SpeedLabs

MATHS

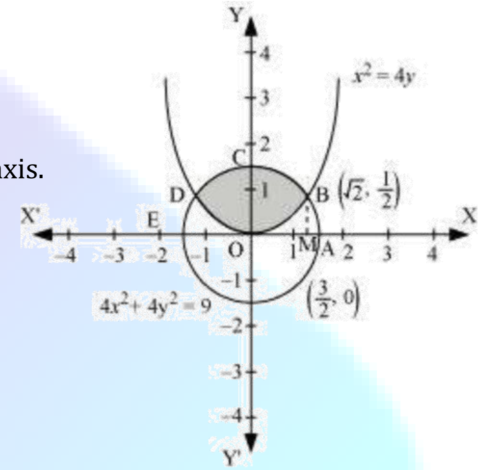
CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. Find the area of the circle 4x2 + 4y2 = 9 which is interior to the parabola x2 = 4y

Ans. The required area is represented by the shaded area OBCDO.

Solving the given equation of circle, 4x2 + 4y2 = 9, and parabola, x2 = 4y, we obtain



point of intersection as $B\left(\sqrt{2},\frac{1}{2}\right)$ and $D\left(-\sqrt{2},\frac{1}{2}\right)$.

It can be observed that the required area is symmetrical about y-axis.

∴ Area OBCDO = 2 × Area OBCO

We draw BM perpendicular to OA.

Therefore, the coordinates of M are $\left(\sqrt{2},0\right)$.

Therefore, Area OBCO = Area OMBCO – Area OMBO

$$=\int_0^{\sqrt{2}}\sqrt{\frac{\left(9-4x^2\right)}{4}}dx-\int_0^{\sqrt{2}}\sqrt{\frac{x^2}{4}}dx$$

$$=\frac{1}{2}\int_0^{\sqrt{2}}\sqrt{9-4x^2}dx-\frac{1}{4}\int_0^{\sqrt{2}}x^2dx$$

$$=\frac{1}{4}\left[x\sqrt{9-4x^2}+\frac{9}{2}\sin^{-1}\frac{2x}{3}\right]_0^{\sqrt{2}}$$

$$=\frac{1}{4}\left[\sqrt{2}\sqrt{9-8}+\frac{9}{2}\sin^{-1}\frac{2\sqrt{2}}{3}\right]-\frac{1}{12}\left(\sqrt{2}\right)^3$$

$$=\frac{\sqrt{2}}{4}+\frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}-\frac{\sqrt{2}}{6}$$

$$=\frac{\sqrt{2}}{12}+\frac{9}{8}\sin^{-1}\frac{2\sqrt{2}}{3}$$

$$=\frac{1}{2}\left(\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right)$$

Therefore, the required area OBCDO is

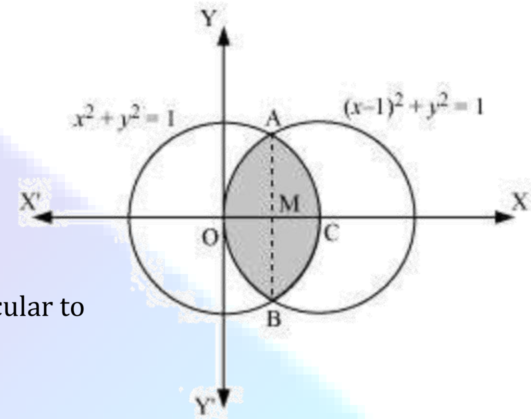
$$\left(2\times\frac{1}{2}\left[\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\right)=\left[\frac{\sqrt{2}}{6}+\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}\right]\text{ units .}$$

2. Find the area bounded by curves (x – 1)2 + y2 = 1 and x2 + y 2 = 1

Ans. The area bounded by the curves, (x – 1)2 + y2 = 1 and x2 + y 2 = 1, is represented the shaded area as

On solving the equations, (x – 1)2 + y2 = 1 and x2 + y 2 = 1, we obtain the point of

intersection as A $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and B $\left(\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$.



It can be observed that the required area is symmetrical about x-axis.

