SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 Differentiate the function with respect to x.

$\cos x . \cos 2x . \cos 3x$

Sol: Let $y = \cos x . \cos 2x . \cos 3x$

Taking logarithm on both the sides, we obtain

$\log y = \log(\cos x . \cos 2x . \cos 3x)$

$\Rightarrow \log y = \log(\cos x . \cos 2x . \cos 3x)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cos x}.\frac{d}{dx}(\cos x) + \frac{1}{\cos 2x}.\frac{d}{dx}(\cos 2x) + \frac{1}{\cos 3x}.\frac{d}{dx}(\cos 3x)$

$\Rightarrow \frac{dy}{dx} = y\left[-\frac{\sin x}{\cos x} - \frac{\sin 2x}{\cos 2x}\frac{d}{dx}(2x) - \frac{\sin 3x}{\cos 3x}.\frac{d}{dx}(3x)\right]$

$\therefore \frac{dy}{dx} = -\cos x . \cos 2x . \cos 3x\,[\tan x + 2\tan 2x + 3\tan 3x]$

Q.2 Differentiate the function with respect to x.

$\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Sol: Let $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Taking logarithm on both the sides, we obtain

$\log y = \log\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

$\Rightarrow \log y = \frac{1}{2}\log\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right]$

$\Rightarrow \log y = \frac{1}{2}[\log\{(x-1)(x-2)\} - \log\{(x-3)(x-4)(x-5)\}]$

$\Rightarrow \log y = \frac{1}{2}[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1}.\frac{d}{dx}(x-1) + \frac{1}{x-2}.\frac{d}{dx}(x-2) - \frac{1}{x-3}.\frac{d}{dx}(x-3) - \frac{1}{x-4}.\frac{d}{dx}(x-4) - \frac{1}{x-5}.\frac{d}{dx}(x-5)\right]$

$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$

$\therefore \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}\left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right]$

Q.3 Differentiate the function with respect to x.

$(\log x)^{\cos x}$

Sol: Let $y = (\log x)^{\cos x}$

Taking logarithm on both the sides, we obtain

$\log y = \cos x . \log(\log x)$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(\cos x) \times \log(\log x) + \cos x \times \frac{d}{dx}[(\log x)]$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\sin x \log(\log x) + \cos x \times \frac{1}{\log x}\frac{d}{dx}(\log x)$

$\Rightarrow \frac{dy}{dx} = -\sin x \log(\log x) + \frac{\cos x}{\log x} \times \frac{1}{x}$

$\therefore \frac{dy}{dx} = (\log x)^{\cos x} - \left[\frac{\cos x}{\log x} - \sin x \log(\log x)\right]$

Q.4 Differentiate the function with respect to x.

$x^x - 2^{\sin x}$

Sol: Let $y = x^x - 2^{\sin x}$

Also, let $x^x = u$ and $2^{\sin x} = v$

$\therefore y = u - v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$

$u = x^x$

Taking logarithm on both the sides, we obtain

$\log u = x \log x$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \left[\frac{d}{dx}(x) \times \log x + x \times \frac{d}{dx}(\log x)\right]$

$\Rightarrow \frac{du}{dx} = u\left[1 \times \log x + x \times \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = u\left[1 \times \log x + x \times \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^x (1 + \log x)$

$v = 2^{\sin x}$

Taking logarithm on both the sides with respect to x, we obtain

$\log v = \sin x . \log 2$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \log 2 . \frac{d}{dx}(\sin x)$

$\Rightarrow \frac{dv}{dx} = v \log 2 \cos x$

$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \log 2 \cos x$

$\therefore \frac{dy}{dx} = x^x (1 + \log x) - 2^{\sin x} \cos x \log 2$

Q.5 Differentiate the function with respect to x.

$(x + 3)^2 . (x + 4)^3 . (x + 5)^4$

Sol: Let $y = (x+3)^2.(x+4)^3.(x+5)^4$

Taking logarithm on both the sides, we obtain

$\log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$

$\Rightarrow \log y = \log(x+3)^2 + \log(x+4)^3 + \log(x+5)^4$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 2.\frac{1}{x+3}.\frac{d}{dx}(x+3) + 3.\frac{1}{x+4}.\frac{d}{dx}(x+4) + 4.\frac{1}{x+5}.\frac{d}{dx}(x+5)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$

$\Rightarrow \frac{dy}{dx} = (x+3)^2.(x+4)^3.(x+5)^4.\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$

