SpeedLabs

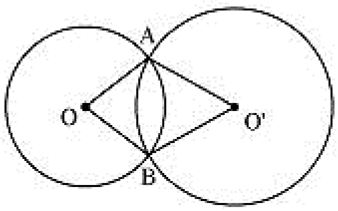
MATHS

CBSE 9th

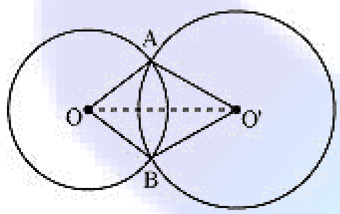
TEEVRA EDUTECH PVT. LTD.

Q.1 Prove that line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Ans.



Let two circles having their centres as O and intersect each other at point A and B respectively. Let us join O.



In ΔAO O and BO O',

OA = OB (Radius of circle 1)

O' A = O' B (Radius of circle 2)

O O' = O O' (Common)

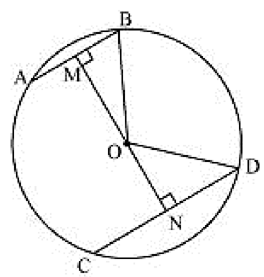
ΔAO O' ≅ ΔBO O' (By SSS congruence rule)

∠OA O'= ∠OB O' (By CPCT)

Therefore, line of centres of two intersecting circles subtends equal angles at the two points of intersection.

Q.2 Two chords AB and CD of lengths 5 cm 11cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6 cm, find the radius of the circle.

Ans. Draw OM ⊥ AB and ON ⊥ CD. Join OB and OD.



$BM = \frac{AB}{2} = \frac{5}{2}$ (Perpendicular from the centre bisects the chord)

$$ND = \frac{CD}{2} = \frac{11}{2}$$

Let ON be x. Therefore, OM will be 6− x.

In ΔMOB,

$$OM^2 + \left(\frac{5}{2}\right)^2 = OB^2$$

$$(6 - x)^2 - 12x + \frac{25}{4} = OB^2$$

$$36 + x^2 - 12x + \frac{25}{4} = OB^2 \text{.................(i)}$$

In ΔNOD,

$$ON^2 + ND^2 = OD^2$$

$$x^2 + \left(\frac{11}{2}\right)^2 = OD^2$$

$$x^2 + \frac{121}{4} = OD^2 \text{ ....(2)}$$

We have OB = OD (Radii of the same circle)

Therefore, from equation (1) and (2),

$$36 + x^2 - 12x + \frac{25}{4} = x^2 + \frac{121}{4}$$

$$12x = 36 + \frac{25}{4} - \frac{121}{4}$$

$$= \frac{144 + 25 - 121}{4} = \frac{48}{4} = 12$$

$$x = 1$$

From equation (2),

$$(1)^2 + \left(\frac{121}{4}\right) = OD^2$$

$$OD^2 = 1 + \frac{121}{4} = \frac{125}{4}$$

$$OD = \frac{5}{2}\sqrt{5}$$

Therefore, the radius of the circle is $\frac{5}{2}\sqrt{5}$ cm.

Q.3 The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at distance 4 cm from the centre, what is the distance of the other chord from the centre?

Ans- Let AB and CD be two parallel chords in a circle centered at O. Join OB and OD.

Distance of smaller chord AB from the centre of the circle = 4 cm

OM = 4 cm

MB = OM = 4 cm

In ΔOMB,

$$OM^2 + MB^2 = OB^2$$

$$(4)^2 + (3)^2 = OB^2$$

$$16 + 9 = OB^2$$

$$OB = \sqrt{25}$$

$$OB = 5cm$$

In ΔOND,

$$OD = OB = 5 \text{ cm} \qquad \text{(Radii of the circle)}$$

$$ND = \frac{CD}{2} = \frac{8}{2} = 4cm$$

$$ON^2 + ND^2 = OD^2$$

$$ON^2 + (4)^2 = (5)^5$$

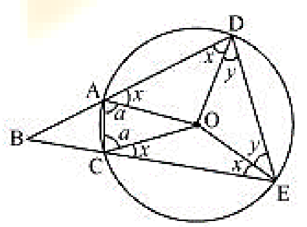
$$ON^2 = 25 - 16 = 9$$

$$ON = 3$$

Therefore, the distance of the bigger chord from the centre is 3 cm.

Q.4 Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is equal to half the difference of the angles subtended by the chord's AC and DE at the centre.

Ans.



