



SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

Conic Sections

Exercise- 11.3

1. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and

the length of the latus rectum of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Ans The given equation is $\frac{x^2}{36} + \frac{y^2}{16} = 1$.

Here, the denominator of $\frac{x^2}{36}$ is greater than the denominator of $\frac{y^2}{16}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 6$ and $b = 4$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 16} = \sqrt{20} = 2\sqrt{5}$$

Therefore,

The coordinates of the foci are $(2\sqrt{5}, 0)$ and $(-2\sqrt{5}, 0)$.

The coordinates of the vertices are $(6, 0)$ and $(-6, 0)$.

Length of major axis = $2a = 12$

Length of minor axis = $2b = 8$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 16}{6} = \frac{16}{3}$$

2. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis the eccentricity and

the length of the latus rectum of the ellipse $\frac{x^2}{4} + \frac{y^2}{25} = 1$

Ans. The given equation is $\frac{x^2}{4} + \frac{y^2}{25} = 1$ or $\frac{x^2}{2^2} + \frac{y^2}{5^2} = 1$.

Here, the denominator of $\frac{y^2}{25}$ is greater than the denominator of $\frac{x^2}{4}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 2$ and $a = 5$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{25 - 4} = \sqrt{21}$$

Therefore,

The coordinates of the foci are $(0, \sqrt{21})$ and $(0, -\sqrt{21})$.

The coordinates of the vertices are $(0, 5)$ and $(0, -5)$

Length of major axis = $2a = 10$

Length of minor axis = $2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{21}}{5}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{5} = \frac{8}{5}$$

3. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and

the length of the latus rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Ans The given equation is $\frac{x^2}{16} + \frac{y^2}{9} = 1$ or $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$.

Here, the denominator of $\frac{x^2}{16}$ is greater than the denominator of $\frac{y^2}{9}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 4$ and $b = 3$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{16 - 9} = \sqrt{7}$$

Therefore,

The coordinates of the foci are $(\pm\sqrt{7}, 0)$.

The coordinates of the vertices are $(\pm 4, 0)$.

Length of major axis = $2a = 8$

Length of minor axis = $2b = 6$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{7}}{4}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 9}{4} = \frac{9}{2}$$

4. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and

the length of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{100} = 1$

Ans. The given equation is $\frac{x^2}{25} + \frac{y^2}{100} = 1$ or $\frac{x^2}{5^2} + \frac{y^2}{10^2} = 1$.

Here, the denominator of $\frac{y^2}{100}$ is greater than the denominator of $\frac{x^2}{25}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 5$ and $a = 10$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3}$$

Therefore,

The coordinates of the foci are $(0, \pm 5\sqrt{3})$.

The coordinates of the vertices are $(0, \pm 10)$.

Length of major axis = $2a = 20$

Length of minor axis = $2b = 10$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{5\sqrt{3}}{10} = \frac{\sqrt{3}}{2}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 25}{10} = 5$$

5. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and

the length of the latus rectum of the ellipse $\frac{x^2}{49} + \frac{y^2}{36} = 1$

Ans. The given equation is $\frac{x^2}{49} + \frac{y^2}{36} = 1$ or $\frac{x^2}{7^2} + \frac{y^2}{6^2} = 1$.

Here, the denominator of $\frac{x^2}{49}$ is greater than the denominator of $\frac{y^2}{36}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $a = 7$ and $b = 6$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{49 - 36} = \sqrt{13}$$

Therefore,

The coordinates of the foci are $(\pm\sqrt{13}, 0)$.

The coordinates of the vertices are $(\pm 7, 0)$.

$$\text{Length of major axis} = 2a = 14$$

$$\text{Length of minor axis} = 2b = 12$$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{\sqrt{3}}{7}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 36}{7} = \frac{72}{7}$$

6. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{100} + \frac{y^2}{400} = 1$

Ans The given equation is $\frac{x^2}{100} + \frac{y^2}{400} = 1$ or $\frac{x^2}{10^2} + \frac{y^2}{20^2} = 1$.

Here, the denominator of $\frac{y^2}{400}$ is greater than the denominator of $\frac{x^2}{100}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing the given equation with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $b = 10$ and $a = 20$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{400 - 100} = \sqrt{300} = 10\sqrt{3}$$

Therefore,

The coordinates of the foci are $(0, \pm 10\sqrt{3})$.

The coordinates of the vertices are $(0, \pm 20)$

$$\text{Length of major axis} = 2a = 40$$

$$\text{Length of minor axis} = 2b = 20$$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 100}{20} = 10$$

7. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $36x^2 + 4y^2 = 144$

Ans The given equation is $36x^2 + 4y^2 = 144$.

