



SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Conic Sections

Exercise- 11.1

1. Find the equation of the circle with centre (0, 2) and radius 2

Ans. The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h, k) = (0, 2) and radius (r) = 2.

Therefore, the equation of the circle is

$$(x - 0)^2 + (y - 2)^2 = 2^2$$

$$x^2 + y^2 + 4 - 4y = 4$$

$$x^2 + y^2 - 4y = 0$$

2. Find the equation of the circle with centre (-2, 3) and radius 4

Ans. The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h, k) = (-2, 3) and radius (r) = 4.

Therefore, the equation of the circle is

$$(x + 2)^2 + (y - 3)^2 = (4)^2$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 = 16$$

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

3. Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

Ans. The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h, k) = $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius (r) = $\frac{1}{12}$

Therefore, the equation of the circle is

$$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} - \frac{1}{144}$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 - 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

4. Find the equation of the circle with centre (1, 1) and radius

Ans The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h, k) = (1, 1) and radius (r) = $\sqrt{2}$.

Therefore, the equation of the circle is

$$(x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 2$$

$$x^2 + y^2 - 2x - 2y = 0$$

5. Find the equation of the circle with centre (-a, -b) and radius $\sqrt{a^2 - b^2}$

Ans The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre (h, k) = (-a, -b) and radius (r) = $\sqrt{a^2 - b^2}$.

Therefore, the equation of the circle is

$$(x + a)^2 + (y + b)^2 = (\sqrt{a^2 - b^2})^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

6. Find the centre and radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$

Ans The equation of the given circle is $(x + 5)^2 + (y - 3)^2 = 36$.

$$(x + 5)^2 + (y - 3)^2 = 36$$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = -5$, $k = 3$, and $r = 6$.

Thus, the centre of the given circle is (-5, 3), while its radius is 6.

7. Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 45 = 0$

Ans The equation of the given circle is $x^2 + y^2 - 4x - 8y - 45 = 0$.

$$x^2 + y^2 - 4x - 8y - 45 = 0$$

$$\Rightarrow (x^2 - 4x) + (y^2 - 8y) = 45$$

$$\Rightarrow \{x^2 - 2(x)(2) + 2^2\} + \{y^2 - 2(y)(4) + 4^2\} - 4 - 16 = 45$$

$$\Rightarrow (x - 2)^2 + (y - 4)^2 = 65$$

$\Rightarrow (x - 2)^2 + (y - 4)^2 = (\sqrt{65})^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = 2$, $k = 4$, and

$$r = (\sqrt{65}).$$

Thus, the centre of the given circle is $(2, 4)$, while its radius is $\sqrt{65}$.

8. Find the centre and radius of the circle $x^2 + y^2 - 8x + 10y - 12 = 0$

Ans The equation of the given circle is $x^2 + y^2 - 8x + 10y - 12 = 0$.

$$x^2 + y^2 - 8x + 10y - 12 = 0$$

$$\Rightarrow (x^2 - 8x) + (y^2 + 10y) = 12$$

$$\Rightarrow \{x^2 - 2(x)(4) + 4^2\} + \{y^2 + 2(y)(5) + 5^2\} - 16 - 25 = 12$$

$$\Rightarrow (x - 4)^2 + (y + 5)^2 = 53$$

$$\Rightarrow (x - 4)^2 + \{y - (-5)\}^2 = (\sqrt{53})^2, \text{ which is of the form } (x - h)^2 + (y - k)^2 = r^2, \text{ where } h$$

$$= 4, k = -5, \text{ and } r = \sqrt{53}.$$

Thus, the centre of the given circle is $(4, -5)$, while its radius is $\sqrt{53}$.

