SpeedLabs

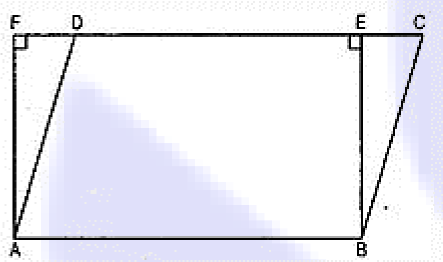
MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

Q.1 Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Ans. Given: Parallelogram ABCD and rectangle ABEF are on same base AB and between the same parallels AB and CF.



∴ ar (∥ gm ABCD) = ar (rect. ABEF)

To prove: AB + BC + CD + AD > AB + BE + EF + AF

Proof: AB = CD [∴ opposites sides of a parallelogram are always equal]

AB = EF [∴ opposites sides of a rectangle are always equal]

∴ CD = EF

Adding AB both sides,

AB + CD = AB + EF ……….(i)

∴ Off all the segments that can be drawn to a given line from a point not lying on it, the perpendicular Segment is the shortest.

∴ BE < BC and AF < AD

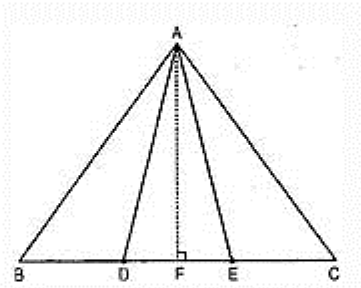
⟹ BC > BE and AD > AF

BC + AD > BE + AF ……….(ii)

From eq. (i) and (ii),

AB + CD + BC + AD = AB + EF + BE + AF

Q.2 In figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC). Can you know answer the question that you have left in the 'introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?



Ans. In Δ ABC, points D and E divides BC in three equal parts such that BD = DE = EC.

$\therefore$ BD = DE = EC = $\frac{1}{3}$ BC

Draw AF $\perp$ BC

ar (Δ ABC) = $\frac{1}{2} \times BC \times AF$ .... (i)

and ar (Δ ABD) = $\frac{1}{2} \times BC \times AF$ .... (ii)

$= \frac{1}{2} \times \frac{BC}{3} \times AF = \frac{1}{3} \times [\frac{1}{2} \times BC \times AF]$

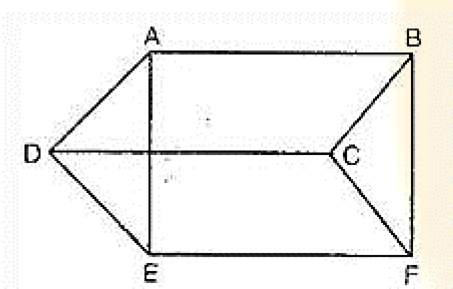
$= \frac{1}{3}$ ar (Δ ABC) .... (iii)

And ar (Δ AEC) = $\frac{1}{3}$ ar (Δ ABC) ....(iv)

From (ii), (iii) and (iv),

ar (ΔABD) = ar (Δ ADE) = ar (Δ AEC)

Q.3 In figure, ABCD, DCFE and ABFE are parallelograms. Show that ar (ADE) = ar (BCF).



Ans. As we know that opposite sides of a parallelogram are always equal.

$\therefore$ In parallelogram ABFE, AE = BF and AB = EF

In parallelogram DCFE, DE = CF and DC = EF

In parallelogram ABCD, AD = BC and AB = DC

Now in Δ ADE and Δ BCF,

AE = BF [Opposite sides of parallelogram ABFE]

DE = CF [Opposite sides of parallelogram DCFE]

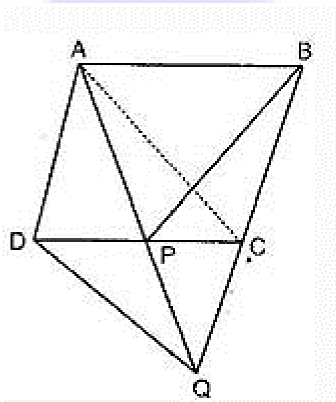
And AD = BC [Opposite sides of parallelogram ABCD]

∴ Δ ADE ≅ Δ BCF [By SSS congruency]

∴ ar (ADE) = ar (BCF)

[∵ Area of two congruent figures is always equal]

Q.4 In figure, ABCD is a parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersects DC at P, show that ar (BPC) = ar (DPQ).