$\Rightarrow \frac{dy}{dx} = (x+3)^2.(x+4)^3.(x+5)^4.\left[\frac{2(x+4)(x+5)+3(x+3)(x+5)+4(x+3)(x+4)}{(x+3)(x+4)(x+5)}\right]$

$\Rightarrow \frac{dy}{dx} = (x+3)^2.(x+4)^3.(x+5)^4.[2(x^2+9x+20)+3(x^2+8x+15)+4(x^2+7x+12)]$

$\Rightarrow \frac{dy}{dx} = (x+3)^2.(x+4)^3.(x+5)^4\,(9x^2+70\,x+133)$

Q.6 Differentiate the function with respect to x.

$\left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Sol: Let $y = \left(x+\frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Also, let $\left(x+\frac{1}{x}\right)^x = u$ and $v = x^{\left(1+\frac{1}{x}\right)}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ …….. (1)

Then, $u = \left(x+\frac{1}{x}\right)^x$

$\Rightarrow \log u = \log\left(x+\frac{1}{x}\right)^x$

$\Rightarrow \log u = x\log\left(x+\frac{1}{x}\right)$

Taking logarithm on both the sides, we obtain

$\log u = x\log x$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x)\times\log\left(x+\frac{1}{x}\right) + x\times\frac{d}{dx}\left[\log\left(x+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = 1\times\log\left(x+\frac{1}{x}\right) + x\times\frac{1}{\left(x+\frac{1}{x}\right)}.\frac{d}{dx}\left(x+\frac{1}{x}\right)$

$\Rightarrow \frac{du}{dx} = u\left[\log\left(x+\frac{1}{x}\right) + x\times\frac{1}{\left(x+\frac{1}{x}\right)}.\frac{d}{dx}\left(x-\frac{1}{x^2}\right)\right]$

$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\log\left(x+\frac{1}{x}\right) + \frac{\left(x-\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)}\right]$

$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x \left[\log\left(x+\frac{1}{x}\right)+\frac{x^2-1}{x^2+1}\right]$

$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right]$ .... (2)

$v = x^{\left(1+\frac{1}{x}\right)}$

$\Rightarrow \log v = \log\left[x^{\left(1+\frac{1}{x}\right)}\right]$

$\Rightarrow \log v = \left(1+\frac{1}{x}\right)\log x$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \left[\frac{d}{dx}\left(1+\frac{1}{x}\right)\right] \times \log x + \left(1+\frac{1}{x}\right).\frac{d}{dx}\log x$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left(-\frac{1}{x^2}\right)\log x + \left(1+\frac{1}{x}\right).\frac{1}{x}$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = -\frac{\log x}{x^2}+\frac{1}{x}+\frac{1}{x^2}$

$\Rightarrow \frac{dv}{dx} = v\left[\frac{-\log x+x+1}{x^2}\right]$

$\Rightarrow \frac{dv}{dx} = x^{\left(1+\frac{1}{x}\right)}\left(\frac{-\log x+x+1}{x^2}\right)$ .... (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{x^2-1}{x^2+1}+\log\left(x+\frac{1}{x}\right)\right] + x^{\left(1+\frac{1}{x}\right)}\left(\frac{-\log x+x+1}{x^2}\right)$

Q.7 Differentiate the function with respect to x.

$(\log x)^x + x^{\log x}$

Sol: Let $y = (\log x)^x + x^{\log x}$

Also, let $u = (\log x)^x$ and $v = x^{\log x}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx}$ .... (1)

Then, $u = (\log x)^x$

$\Rightarrow \log u = [\log(\log x)^x]$

$\Rightarrow \log u = x[\log(\log x)]$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x) \times \log(\log x) + x.\frac{1}{\log x}\frac{d}{dx}[\log(\log x)]$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = u\left[1 \times \log(\log x) + x.\frac{1}{\log x}\frac{d}{dx}(\log x)\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x}.\frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x \left[\frac{\log(\log x).\log x+1}{\log x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^{x-1}[1 + \log(\log x) . \log x]$ .... (2)

$v = x^{\log x}$

$\Rightarrow \log v = \log\left(x^{\log x}\right)$

$\Rightarrow \log v = \log x \log x = (\log x)^2$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}[(\log x)^2]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = 2(\log x)\ \frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 2v(\log x).\frac{1}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\ \frac{\log x}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x-1} . \log x$ .... (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = (\log x)^{x-1}[1 + \log(\log x) . \log x] + 2x^{\log x-1} . \log x$

Q.8 Differentiate the function with respect to x.

