

Board – CBSE

Class – 12

Topic – Continuity and Differentiability

2. Continuity

LEVEL-I

1. Examine the continuity of the function $f(x) = x^2 + 5$ at $x = -1$.
2. Examine the continuity of the function $f(x) = \frac{1}{x+3}$, $x \in \mathbb{R}$.
3. Show that $f(x) = 4x$ is a continuous for all $x \in \mathbb{R}$.

LEVEL-II

4. Give an example of a function which is continuous at $x = 1$, but not differentiable at $x = 1$.
5. For what value of k , the function $\begin{cases} kx^2, & \text{if } x \leq 2 \\ 3, & \text{if } x > 2 \end{cases}$ is continuous at $x = 2$.
6. Find the relationship between “a” and “b” so that the function ‘f’ defined by: [CBSE 2011]
 $f(x) = \begin{cases} ax + 1, & \text{if } x \leq 3 \\ bx + 3, & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$.
7. If $f(x) = \begin{cases} \frac{\sin 3x}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$. Find whether $f(x)$ is continuous at $x = 0$.

LEVEL-III

8. For what value of k , the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$?
9. If function $f(x) = \frac{2x+3 \sin x}{3x+2 \sin x}$, for $x \neq 0$ is continuous at $x = 0$, then. Find $f(0)$.
10. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3 \cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ a, & \text{if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$ = If $f(x)$ be a continuous function at $x = \frac{\pi}{2}$, find a and b.
11. For what value of k , is the function $f(x) = \begin{cases} \frac{\sin x + x \cos x}{x}, & \text{when } x \neq 0 \\ k, & \text{when } x = 0 \end{cases}$ continuous at $x = 0$?

3. Differentiation

LEVEL-I

12. Discuss the differentiability of the function $f(x) = (x - 1)^{2/3}$ at $x = 1$
13. Differentiate $y = \tan^{-1} \frac{2x}{1-x^2}$

14. If $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$, Find $\frac{dy}{dx}$

LEVEL-II

15. Find $\frac{dy}{dx}$, $y = \cos(\log x)^2$

16. Find $\frac{dy}{dx}$ of $y = \tan^{-1} \left[\frac{\sqrt{1+x^2}-1}{x} \right]$

17. If $y = e^{ax} \sin bx$, then prove that $\frac{d^2y}{dx^2} - 2a \frac{dy}{dx} + (a^2 + b^2)y = 0$

18. Find $\frac{d^2y}{dx^2}$, if $y = \frac{3at}{1+t}$, $x = \frac{2at^2}{1+t}$.

LEVEL-III

19. Find $\frac{dy}{dx}$, if $y = \tan^{-1} \left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}} \right]$

20. Find $\frac{dy}{dx} y = \cot^{-1} \left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \right]$, $0 < x < \frac{\pi}{2}$

21. If $y = \sin^{-1} \left(\frac{a+b \cos x}{b+a \cos x} \right)$, show that $\frac{dy}{dx} = \frac{-\sqrt{b^2-a^2}}{b+a \cos x}$

22. Prove that $\frac{d}{dx} \left[\frac{1}{4\sqrt{2}} \log \left| \frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1} \right| + \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{\sqrt{2}x}{1-x^2} \right) \right] = \frac{1}{1+x^4}$

4. Logarithmic Differentiation

LEVEL-I

23. Differentiate $y = \log_7 (\log x)$.

24. Differentiate, $\sin(\log x)$, with respect to x .

25. Differentiate $y = \tan^{-1}(\log x)$