SpeedLabs

MATHS

# CBSE 11$^{th}$

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**1.** Expand the expression (1– 2x)5

**Ans.** By using Binomial Theorem, the expression (1– 2x)5 can be expanded as

$$(1-2x)^5$$
$$= {}^5C_0(1)^5 - {}^5C_1(1)^4(2x) + {}^5C_2(1)^3(2X)^2 - {}^5C_3(1)^2(2x)^4 - {}^5C_5(2x)^5$$
$$= 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - (32x^5)$$
$$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$$

**2.** Expand the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$

**Ans.** By using Binomial Theorem, the expression $\left(\frac{2}{x}-\frac{x}{2}\right)^5$ can be expanded as

$$\left(\frac{2}{x}-\frac{x}{2}\right)^5 = {}^5C_0\left(\frac{2}{x}\right)^2 - {}^5C_1\left(\frac{2}{x}\right)^4\left(\frac{x}{2}\right) + {}^5C_2\left(\frac{2}{x}\right)^3\left(\frac{x}{2}\right)^2$$
$$- {}^5C_3\left(\frac{2}{x}\right)^2\left(\frac{x}{2}\right)^3 + {}^5C_4\left(\frac{2}{x}\right)\left(\frac{x}{2}\right)^4 - {}^5C_5\left(\frac{x}{2}\right)^5$$
$$= \frac{32}{x^5} - 5\left(\frac{16}{x^4}\right)\left(\frac{x}{2}\right) + 10\left(\frac{8}{x^3}\right)\left(\frac{x^2}{4}\right) - 10\left(\frac{4}{x^2}\right)\left(\frac{x^3}{8}\right) + 5\left(\frac{2}{x}\right)\left(\frac{x^4}{16}\right) - \frac{x^5}{32}$$
$$= \frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$$

**3.** Expand the expression $(2x-3)^6$

**Ans.** By using Binomial Theorem, the expression $(2x-3)^6$ can be expanded as

$$(2x-3)^6 = {}^6C_0(2x)^6 - {}^6C_1(2x)^5(3) + {}^6C_2(2x)^4(3)^2 - {}^6C_3(2x)^3(3)^3$$
$$+ {}^6C_4(2x)^2(3)^4 - {}^6C_5(2x)(3)^5 + {}^6C_6(3)^6$$
$$= 64x^6 - 6(32x^5)(3) + 15(16x^4)(9) - 20(8x^3)(27)$$
$$+ 15(4x^2)(81) - 6(2x)(243) + 729$$
$$= 64x^6 - 57x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

**4.** Expand the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$

**Ans.** By using Binomial Theorem, the expression $\left(\frac{x}{3}+\frac{1}{x}\right)^5$ can be expanded as

$$\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_5\left(\frac{1}{x}\right)^5$$

$$+ {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{1}{x}\right)^5$$

$$= \frac{x^5}{243} + 5\left(\frac{x^4}{81}\right)\left(\frac{1}{x}\right) + 10\left(\frac{x^3}{27}\right)\left(\frac{1}{x^2}\right) + 10\left(\frac{x^2}{9}\right)\left(\frac{1}{x^3}\right) + 5\left(\frac{x}{3}\right)\left(\frac{1}{x_4}\right) + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5x^3}{81} + \frac{10x}{27} + \frac{10x}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$$

**5.** Expand $\left(x+\frac{1}{x}\right)^6$

**Ans.** By using Binomial Theorem, the expression $\left(x+\frac{1}{x}\right)^6$ can be expanded as

$$\left(x+\frac{1}{x}\right)^6 = {}^6C_0(x)^6 + {}^6C_1(x)^5\left(\frac{1}{x}\right) + {}^6C_2(x)^4\left(\frac{1}{x}\right)^2$$

$$+ {}^6C_3(x)^5\left(\frac{1}{x}\right)^3 + {}^6C_4(x)^2\left(\frac{1}{x}\right)^4 + {}^6C_5(x)\left(\frac{1}{x}\right)^5 + {}^6C_6\left(\frac{1}{x}\right)^6$$

