SpeedLabs

MATHS

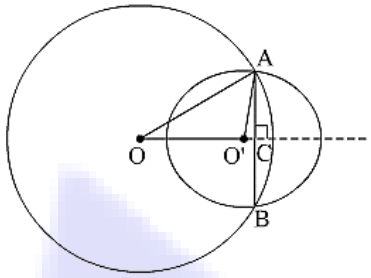
CBSE 9th

TEEVRA EDUTECH PVT. LTD.

CIRCLE

Exercise- 10.4

Q.1 Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm.Find the length of the common chord.



Ans- Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively.

$OA = OB = 5$ cm

$O'A = O'B = 3$ cm

OO' will be the perpendicular bisector of chord AB.

$\therefore AC = CB$

It is given that, $OO' = 4$ cm

Let OC be x. Therefore, O'C will be $x - 4$

In ΔOAC,

$OA^2 = AC^2 + OC^2$

$\Rightarrow 5^2 = AC^2 + x^2$

$\Rightarrow 25 - x^2 = AC^2 \ldots (1)$

In $\Delta O'AC$,

$O'A^2 = AC^2 + O'C^2$

$\Rightarrow 3^2 = AC^2 + (x - 4)^2$

$\Rightarrow 9 = AC^2 + x^2 + 16 - 8x$

$\Rightarrow AC^2 = -x^2 - 7 + 8x \ldots (2)$

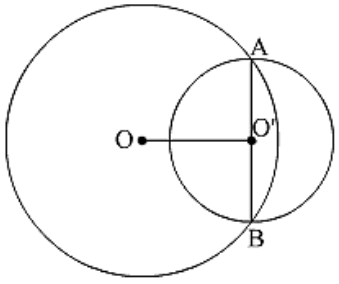
From equations (1) and (2), we obtain

$25 - x^2 = -x^2 - 7 + 8x$

$8x = 32$

$x = 4$

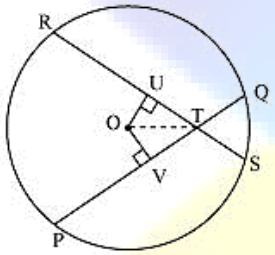
Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle.



Length of the common chord AB = 2 O'A = (2 × 3) cm = 6 cm

Q.2 If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Ans- Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T.



Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT,

OV = OU (Equal chords of a circle are equidistant from the centre)

∠OVT = ∠OUT (Each 90°)

OT = OT (Common)

∴ ΔOVT ≅ ΔOUT (RHS congruence rule)

∴ VT = UT (By CPCT) ... (1)

It is given that,

PQ = RS ... (2)

$\Rightarrow \frac{1}{2}PQ = \frac{1}{2}$

⇒ PV = RU ... (3)

On adding equations (1) and (3), we obtain

$PV + VT = RU + UT$

$\Rightarrow PT = RT$ ... (4)

On subtracting equation (4) from equation (2), we obtain

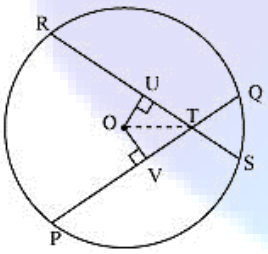
$PQ - PT = RS - RT$

$\Rightarrow QT = ST$ ... (5)

Equations (4) and (5) indicate that the corresponding segments of chords PQ and RS are congruent to each other.

Q.3 If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Ans-



Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T.

Draw perpendiculars OV and OU on these chords.

In ΔOVT and ΔOUT,

$OV = OU$ (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90°)

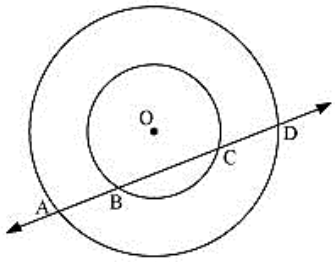
$OT = OT$ (Common)

$\therefore \Delta OVT \cong \Delta OUT$ (RHS congruence rule)

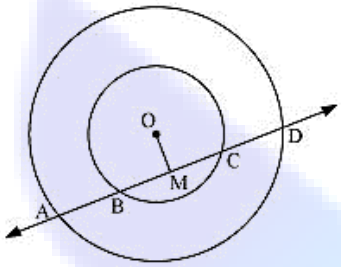
$\therefore \angle OTV = \angle OTU$ (By CPCT)

Therefore, it is proved that the line joining the point of intersection to the centre makes equal angles with the chords.

Q.4 If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD (see figure 10.25).



Ans. Let us draw a perpendicular OM on line AD.



It can be observed that BC is the chord of the smaller circle and AD is the chord of the bigger circle.

We know that perpendicular drawn from the centre of the circle bisects the chord.

$\therefore BM = MC$ ... (1)

And, $AM = MD$ ... (2)

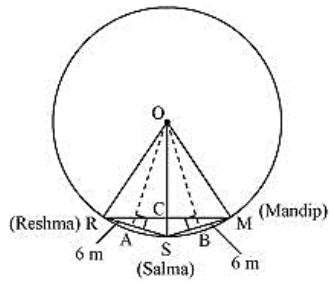
On subtracting equation (2) from (1), we obtain

$AM - BM = MD - MC$

$\Rightarrow AB = CD$

Q.5 Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?

Ans. Draw perpendiculars OA and OB on RS and SM respectively.



$OR = OS = OM = 5$ m. (Radii of the circle)

In ΔOAR,

$OA^2 + AR^2 = OR^2$

$OA^2 + (3\text{ m})^2 = (5\text{ m})^2$

$OA^2 = (25 - 9)\text{ m}^2 = 16\text{ m}^2$

$OA = 4$ m

ORSM will be a kite ($OR = OM$ and $RS = SM$). We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal.

$\therefore \angle RCS$ will be of $90°$ and $RC = CM$

Area of $\Delta ORS = \frac{1}{2} \times OA \times RS$

$\frac{1}{2} \times RC \times OS = \frac{1}{2} \times 4 \times 6$

$RC \times 5 = 24$

$RC = 4.8$

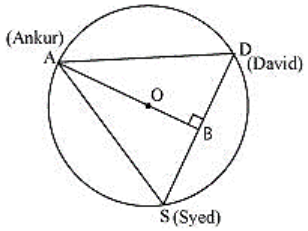
$RM = 2$

$RC = 2(4.8) = 9.6$

Therefore, the distance between Reshma and Mandip is 9.6 m.

Q.6 A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Ans.



It is given that AS = SD = DA

Therefore, ΔASD is an equilateral triangle.

OA (radius) = 20 m

Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ASD.

We also know that medians intersect each other in the ratio 2: 1. As AB is the median of equilateral triangle ASD, we can write

$\Rightarrow \frac{OA}{OB} = \frac{2}{1}$

$\Rightarrow \frac{20\ m}{OB} = \frac{2}{1}$

$\Rightarrow OB = \left(\frac{20}{2}\right) m = 10m$

$\therefore AB = OA + OB = (20 + 10)\ m = 30\ m$

In ΔABD,

$AD^2 = AB^2 + BD^2$

$AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$

$AD^2 = 900 + \frac{1}{4}$

$AD^2 \frac{3}{4} AD^2 = 900$

$AD^2 = 1200$

$AD = 20\sqrt{3}$

Therefore, the length of the string of each phone will be $20\sqrt{3}$ m.