SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Q.1 Express the given complex number in the form a + ib: $5(i)\left(-\frac{3}{5}i\right)$

Ans. $5(i)\left(-\frac{3}{5}i\right) = -5\times\frac{3}{5}\times i\times i$

$= -3i^2$

$= -3(-1)$

$= 3$

Q.2 Express the given complex number in the form a + ib: $i^9 + i^{19}$

Ans. $i^9 + i^{19} = i^{4\times2\times1} + i^{4\times4\times3}$

$= (i^4)^2.i + (i^4)^4.i^3$

$= 1\times i + 1\times(-i)$

$= 0$

Q.2 Express the given complex number in the form a + ib: i^{-39}

Ans. $i^{39} = i^{-4\times9-3} = (i^4)^{-9}.i^{-3}$

$= (1)^{-9}.i^{-3} \quad [i^4 = 1]$

$= \frac{1}{i^3} = \frac{1}{-i} \quad [i^3 = -i]$

$= \frac{-1}{i}\times\frac{i}{i}$

$= \frac{-i}{i^2} = \frac{-i}{-1} = i \quad [i^2 = -1]$

Q.4 Express the given complex number in the form a + ib: $3(7 + i7) + i(7 + i7)$

Ans. $3(7+i7)+i(7+i7) = 21 + 21i + 7i + 7i^2$

$= 21 + 28i + 7\times(-1)$

$= 14 + 28i$

Q.5 Express the given complex number in the form a + ib: (1 – i) – (–1 + i6)

Ans. $(1-i)-(-1+i6) = 1-i+1-6i$

$= 2 - 7i$

Q.6 Express the given complex number in the form a + ib: $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

Ans. $\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+i\frac{5}{2}\right)$

$=\frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i$

$=\left(\frac{1}{5}-4\right)+i\left(\frac{2}{5}-\frac{5}{2}\right)$

$=\frac{-19}{5}-\frac{21}{10}i$

Q.7 Express the given complex number in the form a + ib: $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

Ans. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right)$

$=\frac{1}{3}+\frac{7}{3}i+4+\frac{1}{3}i+\frac{4}{3}-i$

$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$

$=\frac{17}{3}+i\frac{5}{3}$

Q.8 Express the given complex number in the form a + ib: $(1-i)^4$

Ans. $(1-i)^4=\left[(1-i)^2\right]^2$

$=\left[1^2+i^2-2i\right]^2$

$=\left[1-1-2i\right]^2$

$=(-2i)\times(-2i)$

$=4i^2=-4\ \left[i^2=-1\right]$

Q.9 Express the given complex number in the form a + ib: $\left(\frac{1}{3}+3i\right)^3$

Ans. $\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$

$$= \frac{1}{27} + 27i^3 + 3^i\left(\frac{1}{3} + 3i\right)$$

$$= \frac{1}{27} + 27(-i) + i + 9i^2 \qquad [i^3 = -i]$$

$$= \frac{1}{27} - 27i + i - 9 \qquad [i^2 = -1]$$

$$= \left(\frac{1}{27} - 9\right) + i(-27 + 1)$$

$$= \frac{-242}{27} - 26i$$

Q.10 Express the given complex number in the form a + ib: $\left(-2 - \frac{1}{3}i\right)^3$

Ans. $\left(-2 - \frac{1}{3}i\right)^3 = (-1)^3\left(2 + \frac{1}{3}i\right)^3$

$$= -\left[2^3 + \left(\frac{i}{3}\right)^3 + 3(2)\left(\frac{i}{3}\right)\left(2 + \frac{i}{3}\right)\right]$$

$$= -\left[8 + \frac{i^3}{27} + 2i\left(2 + \frac{i}{3}\right)\right]$$

$$= -\left[8 - \frac{i^3}{27} + 4i + \frac{2i^2}{3}\right] \qquad [i^3 = -i]$$

$$= -\left[8 - \frac{i}{27} + 4i - \frac{2}{3}\right] \qquad [i^2 = -i]$$

$$= -\left[\frac{22}{3} + \frac{107i}{27}\right]$$

$$= -\frac{22}{3} - \frac{107}{27}$$

Q.11 Find the multiplicative inverse of the complex number 4 – 3i

Ans. Let $z = 4 - 3i$

Then $\bar{z}$, $= 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of 4 – 3i is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4 + 3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Q.12 Find the multiplicative inverse of the complex number $\sqrt{5}+3i$

Ans. Let $z=\sqrt{5}+3i$

then, $\bar{z}=\sqrt{5}+3i$ and $|z|^2=\left(\sqrt{5}\right)^2+3^3=5+9=14$

Therefore, the multiplicative inverse of is given by $\sqrt{5}+3i$

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{\sqrt{5-3i}}{14}=\frac{\sqrt{5}}{14}-\frac{3i}{14}$$

Q.13 Find the multiplicative inverse of the complex number –i

Ans. Let $z=-i$

Then, $z^{-1}=i$ and $|z|^2=1^2=1$

Therefore, the multiplicative inverse of –i is given by

$$z^{-1}=\frac{\bar{z}}{|z|^2}=\frac{i}{1}i$$

Q.14 Express the following expression in the form of a + ib

$$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3+\sqrt{2}i}\right)-\left(\sqrt{3-i\sqrt{2}}\right)}$$

Ans. $$\frac{\left(3+i\sqrt{5}\right)\left(3-i\sqrt{5}\right)}{\left(\sqrt{3+\sqrt{2}i}\right)-\left(\sqrt{3-i\sqrt{2}}\right)}$$

$$=\frac{(3)^2-\left(i\sqrt{5}\right)^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i}$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$= \frac{-7i}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{-7\sqrt{2}i}{2}$$