SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Continuity and Differentiability
Exercise-5.4

Q.1 Differentiate the following w.r.t. x:

$\frac{e^x}{\sin x}$

Sol: Let $y = \frac{e^x}{\sin x}$

By using the quotient rule, we obtain

$$\frac{dy}{dx} = \frac{\sin x \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(\sin x)}{\sin^2 x}$$

$$\frac{\sin x . (e^x) - e^x(\cos x)}{\sin^2 x}$$

$$= \frac{e^x(\sin x - \cos x)}{\sin^2 x}, x \neq n\pi, n \in Z$$

Q.2 Differentiate the following w.r.t. x:

$e^{\sin^{-1} x}$

Sol: Let $y = e^{\sin^{-1} x}$

By using the chain rule, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}(e^{\sin^{-1} x})$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1} x} . \frac{d}{dx}(\sin^{-1} x)$$

$$= \frac{e^{\sin^{-1} x}}{\sqrt{1 - x^2}}$$

$$\therefore \frac{dy}{dx} = \frac{e^{\sin^{-1} x}}{\sqrt{1 - x^2}}, x \in (-1, 1)$$

Q.3 Differentiate the following w. r. t. x:

e^{x^3}

Sol: Let $y = e^{x^3}$

Differentiating this relationship with respect to x, we obtain

$$\frac{dy}{dx} = \frac{d}{dx}(e^{x^3}) = e^{x^3} \frac{d}{dx}(x^3) = e^{x^3} . 3x^2 = 3x^2 e^{x^3}$$

Q.4 Differentiate the following w. r. t. x:

$\sin(\tan^{-1} e^{-x})$

Sol: Let $y = e^{x^3}$

Differentiating this relationship with respect to x, we obtain

$\frac{dy}{dx} = \frac{d}{dx}[\sin(\tan^{-1}e^{-x})]$

$= \cos(\tan^{-1}e^{-x}).\frac{d}{dx}(\tan^{-1}e^{-x})$

$= \cos(\tan^{-1}e^{-x}).\frac{d}{dx}(\tan^{-1}e^{-x})$

$= \cos(\tan^{-1}e^{-x}).\frac{1}{1+(e^{-x})^2}\frac{d}{dx}(e^{-x})$

$= \frac{\cos(\tan^{-1}e^{-x})}{1+e^{-2x}}(e^{-x}).\frac{d}{dx}(-x)$

$= \frac{e^{-x}\cos(\tan^{-1}e^{-x})}{1+e^{-2x}} \times (-1)$

$= \frac{-e^{-x}\cos(\tan^{-1}e^{-x})}{1+e^{-2x}}$

Q.5 Differentiate the following w.r.t. x:

$\log(\cos e^x)$

Sol: Let $y = \log(\cos e^x)$

By using the chain rule, we obtain

$\frac{dy}{dx} = \frac{d}{dx}[\log(\cos e^x)]$

$= \frac{1}{\cos e^x}\frac{d}{dx}(\cos e^x)$

$= \frac{1}{\cos e^x}(-\sin e^x)\frac{d}{dx}(e^x)$

$= \frac{-\sin e^x}{\cos e^x}(e^x)$

$= -e^x \tan e^x, e^x \neq (2n+1)\frac{\pi}{2}, n \in N$

Q.6 Differentiate the following w.r.t. x:

$e^x + e^{x^2} + \cdots + e^{x^5}$

Sol: $\frac{d}{dx}(e^x + e^{x^2} + \cdots + e^{x^5})$

$\Rightarrow \frac{d}{dx}(e^x) + \frac{d}{dx}(e^{x^2}) + \frac{d}{dx}(e^{x^3}) + \frac{d}{dx}(e^{x^4}) + \frac{d}{dx}(e^{x^5})$

$\Rightarrow e^x + \left[e^{x^2} \times \frac{d}{dx}(x^2)\right] + \left[e^{x^3}.\frac{d}{dx}(x^3)\right] + \left[e^{x^4}.\frac{d}{dx}(x^4)\right] + \left[e^{x^5}.\frac{d}{dx}(x^5)\right]$

$\Rightarrow e^x + (e^{x^2} \times 2x) + (e^{x^3} \times 3x^2) + (e^{x^4} \times 4x^3) + (e^{x^5} \times 5x^4)$

$\Rightarrow e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$

Q.7 Differentiate the following w. r. t. x:

$\sqrt{e^{\sqrt{x}}}, x > 0$

Sol: Let $y = \sqrt{e^{\sqrt{x}}}$

Then, $y^2 = e^{\sqrt{x}}$

Differentiating this relationship with respect to x, we obtain

$y^2 = e^{\sqrt{x}}$

$\Rightarrow 2y\frac{dy}{dx} = e^{\sqrt{x}}\frac{1}{2}.\frac{d}{dx}(\sqrt{x})$ [By applying the chain rule]

$\Rightarrow 2y\frac{dy}{dx} = e^{\sqrt{x}}\frac{1}{2}\frac{.1}{\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{e^{\sqrt{x}}}\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{e^{\sqrt{x}}}{4\sqrt{xe^{\sqrt{x}}}}, x > 0$

Q.8 Differentiate the following w. r. t. x:

$\log(\log x), x > 1$

Sol: Let $y = \log(\log x)$

Then, $y^2 = e^{\sqrt{x}}$

$\frac{dy}{dx} = \frac{d}{dx}[\log(\log x)]$

$= \frac{1}{\log x}.\frac{d}{dx}(\log x)$

$= \frac{1}{\log x}.\frac{1}{x}$

$= \frac{1}{x\log x}, x > 1$

Q.9 Differentiate the following w. r. t. x:

$\frac{\cos y}{\log x}$

Sol: Lets $y = \frac{\cos y}{\log x}$

By using the quotient rule, we obtain

$\frac{dy}{dx} = \frac{\frac{d}{dx}(\cos x)\log x - \cos x \times \frac{d}{dx}(\log x)}{(\log x)^2}$

$= \frac{-\sin x \log x - \cos x \times \frac{1}{x}}{(\log x)^2}$

$= \frac{-[x\ \log x \sin x + \cos x}{x(\log x)^2}, x > 0$

Q.10 Differentiate the following w. r. t. x:

$\cos(\log x + e^x), x > 0$

Sol: Lets $y = \cos(\log x + e^x)$

By using the quotient rule, we obtain

$\frac{dy}{dx} = -\sin(\log x + e^x)\frac{d}{dx}\ (\log x + e^x)$

$= -\sin(\log x + e^x)\frac{d}{dx}\ (\log x + e^x)$

$= -\sin(\log x + e^x)\ \left(\frac{1}{x} + e^x\right)$

$= -\left(\frac{1}{x}+e^{x}\right)\sin(\log x + e^{x}), x > 0$