SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Binomial Theorem
Exercise- 8.2

1. Find the coefficient of x^5 in $(x + 3)^8$

Ans It is known that (r + 1) th term, (Tr+1), in the binomial expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

Assuming that x^5 occurs in the (r + 1) th term of the expansion $(x + 3)^8$, we obtain

$$T_{r+1} = {}^8C_r (x)^{8-r} (3)^r$$

Comparing the indices of x in x^5 and in Tr +1, we obtain r = 3

Thus, the coefficient of x^5 is $T_{r+1} = {}^8C_r (3)^3 = \frac{8!}{3!5!} \times 3^3 = \frac{8.7.6.5!}{3.2.5!}.3^3 = 1512$

2. Find the coefficient of a^5b^7 in $(a - 2b)^{12}$

Ans. It is known that (r + 1) th term, (Tr+1), in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-1} b^r$

Assuming that a^5b^7 occurs in the (r + 1) th term of the expansion $(a - 2b)^{12}$, we obtain

$$T_{r+1} = {}^{12}C_r a^{12-r} (-2b)^r = {}^{12}C_r (-2)^r (a)^{12-r} (b)^r$$

Comparing the indices of a and b in $a^5 b^7$ and in Tr +1, we obtain

r = 7 Thus, the coefficient of a^5b^7 is

$${}^{12}C_7 (-2)^7 = -\frac{12!}{7!5!}.2^7 = -\frac{12.11.10.9.8.7!}{5.4.3.2.7!}.2^7 = -(792)(128) = -101376$$

3. Write the general term in the expansion of $(x^2 - y)^6$

Ans. It is known that the general term T_{r+1} {which is the (r + 1) th term} in the binomial expansion of $(a + b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-1} b^r$. Thus, the general term in the expansion of $(x^2 - y^6)$ is

$$T_{r+1} = {}^6C_r (x^2)^{6-r} (-y)^r = (-1)^r \, {}^6C_r . x^{12-2r} . y^r$$

4. Write the general term in the expansion of $(x^2 - yx)^{12}$, $x \neq 0$

Ans It is known that the general term Tr+1 {which is the (r + 1) th term} in the binomial expansion of (a + b) n is given by $T_{r+1} = {}^nC_r a^{n-1} b^r$.

Thus, the general term in the expansion of $(x^2 - yx)^{12}$ is

$$T_{r+1} = {}^{12}C_r (x^2)^{12-r} (-yx)^r = (-1)^r \, {}^{12}C_1 . x^{4-2r} . y^r . x^r = (-1)^r \, {}^{12}C_1 . x^{4-r} . y^r$$

5. Find the 4^{th} term in the expansion of $(x - 2y)^{12}$.

Ans It is known that (r + 1) th term, (Tr+1), in the binomial expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^{n}C_{1} a^{n-1} b^{r}$$

Thus, the 4th term in the expansion of $(x - 2y)^{12}$ is

$$T_4 = T_{3+1} = {}^{12}C_3 (x)^{12-3} (-2y)^3 = (-1)^3 . \frac{12!}{3!9!} . x^9 . (2)^3 . y^3 = -\frac{12.11.10}{3.2} . (2)^3 x^9 y^3 = -1760 x^9 y^3$$

6. Find the 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)18, x \neq 0$.

Ans It is known that (r + 1) th term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^{n}C_{1} a^{n-1} b^{r}$$

Thus, 13th term in the expansion of $\left(9x - \frac{1}{3\sqrt{x}}\right)^{18}$ is

$$T_{13} = T_{12+1} = {}^{18}C_{12} (9x)^{18-12} \left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= (-1)^{12} \frac{18!}{12!6!} (9)^6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{\sqrt{x}}\right)^{12}$$

$$= \frac{18.17.16.15.14.13.12!}{12!6.5.3.2} . x^6 . \left(\frac{1}{x^6}\right) . 3^{12} \left(\frac{1}{3^{12}}\right)$$

$$= 18564$$

7. Find the middle terms in the expansions of $\left(3 - \frac{x^3}{6}\right)^7$

Ans. It is known that in the expansion of $(a + b)^n$, if n is odd, then there are two middle terms, namely $\left(\frac{n+1}{6}\right)^{th}$, term and $\left(\frac{n+1}{2} + 1\right)^{th}$ term.