9. Find the centre and radius of the circle $2x^2 + 2y^2 - x = 0$

Ans The equation of the given circle is $2x^2 + 2y^2 - x = 0$.

$$2x^2 + 2y^2 - x = 0$$

$$\Rightarrow (2x^2 - x) + 2y^2 = 0$$

$$\Rightarrow 2 \left[\left(x^2 - \frac{x}{2} \right) + y^2 \right] = 0$$

$$\Rightarrow \left\{ x^2 - 2 \cdot x \left(\frac{1}{4} \right) + \left(\frac{1}{4} \right)^2 \right\} + y^2 - \left(\frac{1}{4} \right)^2 = 0$$

$$\Rightarrow \left(x - \frac{1}{4} \right)^2 + (y - 0)^2 = \left(\frac{1}{4} \right)^2$$

which is of the form $(x - h)^2 + (y - k)^2 = r^2$, where $h = \frac{1}{4}$, $k = 0$, and $r = \frac{1}{4}$

Thus, the centre of the given circle is $\left(\frac{1}{4}, 0 \right)$, while its radius is $\frac{1}{4}$.

10. Find the equation of the circle passing through the points $(4, 1)$ and $(6, 5)$ and whose centre is on the line $4x + y = 16$.

Ans Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points $(4, 1)$ and $(6, 5)$,

$$(4 - h)^2 + (1 - k)^2 = r^2 \dots (1)$$

$$(6 - h)^2 + (5 - k)^2 = r^2 \dots (2)$$

Since the centre (h, k) of the circle lies on line $4x + y = 16$,

$$4h + k = 16 \dots (3)$$

From equations (1) and (2), we obtain

$$(4 - h)^2 + (1 - k)^2 = (6 - h)^2 + (5 - k)^2$$

$$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 25 - 10k + k^2$$

$$\Rightarrow 16 - 8h + 1 - 2k = 36 - 12h + 25 - 10k$$

$$\Rightarrow 4h + 8k = 44$$

$$\Rightarrow h + 2k = 11 \dots (4)$$

On solving equations (3) and (4), we obtain $h = 3$ and $k = 4$.

On substituting the values of h and k in equation (1), we obtain

$$(4 - 3)^2 + (1 - 4)^2 = r^2$$

$$\Rightarrow (1)^2 + (-3)^2 = r^2$$

$$\Rightarrow 1 + 9 = r^2$$

$$\Rightarrow r^2 = 10$$

$$\Rightarrow r = \sqrt{10}$$

Thus, the equation of the required circle is

$$(x - 3)^2 + (y - 4)^2 = (\sqrt{10})^2$$

$$x^2 - 6x + 9 + y^2 - 8y + 16 = 10$$

$$x^2 + y^2 - 6x - 8y + 15 = 0$$

- 11.** Find the equation of the circle passing through the points $(2, 3)$ and $(-1, 1)$ and whose centre is on the line $x - 3y - 11 = 0$.

Ans Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through points $(2, 3)$ and $(-1, 1)$,

$$(2 - h)^2 + (3 - k)^2 = r^2 \dots (1)$$

$$(-1 - h)^2 + (1 - k)^2 = r^2 \dots (2)$$

Since the centre (h, k) of the circle lies on line $x - 3y - 11 = 0$,

$$h - 3k = 11 \dots (3)$$

From equations (1) and (2), we obtain

$$(2 - h)^2 + (3 - k)^2 = (-1 - h)^2 + (1 - k)^2$$

$$\Rightarrow 4 - 4h + h^2 + 9 - 6k + k^2 = 1 + 2h + h^2 + 1 - 2k + k^2$$

$$\Rightarrow 4 - 4h + 9 - 6k = 1 + 2h + 1 - 2k$$

$$\Rightarrow 6h + 4k = 11 \dots (4)$$

On solving equations (3) and (4), we obtain $h = \frac{7}{2}$ and $k = \frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\left(2 - \frac{7}{2}\right)^2 + \left(3 + \frac{5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{4-7}{2}\right)^2 + \left(\frac{6+5}{2}\right)^2 = r^2$$

$$\Rightarrow \left(\frac{-3}{2}\right)^2 + \left(\frac{11}{2}\right)^2 = r^2$$

$$\Rightarrow \frac{9}{4} + \frac{121}{4} = r^2$$

$$\Rightarrow \frac{130}{4} = r^2$$

Thus, the equation of the required circle is

$$\left(x - \frac{7}{2}\right)^2 + \left(y + \frac{5}{2}\right)^2 = \frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2 + \left(\frac{2y+5}{2}\right)^2 = \frac{130}{4}$$

$$4x^2 - 28x + 49 + 4y^2 + 20y + 25 = 130$$

$$4x^2 + 4y^2 - 28x + 20y - 56 = 0$$

$$4(x^2 + y^2 - 7x + 5y - 14) = 0$$

$$x^2 + y^2 - 7x + 5y - 14 = 0$$

12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2, 3).