Ans. Join A and C.

Δ APC and Δ BPC are on the same base PC and between the same parallels PC and AB.

∴ ar (Δ APC) = ar (Δ BPC) ………. (i)

Now ACBD is a parallelogram.

AD = BC [opposite sides of a parallelogram are always equal]

Also, BC = CQ [given]

∴ AD = CQ

Now AD ⊥ CQ [Since CQ is the extension of BC]

And AD = CQ

∴ ADQC is a parallelogram.

[∵ If one pair of opposite sides of a quadrilateral is equal and parallel then it is a parallelogram]

Since diagonals of a parallelogram bisect each other.

∴ AP = PQ and CP = DP

Now in Δ APC and Δ DPQ,

AP = PQ [Proved above]

$\angle$ APC = $\angle$ DPQ [Vertically opposite angles]

PC = PD [Prove above]

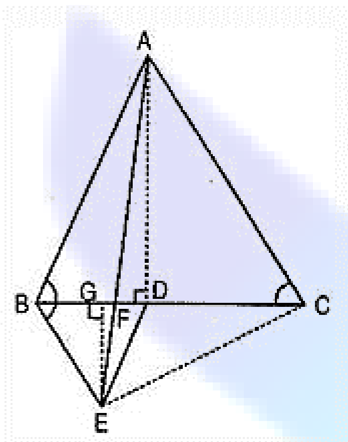
$\therefore$ Δ APC $\cong$ Δ DPQ ……….(ii)

$\Longrightarrow$ ar (APC) = ar (DPQ) [area of congruent figures is always equal]

From eq. (i) and (ii),

ar (Δ BPC) = ar (Δ DPQ)

Q.5 In figure, ABC and BDF are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that:



(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

(ii) ar (BDE) = $\frac{1}{4}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)

Ans. Join EC and AD.

Since Δ ABC is an equilateral triangle.

$\therefore$ $\angle A = \angle B = \angle C = 60°$

Also, Δ BDE is an equilateral triangle.

$\therefore$ $\angle B = \angle D = \angle E = 60°$

If we take two lines, AC and BE and BC as a transversal.

Then $\angle B = \angle C = 60°$ [Alternate angles]

$\Longrightarrow$ BE ∥ AC

Similarly, for lines AB and DE and BF as transversal.

Then ∠ B = ∠ C =60° [Alternate angles]

$\Rightarrow$ BE ∥ AC

(i) Area of equilateral triangle BDE = $\frac{\sqrt{3}}{4}$ (BD)2 ………. (i)

Area of equilateral triangle ABC = $\frac{\sqrt{3}}{4}$ (BC) ………. (ii)

Dividing eq. (i) by (ii),

$$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(BC)^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4}(BD)^2}{\frac{\sqrt{3}}{4}(2BD)^2} \; [\because BD = DC]$$

$$\frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{(BD)^2}{2BD^2} \Rightarrow \frac{ar(\Delta BDE)}{ar(\Delta ABC)} = \frac{1}{4}$$

$\Rightarrow$ ar (Δ BDE) = $\frac{1}{4}$ ar (Δ ABC)

(ii) In Δ BEC, ED is the median.

∴ ar (Δ BEC) = ar (Δ BAE) ………. (i)

[Median divides the triangle in two triangles having equal area]

Now BE ∥ AC

And BEC and BAE are on the same base BE and between the same parallels BE and AC.

∴ ar (BEC) = ar (BAE) ………. (ii)

Using eq. (i) and (ii), we get

Ar (Δ BDE) = $\frac{1}{2}$ ar (Δ BAE)

(iii) We have ar (Δ BDE) = ar (Δ ABC) [Proved in part (i)] ………. (iii)

ar (ΔBDE) = $\frac{1}{4}$ ar (Δ BAE) [Proved in part (ii)]

ar (Δ BDE) = $\frac{1}{4}$ ar (Δ BEC) [Using eq. (iii)] ………. (iv)

From eq. (iii) and (iv), we het

$\frac{1}{4}$ ar (Δ ABC) = $\frac{1}{4}$ ar (Δ BEC)

$\Rightarrow$ ar (Δ ABC) = 2 ar (Δ BEC)

(iv) Δ BDE and Δ AED are on the same base DE and between same parallels AB and DE.