$$= x^6 + 6(x)^5\left(\frac{1}{x}\right) + 15(x)^4\left(\frac{1}{x^2}\right) + 20(x)^3\left(\frac{1}{x^3}\right) + 15(x)^2\left(\frac{1}{x^4}\right) + 6(x)\left(\frac{1}{x^5}\right) + \frac{1}{x^6}$$

$$= x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$$

**6.** Using Binomial Theorem, evaluate $(96)^3$

**Ans** 96 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, binomial theorem can be applied. It can be written that, 96 = 100 – 4

$$\therefore (96)^3 = (100-4)^3$$

$$= {}^3C_0(100)^3 = C_1(100)^2(4) + {}^3C_2(100)(4)^2$$

$$= (100)^3 - 3(100)^2(4) + 3(100)(4)^2 - (4)^3$$

$$= 1000000 - 120000 \_ 4800 - 64$$

$$= 884736$$

**7.** Using Binomial Theorem, evaluate $(102)^5$

**Ans.** 102 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied. It can be written that, 102 = 100 + 2

$$\therefore (102)^5 = (100-2)^5$$

$$= {}^5C_0(100)^3 + C_1(100)^4(2) + {}^5C_2(100)^3(2)^2 + {}^5C_3(100)^2(2)^3$$

$$+{}^5c_4(100)(2)^4 + {}^5C_5(2)^5$$

$$=(100)^5 + 5(100)^4(2) + 10(100)^3(2)^2 + 10(100)^2 + 5(100)(2)^4 + (2)^5$$

$$=10000000000 - 1000000000 + 4000000 + 800000 + 8000 + 32$$

$$=11040808032$$

**8.** Using Binomial Theorem, evaluate (101)4

**Ans.** 101 can be expressed as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied. It can be written that, 101 = 100 + 1.

$$\therefore (101)^5 = (100-1)^4$$

$$= {}^4C_0(100)^4 + {}^4C_1(100)^3(1) + {}^4C_2(100)^2(2)^2 + {}^5C_3(100)^2(1)^2 + {}^4C_3(100)(1)^3 + {}^4C_4(1)^4$$

$$+(100)^4\, 4(100)^3 + 6(100)^2 + (1)^4$$

$$=100000000 - 4000000 + 60000 + 400 + 1$$

$$=104060401$$

**9.** Using Binomial Theorem, evaluate $(99)^5$

**Ans** 99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied. It can be written that, 99 = 100 – 1

$$\therefore (99)^5 = (100-1)^5 - {}^5C_1(100)^4 + {}^5C_2(100)^3(1)^2 - {}^5C_3(100)^2(1)^3$$

$$+ {}^5C_0(100)(1)^4 - {}^5C_5(1)^5$$

$$=(100)^5 - 5(100)^4 + 10(100)^3(100)^2 + 5(100) - 1$$

$$=10000000000 - 500000000 + 10000000 - 100000 + 500 - 1$$

$$=10010000500 - 500100001$$

$$=9509900499$$

**10.** Using Binomial Theorem, indicate which number is larger $(1.1)^{10000}$ or 1000

**Ans.** 99 can be written as the sum or difference of two numbers whose powers are easier to calculate and then, Binomial Theorem can be applied. It can be written that, 99 = 100 – 1

$$(1.1)^{1000} = (1. + 0.1)^{10000}$$

$$= {}^{10000}C_0 + {}^{10000}C_1(1.1) + \text{Other positive term}$$

$$= 1 + 10000 \times 1.1 + \text{Other positive term}$$

$$= 1 + 11000 + \text{Other positive term}$$

$$> 1000$$

Hence, $(1.1)^{10000} > 1000$

**11.** Find (a + b)⁴ – (a – b)⁴. Hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4$

**Ans.** Using Binomial Theorem, the expressions, (a + b)4 and (a – b)4, can be expanded as

$$(a+b)^4 = {}^4C_0a^4 + {}^4C^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$