It can be written as

$$36x^2 + 4y^2 = 144$$

$$\text{or, } \frac{x^2}{4} + \frac{y^2}{36} = 1$$

$$\text{or, } \frac{x^2}{2^2} + \frac{y^2}{6^2} = 1 \dots (1)$$

Here, the denominator of $\frac{y^2}{6^2}$ is greater than the denominator of $\frac{x^2}{2^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we obtain $b = 2$ and $a = 6$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{36 - 4} = \sqrt{32} = 4\sqrt{2}$$

Therefore,

The coordinates of the foci are $(0, \pm 4\sqrt{2})$.

The coordinates of the vertices are $(0, \pm 6)$.

Length of major axis = $2a = 12$

Length of minor axis = $2b = 4$

$$\text{Eccentricity, } e = \frac{c}{a} = \frac{4\sqrt{2}}{6} = \frac{2\sqrt{2}}{3}$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4}{6} = \frac{4}{3}$$

8. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $16x^2 + y^2 = 16$

Ans The given equation is $16x^2 + y^2 = 16$.

It can be written as

$$16x^2 + y^2 = 16$$

$$\text{or, } \frac{x^2}{1} + \frac{y^2}{16} = 1$$

$$\text{or, } \frac{x^2}{1^2} + \frac{y^2}{4^2} = 1 \dots (1)$$

Here, the denominator of $\frac{y^2}{4^2}$ is greater than the denominator of $\frac{x^2}{1^2}$.

Therefore, the major axis is along the y-axis, while the minor axis is along the x-axis.

On comparing equation (1) with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $b = 1$ and $a = 4$.

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{15})$.

The coordinates of the vertices are $(0, \pm 4)$.

Length of major axis = $2a = 8$

Length of minor axis = $2b = 2$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{15}}{4}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 1}{4} = \frac{1}{2}$

9. Find the coordinates of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $4x^2 + 9y^2 = 36$

Ans The given equation is $4x^2 + 9y^2 = 36$.

It can be written as

$$4x^2 + 9y^2 = 36$$

$$\text{or, } \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\text{or, } \frac{x^2}{3^2} + \frac{y^2}{2^2} = 1 \dots (1)$$

Here, the denominator of $\frac{x^2}{3^2}$ is greater than the denominator of $\frac{y^2}{2^2}$.

Therefore, the major axis is along the x-axis, while the minor axis is along the y-axis.

On comparing the given equation with $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, we obtain $a = 3$ and $b = 2$.

$$\therefore c = \sqrt{a^2 - b^2} = \sqrt{9 - 4} = \sqrt{5}$$

Therefore,

The coordinates of the foci are $(\pm\sqrt{5}, 0)$.

The coordinates of the vertices are $(\pm 3, 0)$.

Length of major axis = $2a = 6$

Length of minor axis = $2b = 4$

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$

Length of latus rectum = $\frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

10. Find the equation for the ellipse that satisfies the given conditions: Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Ans Vertices $(\pm 5, 0)$, foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $a = 5$ and $c = 4$.

It is known that $a^2 = b^2 + c^2$

$$\therefore 5^2 + b^2 + 4^2$$

$$\Rightarrow 25 = b^2 + 16$$

$$\Rightarrow b^2 = 25 - 16$$

$$\Rightarrow b = \sqrt{9} = 3$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$

11. Find the equation for the ellipse that satisfies the given conditions: Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

Ans Vertices $(0, \pm 13)$, foci $(0, \pm 5)$

Here, the vertices are on the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$

$$\therefore 13^2 + b^2 + 5^2$$

$$\Rightarrow 169 = b^2 + 25$$

$$\Rightarrow b^2 = 169 - 25$$

$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{12^2} + \frac{y^2}{13^2} = 1$ or $\frac{x^2}{144} + \frac{y^2}{169} = 1$

12. Find the equation for the ellipse that satisfies the given conditions: Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Ans Vertices $(\pm 6, 0)$, foci $(\pm 4, 0)$

Here, the vertices are on the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Accordingly, $a = 6$, $c = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 6^2 + b^2 + b^2$$

$$\Rightarrow 36 = b^2 + 16$$

$$\Rightarrow b^2 = 36 - 16$$

$$\Rightarrow b = \sqrt{20}$$

Thus, the equation of the ellipse is $\frac{x^2}{6^2} + \frac{y^2}{(\sqrt{20})^2} = 1$ or $\frac{x^2}{36} + \frac{y^2}{20} = 1$

- 13.** Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Ans Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

Here, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $a = 3$ and $b = 2$.