Therefore, the middle terms in the expansion $\left(3 - \frac{x^3}{6}\right)^7$ of are $\left(\frac{7+1}{2}\right)^{th} = 4^{th}$ term and $\left(\frac{7+1}{2} + 1\right)^{th} = 5^{th}$ term.

$$T_4 = T_{3+1} = {}^{7}C_3 (3)^{7-3} \left(-\frac{x^3}{6}\right) = (-1)^3 \frac{7!}{3!4!} . 3^4 . \frac{x^9}{6^3}$$

$$= -\frac{7.6.5.4!}{3.2.4!} . 3^4 . \frac{1}{2^3 . 3^3} . x^9 = -\frac{105}{8} x^9$$

$$T_5 = T_{4+1} = {}^{7}C_4 (3)^{7-4} \left(-\frac{x^3}{6}\right)^4 = (-1)^4 \frac{7!}{4!3!} (3)^3 . \frac{x^{12}}{6^4}$$

$$\frac{7.6.5.4!}{4!3.2}.\frac{3^3}{2^4.3^4}.x^{12}=\frac{35}{48}x^{12}$$

Thus, the middle terms in the expansion of $\left(3-\frac{x^3}{6}\right)^7$ are $-\frac{105}{8}x^9$ and $\frac{35}{48}x^{12}$

8. Find the middle terms in the expansions of $\left(\frac{x}{3}+9y\right)^{10}$

Ans. It is known that in the expansion $(a+b)^n$, if n is even, then the middle term $\left(\frac{n}{2}+1\right)^{th}$ is term.

Therefore, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is $\left(\frac{10}{2}+1\right)^{th}=6^{th}$ term is term

$$T_6=T_{5+1}={}^{10}C_5\left(\frac{x}{3}\right)^{10-5}(9y)^5=\frac{10!}{5!5!}.\frac{x^5}{3^5}.9^5.y^5$$

$$=\frac{10.9.8.7.6.5!}{5.4.4.2.5!}.\frac{1}{3^5}.3^{10}.x^5y^5 \qquad \left[9^5=\left(3^2\right)^5=3^0\right]$$

$$=252\times3^5.x^5.y^5=61236x^5y^5$$

Thus, the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$ is 61236 x^5y^5

9. In the expansion of $(1+a)^{m+n}$, prove that coefficients of a^m and a^n are equal

It is known that $(r+1)^{th}$ term, (T_{r+1}), in the binomial expansion of $(a+b)^n$ is given by

$$T_{r+1}={}^nC_ra^{n-r}b^r$$

Assuming that am occurs in the $(r+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$$T_{r+1}={}^{m+n}C_r(1)^{m+n-r}(a)^r={}^{m+n}C_ra^r$$

Comparing the indices of a in a^m and in T_{r+1}, we obtain r = m

Therefore, the coefficient of a^m is

$${}^{m+n}C_m=\frac{(m+n)!}{m!(m+n-m)!}=\frac{(m+n)!}{m!n!}.......(1)$$

Assuming that a^n occurs in the $(k+1)^{th}$ term of the expansion $(1+a)^{m+n}$, we obtain

$$T_{k+1}={}^{m+n}C_k(1)^{m+n-k}(a)^k={}^{m+n}C_k(a)^k$$

Comparing the indices of a in an and in T^{k+1}, we obtain

k = n

Therefore, the coefficient of a^n is

$$^{m+n}C_n = \frac{(m+n)!}{n!(m+n-n)!} = \frac{(m+n)!}{n!m!} \ldots\ldots..(2)$$

Thus, from (1) and (2), it can be observed that the coefficients of a^m and a^n in the expansion of $(1 + a)^{m+n}$ are equal.

10. The coefficients of the $(r - 1)^{th}$, r^{th} and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are in the ratio 1:3:5. Find n and r.

Ans. It is known that $(k + 1)^{th}$ term, (T_{k+1}), in the binomial expansion of $(a + b)^n$ is given by

$$T_{k+1} = {}^nC_k a^{n-k} b^k$$

Therefore, $(r - 1)^{th}$ term in the expansion of $(x + 1)^n$ is

$$T_{r-1} = {}^nC_{r-2}(x)^{n-(r-2)}(1)^{(r-2)} = {}^nC_{1-2}x^{n-r+2}$$

r th term in the expansion of $(x + 1)^n$ is $T_r = {}^nC_{r-1}(x)^{n-(r-1)}(1)^{(r-2)} = {}^nC_{r-1}x^{n-r+1}$

Therefore, the coefficients of the $(r - 1)^{th}$, r^{th}, and $(r + 1)^{th}$ terms in the expansion of $(x + 1)^n$ are ${}^nC_{r-2}$, ${}^nC_{r-1}$, and ${}^nC_r$ respectively. Since these coefficients are in the ratio 1:3:5, we obtain.