$(\sin x)^x + \sin^{-1}\sqrt{x}$

Sol: Let $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

Also, let $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ .... (1)

Then, $u = (\sin x)^x$

$\Rightarrow \log u = x\ \log(\sin x)$

Differentiating this relationship with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x) \times \log(\sin x) + x \times \frac{d}{dx}[\log(\sin x)]$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = u\left[1 \times \log(\sin x) + x.\frac{1}{\sin x}\ \frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x\left[\log(\sin x) + \frac{x}{\sin x}.\cos x\right]$

$\Rightarrow \frac{du}{dx} = (\sin x)^x\ (x\cot x + \log\sin x)$ .... (2)

$v = \sin^{-1}\sqrt{x}$

$\Rightarrow \log v = \log\left(x^{\log x}\right)$

$\Rightarrow \log v = \log x \log x = (\log x)^2$

Differentiating both sides with respect to x, we obtain

$\frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-(\sqrt{x})^2}} \frac{d}{dx}(\sqrt{x})$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x}} . \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{2\sqrt{x-x^2}}$ …….. (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = (\sin x)^x (x \cot x + \log \sin x) + \frac{1}{2\sqrt{x-x^2}}$

Q.9 Differentiate the function with respect to x.

$x^{\sin x} + (\sin x)^{\cos x}$

Sol: Let $y = x^{\sin x} + (\sin x)^{\cos x}$

Also, let $u = x^{\sin x}$ and $v = (\sin x)^{\cos x}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ …….. (1)

Then, $u = x^{\sin x}$

$\Rightarrow \log u = \log(x^{\sin x})$

$\Rightarrow \log u = \sin x \log x$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\sin x). \log x + \sin x \times \frac{d}{dx}(\log x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = u\left[\cos x \log x + \sin x . \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right]$ …….. (2)

$v = (\sin x)^{\cos x}$

$\Rightarrow \log v = \log(\sin x)^{\cos x}$

$\Rightarrow \log v = \cos x \log(\sin x)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \cos x \log(\sin x)$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}(\cos x). \log(\sin x) + \cos x \times \frac{d}{dx}[\log(\sin x)]$

$\Rightarrow \frac{dv}{dx} = v\left[- \sin x . \log(\sin x) + \cos x . \frac{1}{\sin x} . \frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\left[- \sin x . \log(\sin x) + \frac{\cos x}{\sin x} \cos x\right]$

$\Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} [\cot x \cos x + \sin x \log \sin x)]$ …….. (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left[\cos x \log x + \frac{\sin x}{x}\right] + (\sin x)^{\cos x}\ [\cot x \cos x + \sin x \log \sin x)]$

Q.10 Differentiate the function with respect to x.

$x^{x \cos x} + \frac{x^2+1}{x^2-1}$

Sol: Let $y = x^{x \cos x} + \frac{x^2+1}{x^2-1}$

Also, let $u = x^{x \cos x}$ and $v = \frac{x^2+1}{x^2-1}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ …….. (1)

Then, $u = x^{x \cos x}$

$\Rightarrow \log u = \log(x^{x \cos x})$

$\Rightarrow \log u = x \cos x \log x$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x).\cos x + \frac{d}{dx}(\cos x).\log x + x \cos x.\frac{d}{dx}(\log x)$

$\Rightarrow \frac{1}{u}\frac{du}{dx} = u\left[1.\cos x + x.(-\sin x).\log x + x \cos x.\frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^{x \cos x}(\cos x \log x - x \sin x \log x + \cos x)$

$\Rightarrow \frac{du}{dx} = x^{x \cos x}[\cos x (1 + \log x) - x \sin x \log x]$ …….. (2)

$v = \frac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log \frac{x^2+1}{x^2-1}$

$\Rightarrow \log v = \log(x^2 + 1) - \log(x^2 - 1)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \frac{2x}{x^2+1} - \frac{2x}{x^2-1}$

$\Rightarrow \frac{dv}{dx} = v\left[\frac{2x(x^2-1)-2x(x^2+1)}{(x^2+1)(x^2-1)}\right]$

$\Rightarrow \frac{dv}{dx} = \frac{x^2+1}{x^2-1} \times \left[\frac{-4x}{(x^2+1)(x^2-1)}\right]$

$\Rightarrow \frac{dv}{dx} = \frac{-4x}{(x^2-1)^2}$ …….. (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = x^{x \cos x}[\cos x (1 + \log x) - x \sin x \log x] - \frac{-4x}{(x^2-1)^2}$

Q.11 Differentiate the function with respect to x.

