SpeedLabs

MATHS

CBSE 12th

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Q.1 Differentiate the functions with respect to x.

$\sin(x^2+5)$

Sol: Let $f(x)=\sin(x^2+5)$, $u(x)=x^2+5$, and $v(t)=\sin t$

Then, $(vou)(x)=v(u(x))=v(x^2+5)=\tan(x^2+5)=f(x)$

Thus, f is a composite of two functions.

Put $t=u(x)=x^2+5$

Then, we obtain

$\frac{dv}{dt}=\frac{d}{dt}(\sin t)=\cos t=\cos(x^2+5)$

$\frac{dv}{dt}=\frac{d}{dt}(x^2+5)=\frac{d}{dx}(x^2)+\frac{d}{dx}(5)=2x+0=2x$

Therefore, by chain rule, $\frac{df}{dx}=\frac{dv}{dx}.\frac{dt}{dx}=\cos(x^2+5)\times 2x=2x\cos(x^2+5)$

Alternate method

$$\frac{d}{dt}[\sin(x^2+5)]=\cos(x^2+5).\frac{d}{dx}(x^2+5)$$

$$=\cos(x^2+5).\left[\frac{d}{dx}(x^2)+\frac{d}{dx}(5)\right]$$

$$=\cos(x^2+5).[2x+0]$$

$$=2x\cos(x^2+5)$$

Q.2 Differentiate the functions with respect to x.

$\cos(\sin x)$

Sol: Let $f(x)=\cos(\sin x)$, $u(x)=\sin x$, and $v(t)=\cos t$

Then, $(vou)(x)=v(u(x))=v(\sin x)=\cos(\sin x)=f(x)$

Thus, f is a composite function of two functions.

Put $t=u\,(x)=\sin x$

Therefore,

$\frac{dv}{dt}=\frac{d}{dt}(\cos t)=-\sin t=-\sin(\sin x)$

$\frac{dv}{dt}=\frac{d}{dt}(\sin x)=\cos x$

By chain rule, $\frac{df}{dx}=\frac{dv}{dx}.\frac{dt}{dx}=-\sin(\sin x).\cos x=-\cos x\sin(\sin x)$

Alternate method

$$\frac{d}{dt}[\cos(\sin x)]=-\sin(\sin x).\frac{d}{dx}(\sin x)=-\sin(\sin x).\cos x=-\cos x\sin(\sin x)$$

Q.3 Differentiate the functions with respect to x.

$\sin(ax+b)$

Sol: Let $f(x) = \sin(ax + b), u(x) = ax + b,$ and $v(t) = \sin t$

Then, $(vou)(x) = v(u(x)) = v(ax + b) = \sin(ax + b) = f(x)$

Thus, f is a composite function of two functions, u and v.

Put $t = u(x) = ax + b$

Therefore,

$\frac{dv}{dt} = \frac{d}{dt}(\sin t) = \cos t = \cos(ax+b)$

$\frac{dt}{dx} = \frac{d}{dx}(ax+b) = \frac{d}{dx}(ax) + \frac{d}{dx}(b) = a + 0 = a$

By chain rule, $\frac{df}{dx} = \frac{dv}{dx}.\frac{dt}{dx} = \cos(ax+b).a = a\cos(ax+b)$

Alternate method

$\frac{d}{dt}[\sin(ax+b)] = -\cos(ax+b).\frac{d}{dx}(ax+b)$

$= \cos(ax+b)\left[\frac{d}{dx}(ax) + \frac{d}{dx}(b)\right]$

$= \cos(ax+b).(a+0)$

$= a.\cos(ax+b)$

Hence, by chain rule, we obtain

Q.4 Differentiate the functions with respect to x.

$\sec(\tan(\sqrt{x}))$

Sol: Let $f(x) = sec(tan(\sqrt{x}))\,, u(x) = \sqrt{x},\ v(t) = tan\, t\,, and\ w(s) = sec\, s$

Then, $(wovou)(x) = w[v(u(x))] = w[v((x))] = sec(tan(\sqrt{x}) = f(x)$

Thus, f is a composite function of three functions, u, v, and w.

Put $s = v(t) = tan\, t\ \ and\ t = u(x) = \sqrt{x}$

Then, $\frac{dw}{ds} = \frac{d}{ds}(sec\ s) = sec\, s = sec\, s\, tan\, s = sec(tan\, t).tan(tan\, t)$ $[s = tan\, t]$

$= sec(tan\sqrt{x}).tan(tan\sqrt{x})$ [t=$\sqrt{x}$]

$\frac{ds}{dt} = \frac{d}{dt}[tan\, t] = sec^2 t = sec^2\sqrt{x}$

$\frac{dt}{dx} = \frac{d}{dx}[\sqrt{x}] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}\ x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

Hence, by chain rule, we obtain

$\frac{dt}{dx} = \frac{dw}{ds}.\frac{ds}{dt}.\frac{dt}{dx}$

$= sec(tan\sqrt{x}).tan(tan\sqrt{x}) \times sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}}$

$= \ \frac{1}{2\sqrt{x}}\ sec^2\sqrt{x}\, sec(tan\sqrt{x}).tan(tan\sqrt{x})$

$= \frac{sec^2\sqrt{x}sec(tan\sqrt{x})\, tan(tan\sqrt{x})}{2\sqrt{x}}$

Alternate method.

