0.015$$

Hence, the approximate value of  $(26.57)^{\frac{1}{3}}$  is  $3 + (-0.015) = 2.984$ .

(xiii)  $(81.5)^{\frac{1}{4}}$

Consider  $y = (x)^{\frac{1}{4}}$ . Let  $x = 81$  and  $\Delta x = 0.5$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{4}} - (x)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - (81)^{\frac{1}{4}} = (81.5)^{\frac{1}{4}} - 3$$

$$\Rightarrow (81.5)^{\frac{1}{4}} = 3 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{4(x)^{\frac{3}{4}}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{4}} \right]$$

$$= \frac{1}{4(3)^3} (0.5) = \frac{0.5}{108} = 0.0046$$

Hence, the approximate value of  $(81.5)^{\frac{1}{4}}$  is  $3 + 0.0046 = 3.0046$ .

(xiv)  $(3.968)^{\frac{3}{2}}$

Consider  $y = x^{\frac{3}{2}}$ . Let  $x = 4$  and  $\Delta x = -0.032$ .

Then,

$$\Delta y = (x + \Delta x)^{\frac{3}{2}} - x^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - (4)^{\frac{3}{2}} = (3.968)^{\frac{3}{2}} - 8$$

$$\Rightarrow (3.968)^{\frac{3}{2}} = 8 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{3}{2} (x)^{\frac{1}{2}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{3}{2}} \right]$$

$$\frac{3}{2} (2) (-0.032)$$

$$= -0.096$$

Hence, the approximate value of  $(3.968)^{\frac{3}{2}}$  is  $8 + (-0.096) = 7.904$ .

$$(xv) (32.15)^{\frac{1}{5}}$$

Consider  $y = x^{\frac{1}{5}}$ . Let  $x = 32$  and  $\Delta x = 0.15$ . Then,

$$\Delta y = (x + \Delta x)^{\frac{1}{5}} - x^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - (32)^{\frac{1}{5}} = (32.15)^{\frac{1}{5}} - 2$$

$$\Rightarrow (32.15)^{\frac{1}{5}} = 2 + \Delta y$$

Now,  $dy$  is approximately equal to  $\Delta y$  and is given by,

$$dy = \left( \frac{dy}{dx} \right) \Delta x = \frac{1}{5(x)^{\frac{4}{5}}} (\Delta x) \quad \left[ \text{as } y = (x)^{\frac{1}{5}} \right]$$

$$= \frac{1}{5 \times (2)^4} (0.15) = \frac{0.15}{80} = 0.001875$$

Hence, the approximate value of  $(32.15)^{\frac{1}{5}}$  is  $2 + 0.001875 = 2.001875$ .

2. Find the approximate value of  $f(2.01)$ , where  $f(x) = 4x^2 + 5x + 2$

Sol. Let  $x = 2$  and  $\Delta x = 0.01$ . Then, we have:

$$f(2.01) = f(x + \Delta x) = 4(x + \Delta x)^2 + 5(x + \Delta x) + 2$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\Rightarrow f(2.01) \approx (4x^2 + 5x + 2) + (8x + 5)\Delta x$$

$$= [4(2)^2 + 5(2) + 2] + [8(2) + 5](0.01) \quad [\text{as } x = 2, \Delta x = 0.01]$$

$$= (16 + 10 + 2) + (16 + 5)(0.01)$$

$$= 28 + 0.21 = 28.21$$

Hence, the approximate value of  $f(2.01)$  is 28.21.

3. Find the approximate value of  $f(5.001)$ , where  $f(x) = x^3 - 7x^2 + 15$ .

Sol. Let  $x = 5$  and  $\Delta x = 0.001$ . Then, we have:



$$f(5.001) = f(x + \Delta x) = (x + \Delta x)^3 - 7(x + \Delta x)^2 + 15$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\therefore f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \cdot \Delta x \quad (\text{as } dx = \Delta x)$$

$$\Rightarrow f(5.001) \approx (x^3 - 7x^2 + 15) + (3x^2 - 14x) \Delta x$$

$$= [(5)^3 - 7(5)^2 + 15] + [3(5)^2 - 14(5)](0.001) \quad [x = 5, \Delta x = 0.001]$$

$$= (125 - 175 + 15) + (75 - 70)(0.001)$$

$$= -35 + (5)(0.001)$$

$$= -35 + 0.005$$

$$= -34.995$$

Hence, the approximate value of  $f(5.001)$  is  $-34.995$ .