In ΔAOD and ΔCOE,

OA = OC (Radii of the same circle)

OD = OE (Radii of the same circle)

AD = CE (Given)

∴ ΔAOD ≅ ΔCOE (SSS congruence rule)

$\angle OAD = \angle OCE$ (By CPCT) ... (1)

$\angle ODA = \angle OEC$ (By CPCT) ... (2)

Also, $\angle OAD = \angle ODA$ (As $OA = OD$) ... (3)

From equations (1), (2), and (3), we obtain

$\angle OAD = \angle OCE = \angle ODA = \angle OEC$

Let $\angle OAD = \angle OCE = \angle ODA = \angle OEC = x$

In Δ OAC,

$OA = OC$

$\therefore \angle OCA = \angle OAC$ (Let a)

In Δ ODE,

$OD = OE$

$\angle OED = \angle ODE$ (Let y)

ADEC is a cyclic quadrilateral.

$\therefore \angle CAD + \angle DEC = 180°$ (Opposite angles are supplementary)

$x + a + x + y = 180°$

$2x + a + y = 180°$

$y = 180º - 2x - a$ ... (4)

However, $\angle DOE = 180º - 2y$

And, $\angle AOC = 180º - 2a$

$\angle DOE - \angle AOC = 2a - 2y = 2a - 2\,(180° - 2x - a)$

$= 4a + 4x - 360°$ ... (5)

$\angle BAC + \angle CAD = 180°$ (Linear pair)

$\Rightarrow \angle BAC = 180° - \angle CAD = 180° - (a + x)$

Similarly, $\angle ACB = 180° - (a + x)$

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$ (Angle sum property of a triangle)

$\angle ABC = 180° - \angle BAC - \angle ACB$

$= 180º - (180° - a - x) - (180º - a - x)$

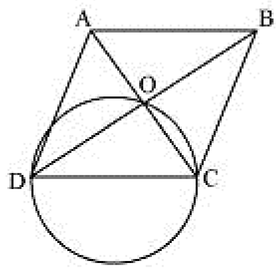
$= 2a + 2x - 180°$

$= \frac{1}{2}\,[4a + 4x - 360°]$

$\angle ABC = \frac{1}{2}\,[\angle DOE - \angle AOC]$ [Using equation (5)]

Q.5 Prove that the circle drawn with any side of a rhombus as diameter passes through the point of intersection of its diagonals.

Ans.



Let ABCD be a rhombus in which diagonals are intersecting at point O and a circle is drawn while taking side CD as its diameter. We know that a diameter subtends 90° on the arc.

∴ ∠COD = 90°

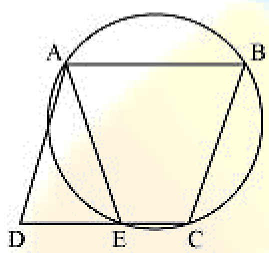
Also, in rhombus, the diagonals intersect each other at 90°.

∠AOB = ∠BOC = ∠COD = ∠DOA = 90°

Clearly, point O has to lie on the circle.

Q.6 ABCD is a parallelogram. The circle through A, B and C intersect CD (produced if necessary) at E. Prove that AE = AD.

Ans.



It can be observed that ABCE is a cyclic quadrilateral and in a cyclic quadrilateral, the sum of the opposite angles is 180°.

∠AEC + ∠CBA = 180°

∠AEC + ∠AED = 180° (Linear pair)

∠AED = ∠CBA ... (1)

For a parallelogram, opposite angles are equal.

∠ADE = ∠CBA ... (2)

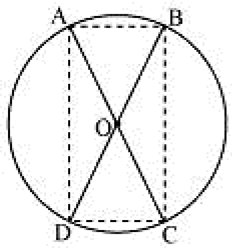
From (1) and (2),

∠AED = ∠ADE

AD = AE (Angles opposite to equal sides of a triangle)

Q.7 AC and BD are chords of a circle which bisect each other. Prove that (i) AC and BD are diameters; (ii) ABCD is a rectangle.

Ans.



Let two chords AB and CD are intersecting each other at point O.

In ΔAOB and ΔCOD,

$OA = OC$ (Given)

$OB = OD$ (Given)

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\Delta AOB \cong \Delta COD$ (SAS congruence rule)

$AB = CD$ (By CPCT)

Similarly, it can be proved that $\Delta AOD \cong \Delta COB$

$\therefore AD = CB$ (By CPCT)

Since in quadrilateral ACBD, opposite sides are equal in length, ACBD is a parallelogram.

