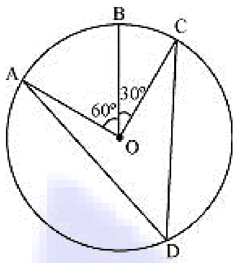
SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

Q.1 In the given figure, A, B and C are three points on a circle with centre O such that ∠BOC = 30° and ∠AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.



Ans. It can be observed that

∠AOC = ∠AOB + ∠BOC

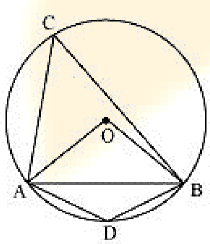
= 60° + 30°

= 90°

We know that angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times 90° = 45°$$

Q.2 A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc.



In ΔOAB,

AB = OA = OB = radius

∴ ΔOAB is an equilateral triangle.

Therefore, each interior angle of this triangle will be of 60°.

∴ ∠AOB = 60°

$$\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60° = 30°$$

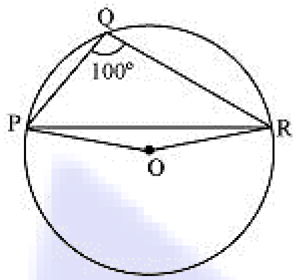
In cyclic quadrilateral ACBD,

$\angle ACB + \angle ADB = 180°$ (Opposite angle in cyclic quadrilateral)

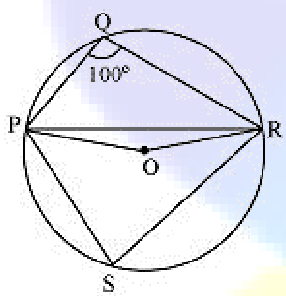
$\Rightarrow \angle ADB = 180° - 30° = 150°$

Therefore, angle subtended by this chord at a point on the major arc and the minor arc are 30° and 150° respectively.

Q.3 In the given figure, $\angle PQR = 100°$, where P, Q and R are points on a circle with centre O. Find $\angle OPR$.



Ans.



Consider PR as a chord of the circle.

Take any point S on the major arc of the circle.

PQRS is a cyclic quadrilateral.

$\angle PQR + \angle PSR = 180°$ (Opposite angles of a cyclic quadrilateral)

$\Rightarrow \angle PSR = 180° - 100° = 80°$

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \angle POR = 2\angle PSR = 2\ (80°) = 160°$

In ΔPOR,

$OP = OR$ (Radii of the same circle)

$\therefore \angle OPR = \angle ORP$ (Angles opposite to equal sides of a triangle)

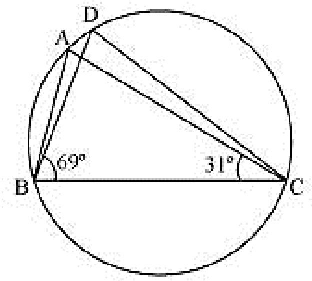
$\angle OPR + \angle ORP + \angle POR = 180°$ (Angle sum property of a triangle)

$2\ \angle OPR + 160° = 180°$

$2\ \angle OPR = 180° - 160° = 20°$

$\angle OPR = 10°$

Q.4 In the given figure, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.



Ans. In ΔABC,

∠BAC + ∠ABC + ∠ACB = 180° (Angle sum property of a triangle)

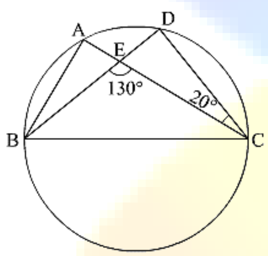
⇒ ∠BAC + 69° + 31° = 180°

⇒ ∠BAC = 180° − 100º

⇒ ∠BAC = 80°

∠BDC = ∠BAC = 80° (Angles in the same segment of a circle are equal)

Q.5 In the given figure, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠BEC = 130° and ∠ECD = 20°. Find ∠BAC.



Ans. In ΔCDE,

∠CDE + ∠DCE = ∠CEB (Exterior angle)

⇒ ∠CDE + 20° = 130°

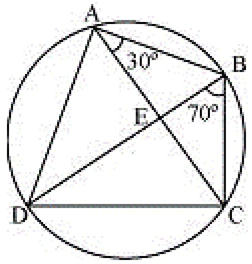
⇒ ∠CDE = 110°

However, ∠BAC = ∠CDE (Angles in the same segment of a circle)

⇒ ∠BAC = 110°

Q.6 ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Ans.



For chord CD,

∠CBD = ∠CAD (Angles in the same segment)

∠CAD = 70°

∠BAD = ∠BAC + ∠CAD = 30° + 70° = 100°

∠BCD + ∠BAD = 180° (Opposite angles of a cyclic quadrilateral)

∠BCD + 100° = 180°

∠BCD = 80°

In ΔABC,

AB = BC (Given)

∴ ∠BCA = ∠CAB (Angles opposite to equal sides of a triangle)

⇒ ∠BCA = 30°

We have, ∠BCD = 80°

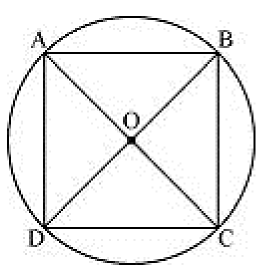
⇒ ∠BCA + ∠ACD = 80°

30° + ∠ACD = 80°

⇒ ∠ACD = 50°

⇒ ∠ECD = 50°

Q.7 If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.



Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O.