$\frac{d}{dx}\left[sec\left(tan\sqrt{x}\right)\right] = sec\left(tan\sqrt{x}\right).tan\left(tan\sqrt{x}\right).\frac{d}{dx}\left(tan\sqrt{x}\right)$

$= sec\left(tan\sqrt{x}\right).tan\left(tan\sqrt{x}\right).sec^2(\sqrt{x})\frac{d}{dx}(\sqrt{x})$

$= sec\left(tan\sqrt{x}\right).tan\left(tan\sqrt{x}\right).sec^2(\sqrt{x})\frac{1}{2\sqrt{x}}$

$= \frac{=sec\left(tan\sqrt{x}\right).tan\left(tan\sqrt{x}\right).sec^2(\sqrt{x})}{2\sqrt{x}}$

Q.5 Differentiate the functions with respect to x.

$\frac{sin(ax+b)}{cos(cx+d)}$

Sol: The given function $f(x) = \frac{sin(ax+b)}{cos(cx+d)} = \frac{g(x)}{h(x)}$, where g (x) = sin (ax + b) and h (x) = cos (cx + d)

$\therefore\ f' = \frac{g'h-gh'}{h^2}$

Consider $g(x) = sin(ax+b)$

Let $u(x) = ax+b, v(t) = sint\ t$

Then, $(vou)(x) = v\left(u(x)\right) = v(ax\ +\ b) = sin(ax\ +\ b) = f(x)$

$\therefore$ g is a composite function of two functions, u and v.

Put $t\ =\ u\ (x)\ =\ ax\ +\ b$

$\frac{dv}{dt} = \frac{d}{dt}(sin\ t) = cos\ t = cos(ax+b)$

$\frac{dt}{dx} = \frac{d}{dx}(ax+b) = \frac{d}{dx}(ax) + \frac{d}{dx}(b) = a + 0 = a$

Therefore, By chain rule,

$g' = \frac{dg}{dx} = \frac{dv}{dx}.\frac{dt}{dx} = cos(ax+b).a = a\,cos(ax+b)$

Consider $h(x)\ =\ cos(cx+d)$

Let $p(x) = cx+d, q(y) = cosy$

Then, $(qop)(x) = q(p(x)) = q(cx+d) = cos(cx+d) = h(x)$

$\therefore$ h is a composite function of two functions, p and q.

Put $y\ =\ p\ (x)\ =\ cx\ +\ d$

$\frac{dq}{dy} = \frac{d}{dy}(cos\ y) = -sin\ y = -sin(cx+d)$

$\frac{dy}{dx} = \frac{d}{dx}(cx+d) = \frac{d}{dx}(cx) + \frac{d}{dx}(d) = c$

Therefore, by chain rule, we obtain

$h' = \frac{dh}{dx} = \frac{dq}{dy}.\frac{dy}{dx} = -\,sin(cx+d)\times c = -csin(cx+d)$

$\therefore f' = \frac{acos(a+b).cos(cx+d)-sin(ax+b)\{-c\,sin(cx+d)\}}{[cos(cx+d)]^2}$

$= \frac{a\,cos(cx+d)}{cos(cx+d)} + sin(ax+b).\frac{a\,sin(cx+d)}{cos(cx+d)} \times \frac{1}{cos(cx+d)}$

$= a\,cos(ax+b)\,sec(cx+d) + c\,sin\,(ax+b)\,tan(cx+d)\,sec(cx+d)$

Q.6 Differentiate the functions with respect to x.

$cos\,x^3 \,.\, sin^2(x^5)$

Sol: The given function is $cos\,x^3 \,.\, sin^2(x^5)$

$\frac{d}{dx}[cos\,x^3 \,.\, sin^2(x^5)] = sin^2(x^5) \times \frac{d}{dx}(cos\,x^3) + cos\,x^3 \times \frac{d}{dx}[sin^2(x^5)]$

$= sin^2(x^5) \times (-\,sin\,x^3) \times \frac{d}{dx}(x^3) + cos\,x^3 \times 2\,sin(x^5)\,.\frac{d}{dx}[sin\,x^5]$

$= -sinx^3\,sin^2(x^5) \times 3x^2 + 2\,sinx^5\,cos\,x^5\,.\,cos\,x^5 \times \frac{d}{dx}(x^5)$

$= -3x^2\,sinx^3\,sin^2(x^5) + 2\,sinx^5\,cos\,x^5\,.\,cos\,x^5 \times 5x^4$

$= 10x^4\,sinx^5\,cos\,x^5\,cos\,x^3 - 3x^2\,sin\,x^3\,sin^2(x^5)$

Q.7 Differentiate the functions with respect to x.