∴ Area OBCAO = 2 × Area OCAO

We join AB, which intersects OC at M, such that AM is perpendicular to OC.

The coordinates of M are

$$\Rightarrow \text{Area OCAO} = \text{Area} + \text{MCAM}$$

$$=\left[\int_0^{\frac{1}{2}}\sqrt{1-(X-1)^2}\,dx+\int_{\frac{1}{2}}^{1}\sqrt{1-x^2}\,dx\right]$$

$$=\left[\frac{x-1}{2}\sqrt{1-(x-1)^2}+\frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}}+\left[\frac{x}{2}\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$=\left[-\frac{1}{4}\sqrt{1-\left(-\frac{1}{2}\right)^2}+\frac{1}{2}\sin^{-1}\left(\frac{1}{2}-1\right)-\frac{1}{2}\sin^{-1}(-1)\right]+\left[\frac{1}{2}\sin^{-1}(1)-\frac{1}{4}\sqrt{1-\left(\frac{1}{2}\right)^2}-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{8}+\frac{1}{2}\left(-\frac{\pi}{6}\right)-\frac{1}{2}\left(\frac{\pi}{2}\right)\right]+\left[\frac{1}{2}\left(\frac{\pi}{2}\right)-\frac{\sqrt{3}}{8}-\frac{1}{2}\left(\frac{\pi}{6}\right)\right]$$

$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{12}+\frac{\pi}{4}+\frac{\pi}{4}-\frac{\pi}{12}\right]$$

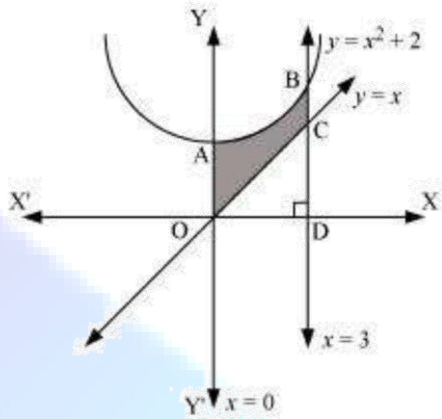
$$=\left[-\frac{\sqrt{3}}{4}-\frac{\pi}{6}+\frac{\pi}{2}\right]$$

$$=\left[\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right]$$

Therefore, required area OBCAO = $2\times\left(\frac{2\pi}{6}-\frac{\sqrt{3}}{4}\right)=\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$units

3. Find the area of the region bounded by the curves y = x2 + 2, y = x, x = 0 and x = 3

Ans. The area bounded by the curves, y = x2 + 2, y = x, x = 0, and x = 3, is represented by the shaded area OCBAO as



Then, Area OCBAO = Area ODBAO – Area ODCO

$$=\int_0^3\left(x^2+2\right)dx-\int_0^3 x\,dx$$

$$=\left[\frac{x^3}{3}+2x\right]_0^3-\left[\frac{x^2}{2}\right]_0^3$$

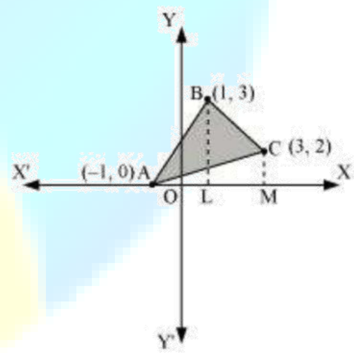
$$=\left[9+6\right]-\left[\frac{9}{2}\right]$$

$$=15-\frac{9}{2}$$

$$=\frac{21}{2}\text{ units}$$

4. Using integration finds the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

Ans. BL and CM are drawn perpendicular to x-axis.