(Consider BD as a chord)

$\angle BCD + \angle BAD = 180°$ (Cyclic quadrilateral)

$\angle BCD = 180° - 90° = 90°$

$\angle ADC = \frac{1}{2}\angle AOC = \frac{1}{2} \times (180)° = 90°$ (Considering AC as a chord)

$\angle ADC + \angle ABC = 180°$ (Cyclic quadrilateral)

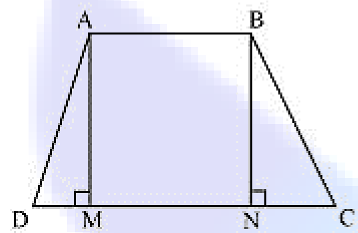
$90° + \angle ABC = 180°$

$\angle ABC = 90°$

Each interior angle of a cyclic quadrilateral is of 90°. Hence, it is a rectangle.

Q.8 If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

Ans.



Consider a trapezium ABCD with AB | |CD and BC = AD.

Draw $AM \perp CD$ and $BN \perp CD$.

In ΔAMD and ΔBNC,

AD = BC (Given)

$\angle AMD = \angle BNC$ (By construction, each is 90°)

AM = BN (Perpendicular distance between two parallel lines is same)

$\therefore \Delta AMD \cong \Delta BNC$ (RHS congruence rule)

$\therefore \angle ADC = \angle BCD$ (CPCT) ... (1)

$\angle BAD$ and $\angle ADC$ are on the same side of transversal AD.

$\angle BAD + \angle ADC = 180°$ ... (2)

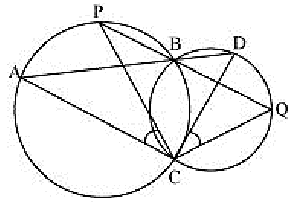
$\angle BAD + \angle BCD = 180°$ [Using equation (1)]

This equation shows that the opposite angles are supplementary.

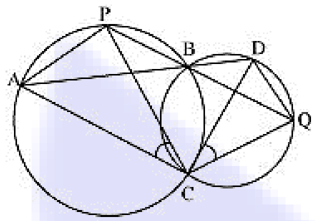
Therefore, ABCD is a cyclic quadrilateral.

Q.9 Two circles intersect at two points B and C. Through B, two-line segments ABD and PBQ are drawn to intersect the circles at A, D and P, Q respectively (see the given figure). Prove that ∠ACP = ∠QCD.

Ans.



Q.10



Ans. Join chords AP and DQ.

For chord AP,

∠PBA = ∠ACP (Angles in the same segment) ... (1)

For chord DQ,

∠DBQ = ∠QCD (Angles in the same segment) ... (2)

ABD and PBQ are line segments intersecting at B.

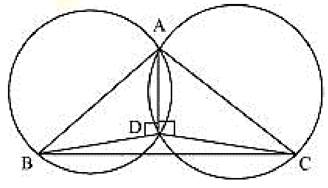
∴ ∠PBA = ∠DBQ (Vertically opposite angles) ... (3)

From equations (1), (2), and (3), we obtain

∠ACP = ∠QCD

Q.11 If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

Ans.



Consider a ΔABC.

Two circles are drawn while taking AB and AC as the diameter.

Let they intersect each other at D and let D not lie on BC.

Join AD.

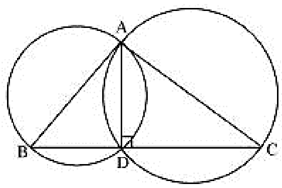
∠ADB = 90° (Angle subtended by semi-circle)

∠ADC = 90° (Angle subtended by semi-circle)

∠BDC = ∠ADB + ∠ADC = 90° + 90° = 180°

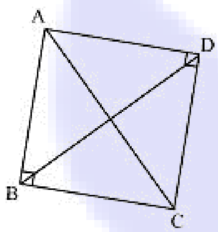
Therefore, BDC is a straight line and hence, our assumption was wrong.

Thus, Point D lies on third side BC of ΔABC.



Q.12 ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD.

Ans.



In ΔABC,

∠ABC + ∠BCA + ∠CAB = 180° (Angle sum property of a triangle)

⇒ 90° + ∠BCA + ∠CAB = 180°

⇒ ∠BCA + ∠CAB = 90° ... (1)

In ΔADC,

∠CDA + ∠ACD + ∠DAC = 180° (Angle sum property of a triangle)

⇒ 90° + ∠ACD + ∠DAC = 180°

⇒ ∠ACD + ∠DAC = 90° ... (2)

Adding equations (1) and (2), we obtain

∠BCA + ∠CAB + ∠ACD + ∠DAC = 180°

⇒ (∠BCA + ∠ACD) + (∠CAB + ∠DAC) = 180°

∠BCD + ∠DAB = 180° ... (3)

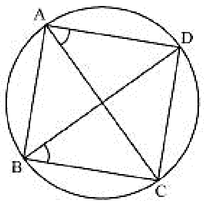
However, it is given that

∠B + ∠D = 90° + 90° = 180° ... (4)

From equations (3) and (4), it can be observed that the sum of the measures of opposite angles of quadrilateral ABCD is 180°. Therefore, it is a cyclic quadrilateral.

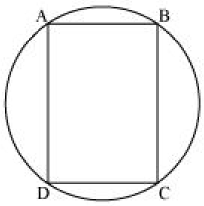
Consider chord CD.

∠CAD = ∠CBD (Angles in the same segment)



Q.13 Prove that a cyclic parallelogram is a rectangle.

Ans.



Let ABCD be a cyclic parallelogram.

$\angle A + \angle C = 180°$ (Opposite angles of a cyclic quadrilateral) ... (1)

We know that opposite angles of a parallelogram are equal.

$\therefore \angle A = \angle C$ and $\angle B = \angle D$

From equation (1),

$\angle A + \angle C = 180°$

$\Rightarrow \angle A + \angle A = 180°$

$\Rightarrow 2\ \angle A = 180° \Rightarrow \angle A = 90°$

Parallelogram ABCD has one of its interior angles as 90°. Therefore, it is a rectangle.