SpeedLabs

MATHS

# CBSE 10th

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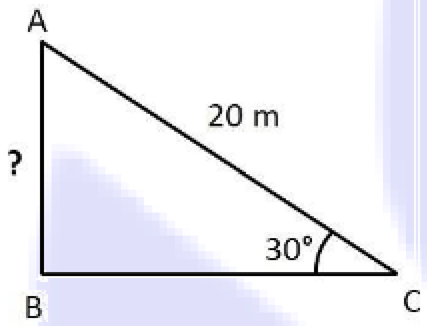
**Q.1** A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.




**Sol:** It can be observed from the figure that AB is the pole. In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

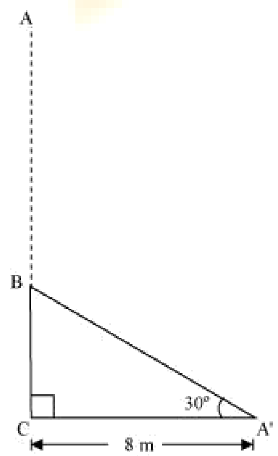
$$\frac{AB}{20} = \frac{1}{2}$$

$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

**Q.2** A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. find the height of the tree.

**Sol:**



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part A'B is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30°$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right) \text{m}$$

$$\frac{A'C}{A'B} = \cos 30°$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right) \text{m}$$

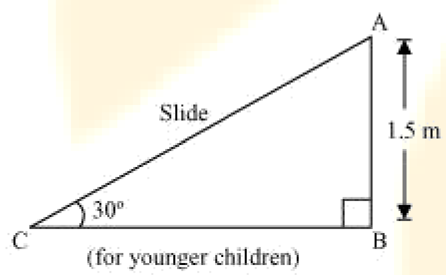
Height of tree $= A'B + BC$

$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) \text{m} = \frac{24}{\sqrt{3}} \text{m} = 8\sqrt{3}$$

Hence, the height of the tree is $8\sqrt{3}$m.

**Q.3** A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, whereas for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

**Sol:** It can be observed that AC and PR are the slides for younger and elder children respectively.

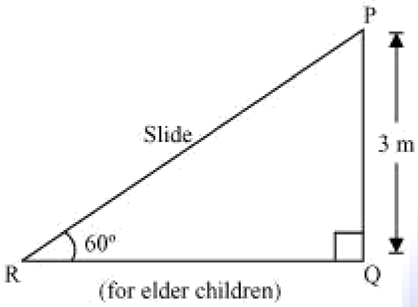



In $\Delta ABC$,

$$\frac{AB}{AC} = \sin 30°$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$




In $\Delta PQR$,

$$\frac{PQ}{PR} = \sin 60°$$

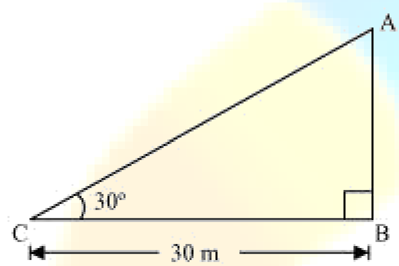
$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$m.

**Q.4** The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

**Sol:**




Let AB be the tower and the angle of elevation from point C (on ground) is 30°.

In $\Delta ABC$,

$$\frac{AB}{BC} = \tan 30°$$

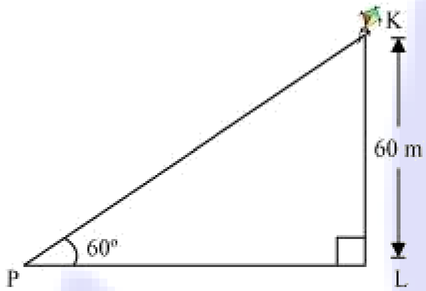
$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}\text{m}$$

Therefore, the height of the tower is $10\sqrt{3}$m.