∴ ar (BDE) = ar (Δ AED)

Subtracting Δ FED from both the sides,

ar (Δ BDE) – ar (Δ FED) = ar (Δ AED) – ar (Δ FED)

⟹ ar (Δ BFE) = ar (Δ AFD) .... (v)

(v) An in equilateral triangle, median drawn is also perpendicular to the side,

∴ AD ⊥ BC

Now ar (Δ AFD) = $\frac{1}{2} \times FD \times AD$ ....(vi)

Draw EG ⊥ BC

∴ ar (Δ FED) = $\frac{1}{2} \times FD \times EG$ .... (vii)

Dividing eq. (vi) by (vii), we get

$$\frac{ar(\Delta AFD)\frac{1}{2}\times FD\times AD}{ar(\Delta FED)\frac{1}{2}\times FD\times EG} \Rightarrow \frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{AD}{EG}$$

$$\frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{\frac{\sqrt{3}}{4}BC}{\frac{\sqrt{3}}{4}BD}$$ [Altitude of equilateral triangle = $\frac{\sqrt{3}}{4}$ side]

$$\frac{ar(\Delta AFD)}{ar(\Delta FED)} = \frac{2BD}{BD}$$ [D is the mid-point of BC]

$$\frac{ar(\Delta AFD)}{ar(\Delta FED)} = 2$$ ⟹ ar (Δ AFD) = 2 ar (Δ FED) ......(viii)

ar (Δ AFD) = 2 ar (Δ FED) ......(viii)

Using the value of eq. (viii) in eq. (v),

Ar (Δ BFE) = 2 ar (Δ FED)

(vi) ar (Δ AFC) = ar (Δ AFD) + ar (Δ ADC) = 2 ar (Δ FED) + $\frac{1}{2}$ ar (Δ ABC) [using (v)

= 2 ar (Δ FED) + $\frac{1}{2}$ [4 × ar (Δ BDE)] [Using result of part (i)]

= 2 ar (Δ FED) + 2 ar (Δ BDE) = 2 ar (Δ FED) + 2 ar (Δ AED)

[Δ BDE and Δ AED are on the same base and between same parallels]

= 2 ar (Δ FED) + 2 [ar (Δ AFD) + ar (Δ FED)]

= 2 ar (Δ FED) + 2 ar (Δ AFD) + 2 ar (Δ FED) [Using (viii)]

$= 4$ ar (Δ FED) $+ 4$ ar (Δ FED)

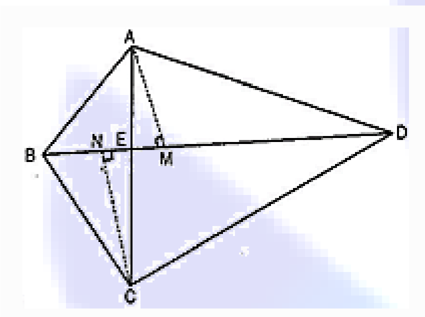
$\Rightarrow$ ar (Δ AFC) $= 8$ ar (Δ FED)

ar (Δ FED) $= \frac{1}{8}$ ar (Δ AFC)

Q.6 Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that:

ar (APB) $\times$ ar (CPD) $=$ ar (APD) $\times$ ar (BPC)

Ans. Given: A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.



To Prove: ar (Δ AED) ar (Δ BEC)

$=$ ar (Δ ABE) $\times$ ar (ΔCDE)

Construction: From A, draw AM $\perp$ BD and from C, draw CN $\perp$ BD.

Proof: ar (Δ ABE) $=$ ....(i)

And ar (Δ AED) $=$ ....(ii)

Dividing eq. (ii) by (i), we get,

$$\frac{ar(\Delta\ AED)}{ar(\Delta\ ABE)} = \frac{\frac{1}{2}\times DE \times AM}{\frac{1}{2}\times BE \times AM} \Rightarrow \frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE} \text{ .... (iii)}$$

Similarly, $\frac{ar(\Delta CDE)}{ar(\Delta BEC)} = \frac{DE}{BE}$ .... (iv)

From eq. (iii) and (iv), we get

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{ar(\Delta CDE)}{ar(\Delta BEC)}$$

$\Rightarrow$ ar (Δ AED) $\times$ ar (ΔBEC) $=$ ar (ΔABE) $\times$ ar (ΔCDE)

Hence proved.