4. Find the approximate change in the volume  $V$  of a cube of side  $x$  metres caused by increasing side by 1%.

Sol. The volume of a cube ( $V$ ) of side  $x$  is given by  $V = x^3$ .

$$\therefore dV = \left(\frac{dV}{dx}\right) \Delta x$$

$$= (3x^2) \Delta x$$

$$= (3x^2)(0.01x) \quad [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$

$$= 0.03x^3$$

Hence, the approximate change in the volume of the cube is  $0.03x^3 \text{ m}^3$ .

5. Find the approximate change in the surface area of a cube of side  $x$  metres caused by decreasing the side by 1%

Sol. The surface area of a cube ( $S$ ) of side  $x$  is given by  $S = 6x^2$ .

$$\therefore \frac{dS}{dx} = \left(\frac{dS}{dx}\right) \Delta x$$

$$= (12x) \Delta x$$

$$= (12x)(0.01x) [\text{as } 1\% \text{ of } x \text{ is } 0.01x]$$

$$= 0.12x^2$$

Hence, the approximate change in the surface area of the cube is  $0.12x^2 \text{ m}^2$ .

6. If the radius of a sphere is measured as 7 m with an error of 0.02m, then find the approximate error in calculating its volume.

Sol. Let  $r$  be the radius of the sphere and  $\Delta r$  be the error in measuring the radius.

Then,

$$r = 7 \text{ m and } \Delta r = 0.02 \text{ m}$$

Now, the volume  $V$  of the sphere is given by,

$$V = \frac{4}{3} \pi r^3$$

$$\therefore \frac{dV}{dr} = 4\pi r^2$$

$$\therefore dV = \left(\frac{dV}{dr}\right) \Delta r$$

$$= (4\pi r^2) \Delta r$$

$$= 4\pi(7)^2 (0.02) \text{m}^3 = 3.92\pi \text{m}^3$$

Hence, the approximate error in calculating the volume is  $3.92\pi \text{m}^3$ .

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating in surface area.

Sol. Let  $r$  be the radius of the sphere and  $\Delta r$  be the error in measuring the radius.

Then,

$$r = 9 \text{ m and } \Delta r = 0.03 \text{ m}$$

Now, the surface area of the sphere ( $S$ ) is given by,

$$S = 4\pi r^2$$

$$\therefore \frac{dS}{dr} = 8\pi r$$

$$\therefore dS = \left(\frac{dS}{dr}\right) \Delta r$$

$$= (8\pi r) \Delta r$$

$$= 8\pi (9) (0.03) \text{m}^2$$

$$= 2.16\pi \text{m}^2$$

Hence, the approximate error in calculating the surface area is  $2.16\pi \text{m}^2$ .

8. If  $f(x) = 3x^2 + 15x + 5$ , then the approximate value of  $f(3.02)$  is  
 (A) 47.66                      (B) 57.66                      (C) 67.66                      (D) 77.66

Sol. Let  $x = 3$  and  $\Delta x = 0.02$ . Then, we have:

$$F(3.02) = f(x + \Delta x) = 3(x + \Delta x)^2 + 15(x + \Delta x) + 5$$

$$\text{Now, } \Delta y = f(x + \Delta x) - f(x)$$

$$\Rightarrow f(x + \Delta x) = f(x) + \Delta y$$

$$\approx f(x) + f'(x) \Delta x \quad (\text{As } dx = \Delta x)$$

$$\Rightarrow f(3.02) \approx (3x^2 + 15x + 5) + (6x + 15)\Delta x$$

$$= [3(3)^2 + 15(3) + 5] + [6(3) + 15](0.02) \quad [\text{As } x = 3, \Delta x = 0.02]$$

$$= (27 + 45 + 5) + (18 + 15)(0.02)$$

$$= 77 + (33)(0.02)$$

$$= 77 + 0.66$$

$$= 77.66$$

Hence, the approximate value of  $f(3.02)$  is 77.66.

9. The approximate change in the volume of a cube of side  $x$  metres caused by increasing the side by 3% is

- (A)  $0.06 x^3 \text{ m}^3$       (B)  $0.6 x^3 \text{ m}^3$       (C)  $0.09 x^3 \text{ m}^3$       (D)  $0.9 x^3 \text{ m}^3$

Sol. The volume of a cube ( $V$ ) of side  $x$  is given by  $V = x^3$ .

$$\begin{aligned}\therefore dV &= \left(\frac{dV}{dx}\right)\Delta x \\ &= (3x^2)\Delta x \\ &= (3x^2)(0.03x) \quad [\text{As } 3\% \text{ of } x \text{ is } 0.03x] \\ &= 0.09x^3 \text{ m}^3\end{aligned}$$

Hence, the approximate change in the volume of the cube is  $0.09x^3 \text{ m}^3$ .