Thus, the equation of the ellipse is $\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$ o.i.e, $\frac{x^2}{9} + \frac{y^2}{4} = 1$

- 14.** Find the equation for the ellipse that satisfies the given conditions: Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Ans. Ends of major axis $(0, \pm\sqrt{5})$, ends of minor axis $(\pm 1, 0)$

Here, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a semi-major axis.

Accordingly, $a = \sqrt{5}$ and $b = 1$.

Thus, the equation of the ellipse is $\frac{x^2}{1^2} + \frac{y^2}{(\sqrt{5})^2} = 1$ or $\frac{x^2}{1} + \frac{y^2}{5} = 1$.

- 15.** Find the equation for the ellipse that satisfies the given conditions: Length of major axis 26, foci $(\pm 5, 0)$

Ans Length of major axis = 26; foci = $(\pm 5, 0)$.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $2a = 26 \Rightarrow a = 13$ and $c = 5$.

It is known that $a^2 = b^2 + c^2$

$$\therefore 13^2 + b^2 + 5^2$$

$$\Rightarrow 169 = b^2 + 25$$

$$\Rightarrow b^2 = 169 - 25$$

$$\Rightarrow b = \sqrt{144} = 12$$

Thus, the equation of the ellipse is $\frac{x^2}{13^2} + \frac{y^2}{12^2} = 1$ or $\frac{x^2}{169} + \frac{y^2}{144} = 1$

16. Find the equation for the ellipse that satisfies the given conditions: Length of minor axis 16, foci $(0, \pm 6)$

Ans Length of minor axis = 16; foci = $(0, \pm 6)$.

Since the foci are on the y-axis, the major axis is along the y-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $2b = 16 \Rightarrow b = 8$ and $c = 6$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore a^2 = 8^2 + 6^2 = 64 + 36 = 100$$

$$\Rightarrow a = \sqrt{100} = 10$$

Thus, the equation of the ellipse is $\frac{x^2}{8^2} + \frac{y^2}{10^2} = 1$ or $\frac{x^2}{64} + \frac{y^2}{100} = 1$

17. Find the equation for the ellipse that satisfies the given conditions: Foci $(\pm 3, 0)$, $a = 4$

Ans Foci $(\pm 3, 0)$, $a = 4$

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a is the semi-major axis.

Accordingly, $c = 3$ and $a = 4$.

It is known that $a^2 = b^2 + c^2$.

$$\therefore 4^2 + b^2 + 3^2$$

$$\Rightarrow 16 = b^2 + 9$$

$$\Rightarrow b^2 = 16 - 9 = 7$$

Thus, the equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{7} = 1$

18. Find the equation for the ellipse that satisfies the given conditions: $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Ans It is given that $b = 3$, $c = 4$, centre at the origin; foci on the x axis.

Since the foci are on the x-axis, the major axis is along the x-axis.

Therefore, the equation of the ellipse will be of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where a is the semi-major axis.

Accordingly, $b = 3$, $c = 4$.

It is known that $a^2 = b^2 + c^2$

$$\therefore a^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$\Rightarrow a = 5$$

Thus, the equation of the ellipse is $\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$ or $\frac{x^2}{25} + \frac{y^2}{9} = 1$

19. Find the equation for the ellipse that satisfies the given conditions: Centre at (0, 0), major axis on the y-axis and passes through the points (3, 2) and (1, 6).

Ans Since the centre is at (0, 0) and the major axis is on the y-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Where, a is the semi – major axis

The ellipse passes through points (3, 2) and (1, 6). Hence,

$$\frac{9}{b^2} + \frac{4}{a^2} = 1 \dots\dots (2)$$

$$\frac{1}{b^2} + \frac{36}{a^2} = 1 \dots\dots (3)$$

On solving equations (2) and (3), we obtain $b^2 = 10$ and $a^2 = 40$

Thus, the equation of the ellipse is $\frac{x^2}{10} + \frac{y^2}{40} = 1$ or $4x^2 + y^2 = 40$

20. Find the equation for the ellipse that satisfies the given conditions: Major axis on the x axis and passes through the points (4, 3) and (6, 2).

Ans Since the major axis is on the x-axis, the equation of the ellipse will be of the form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where, a is the semi – major axis

The ellipse passes through points (4, 3) and (6, 2). Hence,

$$\frac{16}{a^2} + \frac{9}{b^2} = 1 \dots\dots (2)$$

$$\frac{36}{a^2} + \frac{4}{b^2} = 1 \dots\dots (3)$$

On solving equations (2) and (3), we obtain $a^2 = 52$ and $b^2 = 13$.

Thus, the equation of the ellipse is $\frac{x^2}{52} + \frac{y^2}{13} = 1$ or $x^2 + 4y^2 = 52$