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{1}{3} \text{ and } \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{3}{5}$$

$$\frac{{}^nC_{r-2}}{{}^nC_{r-1}} = \frac{n!}{(r-2)!(n-r+2)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{(r-1)(r-2)!(n-r+1)!}{(r-2)!(n-r+2)(n-r+1)!} = \frac{r-1}{n-r+2}$$

$$\therefore \frac{r-1}{n-r+2} = \frac{1}{3}$$

$$\Rightarrow 3r - 3 = n - r + 2$$

$$\Rightarrow n - 4r + 5 = 0$$

$$\frac{{}^nC_{r-1}}{{}^nC_r} = \frac{n!}{(r-1)!(n-r+1)} \times \frac{r!(n-r)!}{n!} = \frac{r(r-1)!(n-r)!}{(r-1)!(n-r+1)(n-r)!}$$

$$= \frac{r}{n-r+1}$$

$$\therefore \frac{r}{n-r+1} = \frac{3}{5}$$

$$\Rightarrow 5r = 3n - 3r + 3$$

$$\Rightarrow 3n - 8r + 3 = 0 \ldots\ldots(2)$$

Multiplying (1) by 3 and subtracting it from (2), we obtain

4r – 12 = 0

⇒ r = 3

Putting the value of r in (1), we obtain

$n - 12 + 5 = 0$

$\Rightarrow n = 7$

Thus, $n = 7$ and $r = 3$

11. Prove that the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of xn in the expansion of $(1 +x)^{2n-1}$.

It is known that (r + 1) th term, (Tr+1), in the binomial expansion of $(a + b)^n$ is given by

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

Assuming that x^n occurs in the (r + 1) th term of the expansion of $(1 + x)^{2n}$, we obtain

$$T_{r+1} = {}^{2n}C_r (1)^{2n-r} (x)^r = {}^{2n}C_r (x)^r$$

Comparing the indices of x in x^n and in T_{r+1}, we obtain

$r = n$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is

$${}^{2n}C_n = \frac{(2n)!}{n!(2n-n)!} = \frac{(2n)!}{n!n!} = \frac{(2n)!}{(n!)^2} \ldots\ldots(1)$$

Assuming that x^n occurs in the (k +1) th term of the expansion $(1 + x)^{2n-1}$, we obtain.

$$T_{k+1} = {}^{2n-1}C_k (1)^{2n-1-k} (X)^k = {}^{2n-1}C_k (X)^k$$

Comparing the indices of x in x^n and T_{k+1}, we obtain

$k = n$

Therefore, the coefficient of xn in the expansion of $(1 + x)^{2n-1}$ is

$${}^{2n-1}C_n = \frac{(2n-1)!}{n!(2n-1-n)!} = \frac{(2n-1)!}{n!(n-1)!}$$

$$= \frac{2n.(2n-1)!}{2n.n!(n-1)!} = \frac{(2n)!}{2.n!n!} = \frac{1}{2}\left[\frac{(2n)!}{(n!)^2}\right] \ldots\ldots(2)$$

From (1) and (2), it is observed that

$$\frac{1}{2}\left({}^{2n-1}C_n\right) = {}^{2n-1}C_n$$

$$\Rightarrow {}^{2n}C_n = 2\left({}^{2n-1}C_n\right)$$

Therefore, the coefficient of x^n in the expansion of $(1 + x)^{2n}$ is twice the coefficient of x^n in the expansion of $(1 + x)2^{n-1}$.

Hence, proved.

12. Find a positive value of m for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

Ans. It is known that (r + 1) th term, (T_{r+1}), in the binomial expansion of $(a + b)^n$ is given by

$T_{r+1} = {}^{n}C_{r}a^{n-r}b^{r}$

Assuming that x^2 occurs in the $(r + 1)^{th}$ term of the expansion $(1 + x)^m$, we obtain

$T_{r+1} = {}^{m}C_{r}(1)^{m-r}(X)^{r} = {}^{m}C_{r}(X)^{r}$

Comparing the indices of x in x^2 and in T_{r+1}, we obtain

r = 2

Therefore, the coefficient of x^2 is ${}^{m}C_{r2}$.

It is given that the coefficient of x^2 in the expansion $(1 + x)^m$ is 6.

$\therefore {}^{m}C_{2} = 6$

$\Rightarrow \frac{m!}{2!(m-2)!} = 6$

$\Rightarrow \frac{m(m-1)(m-2)!}{2\times(m-2)!} = 6$

$\Rightarrow m(m-1) = 12$

$\Rightarrow m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

$\Rightarrow m(m-4) = 0$ or $(m+3) = 0$

$\Rightarrow m = 4$ or $m = -3$

Thus, the positive value of m, for which the coefficient of x^2 in the expansion $(1 + x)^m$ is 6, is 4