Ans. Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the radius of the circle is 5 and its centre lies on the x-axis, $k = 0$ and $r = 5$.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3).

$$\therefore (2 - h)^2 + 3^2 = 25$$

$$\Rightarrow (2 - h)^2 = 25 - 9$$

$$\Rightarrow (2 - h)^2 = 16$$

$$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$$

$$\text{If } 2 - h = 4, \text{ then } h = -2$$

$$\text{If } 2 - h = -4, \text{ then } h = 6$$

When $h = -2$, the equation of the circle becomes

$$(x + 2)^2 + y^2 = 25$$

$$x^2 + 4x + 4 + y^2 = 25$$

$$x^2 + y^2 + 4x - 21 = 0$$

When $h = 6$, the equation of the circle becomes

$$(x - 6)^2 + y^2 = 25$$

$$x^2 - 12x + 36 + y^2 = 25$$

$$x^2 + y^2 - 12x + 11 = 0$$

13. Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the coordinate axes.

Ans Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the centre of the circle passes through $(0, 0)$,

$$(0 - h)^2 + (0 - k)^2 = r^2$$

$$\Rightarrow h^2 + k^2 = r^2$$

The equation of the circle now becomes $(x - h)^2 + (y - k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a, 0)$ and $(0, b)$. Therefore,

$$(a - h)^2 + (0 - k)^2 = h^2 + k^2 \dots (1)$$

$$(0 - h)^2 + (b - k)^2 = h^2 + k^2 \dots (2)$$

From equation (1), we obtain

$$a^2 - 2ah + h^2 + k^2 = h^2 + k^2$$

$$\Rightarrow a^2 - 2ah = 0$$

$$\Rightarrow a(a - 2h) = 0$$

$$\Rightarrow a = 0 \text{ or } (a - 2h) = 0$$

However, $a \neq 0$; hence, $(a - 2h) = 0 \Rightarrow h = \frac{a}{2}$.

From equation (2), we obtain

$$h^2 + b^2 - 2bk + k^2 = h^2 + k^2$$

$$\Rightarrow b^2 - 2bk = 0$$

However, $b \neq 0$; hence, $(b - 2k) = 0 \Rightarrow k = \frac{b}{2}$

Thus, the equation of the required circle is

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$$

$$\Rightarrow \left(\frac{2x - a}{2}\right)^2 + \left(\frac{2y - b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$$\Rightarrow 4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$$

$$\Rightarrow 4x^2 + 4y^2 - 4ax - 4by = 0$$

$$\Rightarrow x^2 + y^2 - ax - by = 0$$

14. Find the equation of a circle with centre $(2, 2)$ and passes through the point $(4, 5)$.

Ans The centre of the circle is given as $(h, k) = (2, 2)$.

Since the circle passes through point $(4, 5)$, the radius (r) of the circle is the distance between the points $(2, 2)$ and $(4, 5)$.

$$\therefore r = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$$

Thus, the equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-2)^2 = (\sqrt{13})^2$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 13$$

$$x^2 + y^2 - 4x - 4y - 5 = 0$$

15. Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

Ans The equation of the given circle is $x^2 + y^2 = 25$.

$$x^2 + y^2 = 25$$

$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2$, which is of the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = 0$, $k = 0$, and $r = 5$.

\therefore Centre = $(0, 0)$ and radius = 5

Distance between point $(-2.5, 3.5)$ and centre $(0, 0)$

$$= \sqrt{(-2.5-0)^2 + (3.5-0)^2}$$

$$= \sqrt{6.25 + 12.25}$$

$$= \sqrt{18.5}$$

$$= 4.3(\text{approx.}) < 5$$

Since the distance between point $(-2.5, 3.5)$ and centre $(0, 0)$ of the circle is less than the radius of the circle, point $(-2.5, 3.5)$ lies inside the circle.