**Q.5** A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

**Sol:**




Let K be the kite and the string is tied to point P on the ground. In ΔKLP,

$$\frac{KL}{KP} = \sin 60°$$

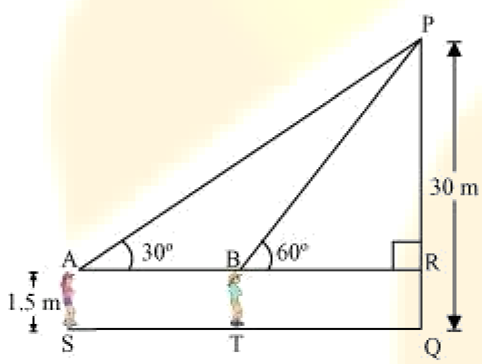
$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3}\text{m}$$

Hence, the length of the string is $40\sqrt{3}$m .

**Q.6** A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

**Sol:**




Let the boy was standing at point S initially. He walked towards the building and reached at point T.

It can be observed that

PR = PQ – RQ

$$= (30 - 1.5)\text{m} = 28.5 \text{ m} = \frac{572}{2}\text{m}$$

In ΔPAR,

$\frac{PR}{AR} = \tan 30°$

$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$

$AR = \left(\frac{57}{2}\sqrt{3}\right) m$

In ΔPRB,

$\frac{PR}{BR} = \tan 60°$

$\frac{57}{2BR} = \sqrt{3}$

$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) m$

$ST = AB$

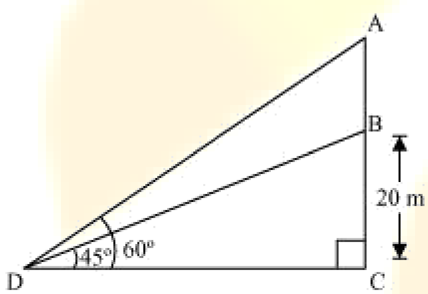
$AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) m$

$= \left(\frac{38\sqrt{3}}{2}\right) m = 19\sqrt{3} m$

Hence, he walked $19\sqrt{3}$m towards the building.

**Q.7** From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

**Sol:**




Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$\frac{BC}{CD} = \tan 45°$

$\frac{20}{CD} = 1$

$CD = 20 \text{ m}$

In ΔACD,

$$\frac{AC}{DC} = \tan 60°$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3}$$

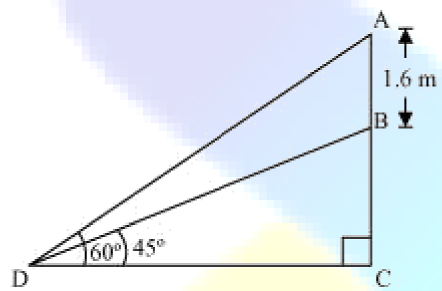
$$AB = \left(20\sqrt{3} - 20\right)\text{m}$$

$$= 20\left(\sqrt{3} - 1\right)\text{m}$$

Therefore, the height of the transmission tower is $20\left(\sqrt{3} - 1\right)$m.

**Q.8** A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

**Sol:**




Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$

$$\frac{BC}{CD} = 1$$

In ΔACD,

$$\frac{AB + BC}{CD} = \tan 60°$$

$$\frac{AB + BC}{BC} = \sqrt{3}$$

$$1.6 + BC = BC\sqrt{3}$$

$$BC\left(\sqrt{3} - 1\right) = 1.6$$

$$BC = \frac{(1.6)\left(\sqrt{3} + 1\right)}{\left(\sqrt{3} - 1\right)\left(\sqrt{3} + 1\right)}$$

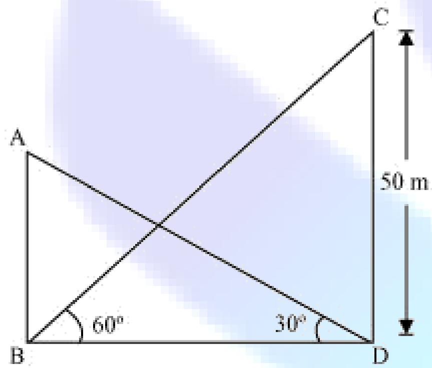
$$= \frac{1.6(\sqrt{3}+1)}{(\sqrt{3})^2 - (1)^2}$$

$$= \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3}+1)$$

Therefore, the height of the pedestal is $0.8(\sqrt{3}+1)$ m.