It can be observed in the following figure that,

Area (0ACB) = Area (ALBA) + Area (BLMCB) – Area (AMCA) ... (1)

Equation of line segment AB is

$$y-0=\frac{3-0}{1+1}\left(x+1\right)$$

$$y=\frac{3}{2}\left(x+1\right)$$

$$\therefore \text{Area}\left(\text{ALBA}\right)=\int_{-1}^{1}\frac{3}{2}\left(x+1\right)dx=\frac{3}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{1}=\frac{3}{2}\left[\frac{1}{2}+1-\frac{1}{2}+1\right]=3\text{ units}$$

Equation of line segment BC is

$$y-0=\frac{2-0}{3+1}\left(x+1\right)$$

$$y=\frac{1}{2}\left(x+1\right)$$

$$\therefore \text{Area}(\text{AMCA}) = \frac{1}{2}\int_{-1}^{3}(x+1)dx = \frac{1}{2}\left[\frac{x^2}{2}+x\right]_{-1}^{3} = \frac{1}{2}\left[\frac{9}{2}+3-\frac{1}{2}+1\right] = 4 \text{ units}$$

Therefore, from equation (1), we obtain

Area (0ABC) = (3 + 5 – 4) = 4 units

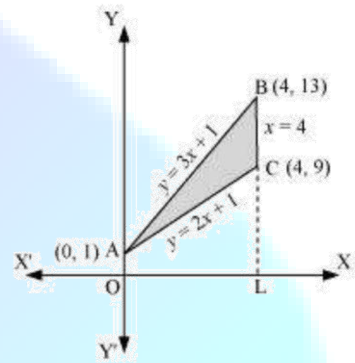
5. Using integration find the area of the triangular region whose sides have the equations y = 2x +1, y = 3x + 1 and x = 4.

Ans. The equations of sides of the triangle are y = 2x +1, y = 3x + 1, and x = 4.

On solving these equations, we obtain the vertices of triangle as A(0, 1), B(4, 13), and C (4, 9).

It can be observed that,

Area (0ACB) = Area (OLBAO) –Area (OLCAO)



$= \int_0^4 (3x+1)dx - \int_0^4 (2x+1)dx$

$= \left[\frac{3x^2}{2}+x\right]_0^4 - \left[\frac{2x^2}{2}+x\right]_0^4$

$= (24+4)-(16+4)$

$= 28-20$

$= 8$ units

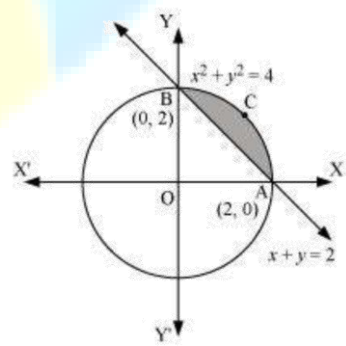
6. Smaller area enclosed by the circle x2 + y2 = 4 and the line x + y = 2 is

(A) $2(\pi-2)$ (B) $\pi-2$ (C) $2\pi-1$ (C) $2(\pi+2)$

Ans. The smaller area enclosed by the circle, x2 + y2 = 4, and the line, x + y = 2, is represented by the shaded area ACBA as

It can be observed that,

Area ACBA = Area OACBO – Area (0OAB)



$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)dx$

$= \left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x-\frac{x^2}{2}\right]_0^2$

$= \left[2\cdot\frac{\pi}{2}\right]-[4-2]$

$= (\pi-2)$ units

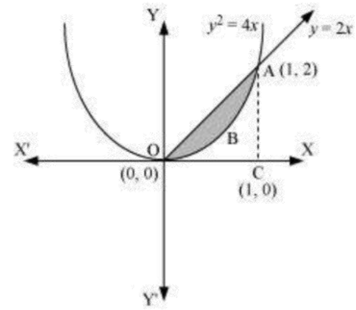
Thus, the correct answer is B.

7. Area lying between the curve y2 = 4x and y = 2x is

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{3}{4}$

Ans. The area lying between the curve, y2 = 4x and y = 2x, is represented by the shaded area OBAO as

The points of intersection of these curves are O (0, 0) and A (1, 2).



We draw AC perpendicular to x-axis such that the coordinates of C are (1, 0).

∴ Area OBAO = Area (0OCA) – Area (OCABO)

$$= \int_0^1 2x\,dx - \int_0^1 2\sqrt{x}\,dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1 - 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$$

$$= \left|1 - \frac{4}{3}\right|$$

$$= \left|-\frac{1}{3}\right|$$

$$= \frac{1}{3} \text{ units}$$

Thus, the correct answer is B.