$2\sqrt{cot(x^2)}$

Sol: $\frac{d}{dx}\left[2\sqrt{cot(x^2)}\,\right]$

$= 2.\frac{1}{2\sqrt{cot(x^2)}} \times \frac{d}{dx}\,[cot(x^2)]$

$= \sqrt{\frac{sin\,(x^2)}{cos(x^2)}} \times \frac{1}{sin\,(x^2)} \times (2x)$

$= \frac{-2x}{\sqrt{2\,sin\,x^2\,cos\,x^2}\,sin\,x^2}$

$= \frac{-2\sqrt{2}\,x}{\sqrt{cos(x^2)}\,\sqrt{sin\,x^2}\,sin\,x^2}$

$= \frac{-2\sqrt{2}\,x}{\sqrt{sin\,2x^2}\,sin\,x^2}$

Q.8 Differentiate the functions with respect to x.

cos $(\sqrt{x})$

Sol: Let f(x)= cos $(\sqrt{x})$

Also, let u(x)= $\sqrt{x}$

And, v(t)=cost

Then, $(vou)(x) = v(u(x))$

$= v(\sqrt{x})$

$= cos\sqrt{x}$

$= f(x)$

Clearly, f is a composite function of two functions, u and v, such that

$t = u(x) = \sqrt{x}$

Then, $\frac{dt}{dx} = \frac{d}{dx}\left[\sqrt{x}\right] = \frac{d}{dx}\left[x^{\frac{1}{2}}\right] = \frac{1}{2}\,x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

And, $\frac{dv}{dt} = \frac{d}{dt}(cos\,t) = -sin\,t = sin(\sqrt{x})$

By using chain rule, we obtain

$\frac{dt}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$

$= sin(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{1}{2\sqrt{x}} sin(\sqrt{x})$

$= -\frac{sin(\sqrt{x})}{2\sqrt{x}}$

Alternate method

$\frac{d}{dx}[cos(\sqrt{x})] = -sin(\sqrt{x}) \cdot \frac{d}{dx}(\sqrt{x})$

$= -sin(\sqrt{x}) \cdot \frac{d}{dx}(x^{\frac{1}{2}})$

$= -sin(\sqrt{x}) \times \frac{1}{2} x^{-\frac{1}{2}}$

$= -\frac{sin(\sqrt{x})}{2\sqrt{x}}$

Q.9 Prove that the function f given by $f(x) = |x-1|, x \in R$ is not differentiable at $x = 1$.

Sol: The given function is $f(x) = |x-1|, x \in R$

It is known that a function f is differentiable at a point x = c in its domain if both

$\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h}$ *and* $\lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at x = 1, consider the left-hand limit of f at x = 1

$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h}$ and $\lim_{h\to 0^+} \frac{|1+h-1|-|1-1|}{h}$

$= \lim_{h\to 0^-} \frac{|h|-0}{h} = \lim_{h\to 0^-} \frac{-h}{h}$ $(h < 0 \Rightarrow |h| = -h)$

$= -1$

Consider the right-hand limit of f at x=1

$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h}$ and $\lim_{h\to 0^+} \frac{|1+h-1|-|1-1|}{h}$

$= \lim_{h\to 0^-} \frac{|h|-0}{h} = \lim_{h\to 0^-} \frac{-h}{h}$ $(h > 0 \Rightarrow |h| = h)$

$= -1$

Since the left- and right-hand limits of f at x = 1 are not equal, f is not differentiable at x = 1

Q.10 Prove that the greatest integer function defined by is not $f(x) = [x], 0 < x < 3$ differentiable at x = 1 and x = 2.

Sol: The given function f is $f(x) = [x], 0 < x < 3$

It is known that a function f is differentiable at a point x = c in its domain if both

$\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h}$ and $\lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$ are finite and equal.

To check the differentiability of the given function at x = 1, consider the left-hand limit of f at x = 1

$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h}$ and $\lim_{h\to 0^-} \frac{|1+h|-|1|}{h}$

$= \lim_{h\to 0^-} \frac{0-1}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$

Since the left- and right-hand limits of f at $x = 1$ are not equal, f is not differentiable at $x = 1$

To check the differentiability of the given function at $x = 2$, consider the left-hand limit of f at $x = 2$

$\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h}$ and $\lim_{h\to 0^-} \frac{|2+h|-|2|}{h}$

$= \lim_{h\to 0^-} \frac{0-1}{h} = \lim_{h\to 0^-} \frac{-1}{h} = \infty$

Consider the right-hand limit of f at x=1

$\lim_{h\to 0^-} \frac{f(2+h)-f(2)}{h}$ and $\lim_{h\to 0^+} \frac{[2+h]-[2]}{h}$

$= \lim_{h\to 0^-} \frac{2-2}{h} = \lim_{h\to 0^-} 0 = 0$

Since the left- and right-hand limits of f at $x = 2$ are not equal, f is not differentiable at x= 2.