**Q.9** The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

**Sol:**




Let AB be the building and CD be the tower.

In ΔCDB,

$$\frac{CD}{BD} = \tan 60°$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔABD,

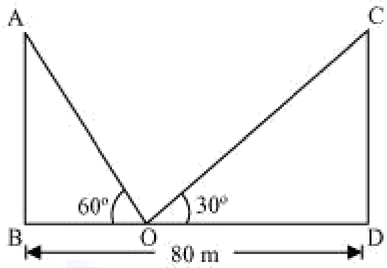
$$\frac{AB}{BD} = \tan 30°$$

$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

**Q.10** Two poles of equal heights are standing opposite each other an either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

**Sol:**




Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60°$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30°$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights, CD = AB

$$CD\left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = 80$$

$$CD\left(\frac{3+1}{\sqrt{3}}\right) = 80$$

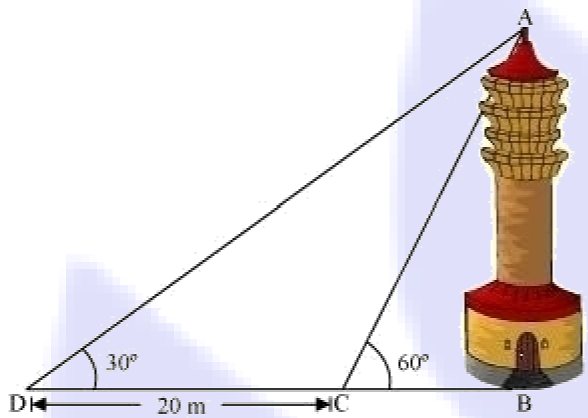
$$CD = 20\sqrt{3}\text{ m}$$

$$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \left(\frac{20\sqrt{3}}{\sqrt{3}}\right)\text{m} = 20\text{m}$$

$$DO = BD - BO = (80 - 20)\text{ m} = 60\text{ m}$$

Therefore, the height of poles is and the point is $20\sqrt{3}$ m and 60 m far from these poles.

**Q.11** A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.




**Sol:** In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

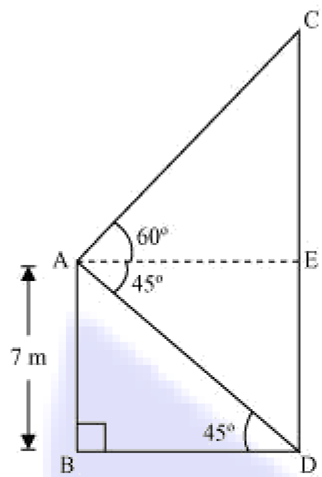
$$AB = 10\sqrt{3} \text{ m}$$

$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right) \text{m}$$

Therefore, the height of the tower is m and the width of the canal is $10\sqrt{3}$ m.

**Q.12** From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

**Sol:**




Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{7}{BD} = 1$$

$BD = 7$ m

In ΔACE,

$AC = BD = 7$ m

$$\frac{CE}{AE} = \tan 60°$$

$$\frac{CE}{7} = \sqrt{3}$$

$CE = 7\sqrt{3}$m

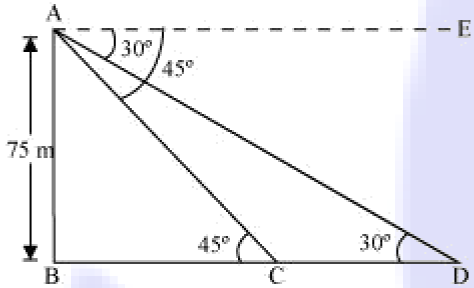
$CD = CE + ED = \left(7\sqrt{3} + 7\right)$m $=$

$7\left(\sqrt{3} + 1\right)$m

Therefore, the height of the cable tower is $7\left(\sqrt{3} + 1\right)$m.