$$(a-b)^4 = {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_3a^3b^2 - {}^4C_3ab^3 + {}^4C_4b^4$$

$$\therefore (a+b)^4 - (a-b)^4 = {}^4C_0 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3ab^3 + {}^4C_4b^4$$

$$-\left[{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4\right]$$

$$= 2\left({}^4C_1a^3b + {}^3C_3ab^3\right) = 2\left(4a^3b + 4ab^3\right)$$

$$= 8ab\left(a^2 + b^2\right)$$

By putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$$\left(\sqrt{3}+\sqrt{2}\right) - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\left(\sqrt{3}\right)\left(\sqrt{3}\right)^2\left\{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right\}$$

$$= 8\left(\sqrt{6}\right)\{3+2\} = 40\sqrt{6}$$

**12.** Find (x + 1)6 + (x – 1)6. Hence or otherwise evaluate $\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6$

**Ans.** Using Binomial Theorem, the expressions, (x + 1)⁶ and (x – 1)⁶, can be expanded as

$$(x-1)^6 = {}^6C_0x^6 + {}^6C_1x^5 + {}^6C_2x^4 + {}^6C_3x^3 + {}^6C_4x^2 + {}^6C_5x + {}^6C_6$$

$$(x-1)^6 = {}^6C_0x^6 - {}^6C_1x^5 + {}^6C_2x^4 - {}^6C_3x^3 + {}^6C_4x^2 - {}^6C_5x + {}^6C_6$$

$$\therefore (x+1)^6 + (x-1)^6 = 2\left[{}^6C_0x^6 + {}^6C_2x^4 + {}^6C_4x^2 + {}^6C_6\right]$$

$$= 2\left[x^6 + 15x^4 + 15x^2 + 1\right]$$

$$\left(\sqrt{2}+1\right)^6 + \left(\sqrt{2}-1\right)^6 = 2\left[\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^6 + 15\left(\sqrt{2}\right)^4 + 15\left(\sqrt{2}\right)^2 + 1\right]$$

$$= 2(8 \times 15 \times 4 + 15 \times 2 + 1)$$

$$= 2(8 + 60 + 30 + 1)$$

$$= 2(99) = 198$$

**13.** Show that $9^{n+1} - 8n - 9$ is divisible by 64, whenever n is a positive integer.

**Ans.** In order to show that $9^{n+1} - 8n - 9$ is divisible by 64, it has to be proved that, $9^{n+1} - 8n - 9 = 64k$ , where k is some natural number

By Binomial Theorem,

$$(1+a)^m = {}^mC_0 + {}^mC_1a + {}^mC_2a^2 + ..... + {}^mC_ma^m$$

For a = 8 and m = n + 1, we obtain

$$(1+8)^{n+1} = {}^{n+1}C_0 + {}^{n+1}C_1(8) + {}^{n+1}C_2(8)^2 + ..... + {}^{n+1}C_{n+1}(8)^{n+1}$$

$$\Rightarrow 9^{n+1} = 9 + (n+1) + 8^2\left[{}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + ..... + {}^{n+1}C_{n+1}(8)^{n-1}\right]$$

$$\Rightarrow 9^{n+1} - 8n - 9 = 64k, \text{where } k = {}^{n+1}C_2 + {}^{n+1}C_3 \times 8 + {}^{n+1}C_{n+1}(8)^{n-1} \text{ is a natural number}$$

Thus, $9^{n+1} - 8n - 9$

is divisible by 64, whenever n is a positive integer.

**14.** Prove that $\sum_{r=0}^{n} 3r\,{}^nC_r = 4^n$.

**Ans** By Binomial Theorem,

$$\sum_{r=0}^{n} {}^nC_r a^{n-r} = (a+b)^n$$

By putting $b = 3$ and $a = 1$ in the above equation, we obtain

$$\sum_{r=0}^{n} {}^nC_r (1)^{n-r} (3)^r = (1+3)^n$$

$$\sum_{r=0}^{n} 3r\,{}^nC_r = 4^n$$

Hence, proved.