SpeedLabs

MATHS

CBSE 11th

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Q.1 Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Ans. $z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})$$

$$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+3$$

$$\Rightarrow r^2=4 \qquad [\cos^2\theta+\sin^2\theta=1]$$

$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{conventinally}, r>0]$$

$$\therefore \text{Modulus}=2$$

$$\therefore 2\cos\theta=-1 \text{ and } 2\sin\theta=-\sqrt{3}$$

$$\Rightarrow \cos\theta=\frac{-1}{2} \text{ and } \sin\theta=\frac{-\sqrt{3}}{2}$$

Since both the values of sin θ and cos θ are negative and sin θ and cos θ are negative in III quadrant,

$$\text{Argument}=-\left(\pi-\frac{\pi}{3}\right)=\frac{-2\pi}{3}$$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $\frac{-2\pi}{3}$ respectively.

Q.2 Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

Ans. $z=-\sqrt{3}+i$

Let $r\cos\theta=-\sqrt{3}$ and $r\sin\theta=1$

On squaring and adding, we obtain

$$r^2\cos^2\theta+r^2\sin^2\theta=(-\sqrt{3})^2+1^2$$

$$\Rightarrow r^2=3+1=4 \qquad [\cos^2\theta+\sin^2\theta=1]$$

$$\Rightarrow r=\sqrt{4}=2 \qquad [\text{conventionaly}, r>0]$$

$$\therefore \text{Modulus}=2$$

$$\therefore 2\cos\theta=-\sqrt{3} \text{ and } 2\sin\theta=1$$

$$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2} \text{ and } \sin\theta = \frac{1}{2}$$

$$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \quad [\text{As } \theta \text{ lies in the II quadrant}]$$

Thus, the modulus and argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Q.3 Convert the given complex number in polar form: 1 – i

Ans. 1 – i

Let r cos θ = 1 and r sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally}, r > 0]$$

$$\therefore \sqrt{2}\cos\theta = 1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$$\therefore \theta - \frac{\pi}{4} \qquad [\text{As } \theta \text{ lies in the IV quadrant}]$$

$$\therefore 1 - i = r\cos\theta + ir\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$$

This is the required polar form.

Q.4 Convert the given complex number in polar form: – 1 + i

Ans. – 1 + i

Let r cos θ = –1 and r sin θ = 1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$$

$$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$$

$$\Rightarrow r^2 = 2$$

$$\Rightarrow r = \sqrt{2} \qquad [\text{Conventionally}, r > 0]$$

$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = 1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = \frac{1}{\sqrt{2}}$$

$\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in the II quadrant]

It can be written,

$$\therefore -1+i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$$

This is the required polar form.

Q.5 Convert the given complex number in polar form: – 1 – i

Ans. – 1 – i

Let r cos θ = –1 and r sin θ = –1

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$$

$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$$

$$\Rightarrow r^2 = 2$$

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r > 0$]

$$\therefore \sqrt{2}\cos\theta = -1 \text{ and } \sqrt{2}\sin\theta = -1$$

$$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \text{ and } \sin\theta = -\frac{1}{\sqrt{2}}$$

$\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$ [As θ lies in the III quadrant]

$$\therefore -1-i = r\cos\theta + i\,r\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4} = \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$$

This is the required polar form.

Q.6 Convert the given complex number in polar form: –3

Ans. –3

Let r cos θ = –3 and r sin θ = 0

On squaring and adding, we obtain

$$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$$

$$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 9$$

$$\Rightarrow r^2 = 9$$

$\Rightarrow r = \sqrt{9} = 3$ [Conventionally, $r > 0$]

$$\therefore 3\cos\theta = -3 \text{ and } 3\sin\theta = 0$$

$$\Rightarrow \cos\theta = -1 \text{ and } \sin\theta = 0$$

$\therefore \theta = \pi$

$\therefore -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + \beta\sin\pi = 3\ (\cos\pi + i\sin\pi)$

This is the required polar form.

Q.7 Convert the given complex number in polar form: $\sqrt{3} + i$

Ans. $\sqrt{3} + i$

Let r cos θ = $\sqrt{3}$ and r sin θ = 1

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3 + 1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \sqrt{4} = 2$ [Conventionally, $r > 0$]

$\therefore 2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}$ [As lies in the I quadrant]

$\therefore \sqrt{3} + i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i2\sin\frac{\pi}{6} = 2\ \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

This is the required polar form.

Q.8 Convert the given complex number in polar form: i

Ans. I, Let r cos θ = 0 and r sin θ = 1

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (0)^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \sqrt{1} = 1$ [Conventionally, $r > 0$]

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \frac{\pi}{2}$

$\therefore i = r\cos\theta + ir\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

This is the required polar form.