**Q.13** As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

**Sol:**




Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$\frac{AB}{BC} = \tan 45°$

$\frac{75}{BC} = 1$

$BC = 75 \text{ m}$

In ΔABD,

$\frac{AB}{BD} = \tan 30°$

$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$

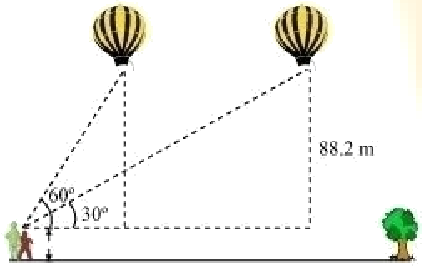
$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$

$75\sqrt{3} = 75 + CD$

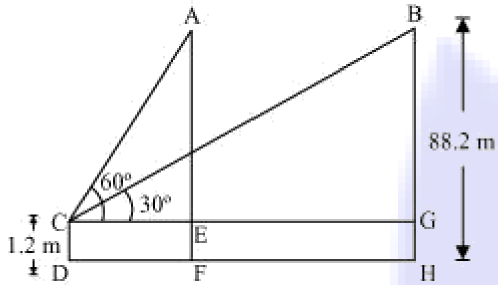
$75(\sqrt{3} - 1)\text{m} = CD$

Therefore, the distance between the two ships is $75(\sqrt{3} - 1)$m.

**Q.14** A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



**Sol:**




Let the initial position A of balloon change to B after some time and CD be the girl.

In $\Delta ACE$,

$\frac{AE}{CE} = \tan 60°$

$\frac{AF - EF}{CE} = \tan 60°$

$\frac{88.2 - 1.2}{CE} = \sqrt{3}$

$\frac{87}{CE} = \sqrt{3}$

$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3}\text{ m}$

In $\Delta BCG$,

$\frac{BG}{CG} = \tan 30°$

$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$

$87\sqrt{3}\text{ m} = CG$

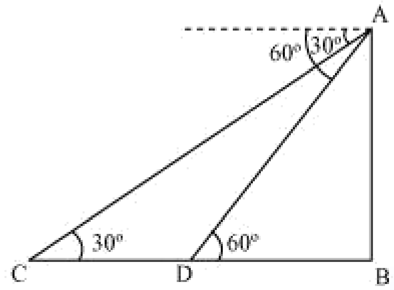
Distance travelled by balloon $= EG = CG - CE$

$(87\sqrt{3} - 29\sqrt{3})\text{m}$

$= 58\sqrt{3}\text{m}$

**Q.15** A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

**Sol:**




Let AB be the tower. Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60°$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30°$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$= \frac{2AB}{\sqrt{3}}$$

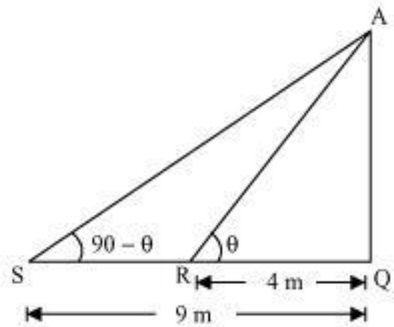
Time taken by the car to travel distance DC $\left(\text{i.e.}, \frac{2AB}{\sqrt{3}}\right) =$ 6 seconds

Time taken by the car to travel distance DB $\left(\text{i.e.}, \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$$= \frac{6}{2} = 3 \text{ seconds.}$$

**Q.16** The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

**Sol:**




Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.

The angles are complementary. Therefore, if one angle is θ, the other will be $90 - \theta$.

In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \ldots\ldots\ldots (i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90 - \theta)$$

$$\frac{AQ}{9} = \cot\theta \ldots\ldots\ldots (ii)$$

On multiplying equations (i) and (ii), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan\theta)(\cot\theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.

Therefore, the height of the tower is 6 m.