We know that opposite angles of a parallelogram are equal.

$\therefore \angle A = \angle C$

However, $\angle A + \angle C = 180°$ (ABCD is a cyclic quadrilateral)

$\Rightarrow \angle A + \angle A = 180°$

$\Rightarrow 2\angle A = 180°$

$\Rightarrow \angle A = 90°$

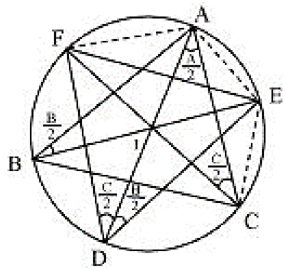
As ACBD is a parallelogram and one of its interior angles is 90°, therefore, it is a rectangle.

$\angle A$ is the angle subtended by chord BD. And as $\angle A = 90°$, therefore, BD should be the diameter of the circle.

Similarly, AC is the diameter of the circle.

Q.8 Bisectors of angles A, B and C of a triangle ABC intersect its circumcircle at D, E and F respectively. Prove that the angles of the triangle DEF are $90° - \frac{1}{2}A$, $90° - \frac{1}{2}B$ and $90° - \frac{1}{2}C$.

Ans.



It is given that BE is the bisector of ∠B.

$\therefore \angle ABE = \frac{\angle B}{2}$

However, ∠ADE = ∠ABE (Angles in the same segment for chord AE)

$\Rightarrow \angle ADE = \frac{\angle B}{2}$

Similarly, $\angle ACF = \angle ADF = \frac{\angle C}{2}$ (Angle in the same segment for chord AF)

$\angle D = \angle ADE + \angle ADF$

$= \frac{\angle B}{2} + \frac{\angle C}{2}$

$= \frac{1}{2}(\angle B + \angle C)$

$= \frac{1}{2}(180° - \angle A)$

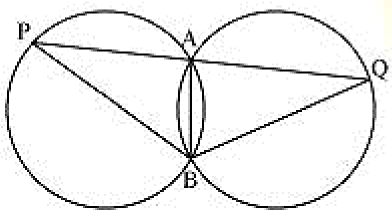
$90° - \frac{1}{2}\angle A$

Similarly, it can be proved that

$\angle E = 90° - \frac{1}{2}\angle B$

$\angle F = 90° - \frac{1}{2}\angle C$

Q.9 Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.



Ans. AB is the common chord in both the congruent circles.

∴ ∠APB = ∠AQB

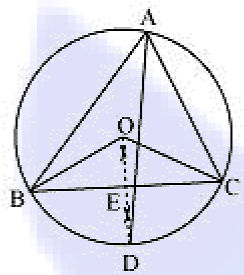
In ΔBPQ,

∠APB = ∠AQB

∴ BQ = BP (Angles opposite to equal sides of a triangle)

Q.10 In any triangle ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.

Ans.



Let perpendicular bisector of side BC and angle bisector of ∠A meet at point D. Let the perpendicular bisector of side BC intersect it at E. Perpendicular bisector of side BC will pass through circumcenter O of the circle. ∠BOC and ∠BAC are the angles subtended by arc BC at the centre and a point A on the remaining part of the circle respectively. We also know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

∠BOC = 2 ∠BAC = 2 ∠A ... (1)

In ΔBOE and ΔCOE,

OE = OE (Common)

OB = OC (Radii of same circle)

∠OEB = ∠OEC (Each 90° as OD ⊥ BC)

∴ ΔBOE ≅ ∠COE (RHS congruence rule)

∠BOE = ∠COE (By CPCT) ... (2)

However, ∠BOE + ∠COE = ∠BOC

⇒ ∠BOE +∠BOE = 2 ∠A [Using equations (1) and (2)]

⇒ 2 ∠BOE = 2 ∠A ⇒ ∠BOE = ∠A

∴ ∠BOE = ∠COE = ∠A

The perpendicular bisector of side BC and angle bisector of ∠A meet at point D.

∴ ∠BOD = ∠BOE = ∠A ... (3)

Since AD is the bisector of angle ∠A,

$\angle BAD = \frac{\angle A}{2} \Rightarrow 2\angle BAD = \angle A$ ... (4)

From equations (3) and (4), we obtain

$\angle BOD = 2\angle BAD$

This can be possible only when point BD will be a chord of the circle. For this, the point D lies on the circum circle. Therefore, the perpendicular bisector of side BC and the angle bisector of ∠A meet on the circum circle of triangle ABC.