$(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

Sol: Let $y = (x \cos x)^x + (x \sin x)^{\frac{1}{x}}$

Also, let $u = (x\cos x)^x$ and $v = (x\sin x)^{\frac{1}{x}}$

$\therefore y = u + v$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ .... (1)

Then, $u = (x\cos x)^x$

$\Rightarrow \log u = x\log(x\cos x)$

$\Rightarrow \log u = x\,[\log x + \log\cos x]$

$\Rightarrow \log u = x\ \log x + x\ \log\cos x$

Differentiating both sides with respect to x, we obtain

$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(x\ \log x) + \frac{d}{dx}(x\log\cos x)$

$\Rightarrow \frac{du}{dx} = u\left[\left\{\log x\,.\frac{d}{dx}(x) + x.\frac{d}{dx}(\log x)\right\} + \left\{\log\cos x\,.\frac{d}{dx}(x) + x.\frac{d}{dx}(\log\cos x)\right\}\right]$

$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[\left\{\log x\,.\,1 + x.\frac{1}{x}\right\} + \left\{\log\cos x\,.\,1 + \frac{1}{\cos x}.\frac{d}{dx}(\cos x)\right\}\right]$

$\Rightarrow \frac{du}{dx} = (x\cos x)^x\left[(\log x\,.\,1) + \left\{\log\cos x\,.\,1 + \frac{x}{\cos x}.(-\sin x)\right\}\right]$

$\Rightarrow \frac{du}{dx} = (x\cos x)^x[(1 + \log x) + (\log\cos x - x\tan x]$

$\Rightarrow \frac{du}{dx} = (x\cos x)^x[1 - x\tan x + (\log x + \log\cos x]$

$\Rightarrow \frac{du}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)]$ .... (2)

$v = (x\sin x)^{\frac{1}{x}}$

$\Rightarrow \log v = \log(x\sin x)^{\frac{1}{x}}$

$\Rightarrow \log v = \frac{1}{x}\log(x\sin x)$

$\Rightarrow \log v = \frac{1}{x}\log x + \frac{1}{x}\log\sin x$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{x}\log x\right) + \frac{d}{dx}\left[\frac{1}{x}\ \log(\sin x)\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x\,.\frac{d}{dx}\left(\frac{1}{x}\right)\frac{1}{x}\frac{d}{dx}(\log x)\right] + \left[\log(\sin x)\frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x}.\frac{d}{dx}\{\log(\sin x)\}\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x\,.\frac{d}{dx}\left(-\frac{1}{x^2}\right)\frac{1}{x}.\frac{1}{x}\right] + \left[\log(\sin x)\left(-\frac{1}{x^2}\right) + \frac{1}{x}.\frac{1}{\sin x}.\frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x\,.\frac{d}{dx}\left(-\frac{1}{x^2}\right)\frac{1}{x}.\frac{1}{x}\right] + \left[\log(\sin x)\left(-\frac{1}{x^2}\right) + \frac{1}{x}.\frac{1}{\sin x}.\frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2} + \frac{-\log(\sin x)+x\cot x}{x^2}\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x-\log(\sin x)+x\cot x}{x^2}\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right]$ .... (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right]$

Q.12 Find $\frac{dy}{dx}$ of function.

$x^y + y^x = 1$

Sol: The given function is $x^y + y^x = 1$

Let $u = x^y$ and $v = y^x$

then, the function becomes $u + v = 1$

$\therefore \frac{du}{dx} + \frac{dv}{dx}$ …….. (1)

Then, $u = x^y$

$\Rightarrow \log u = \log(x^y)$

$\Rightarrow \log u = y\log x$

Differentiating both sides with respect to x, we obtain

$\frac{1}{u}\frac{du}{dx} = \log x\frac{dy}{dx} + y.\frac{d}{dx}(\log x)$

$\Rightarrow \frac{du}{dx} = u\left[\log x\frac{dy}{dx} + y.\frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^y\left(\log x\frac{dy}{dx} + \frac{y}{x}\right)$ …….. (2)

$v = y^x$

$\Rightarrow \log v = \log(y^x)$

$\Rightarrow \log v = x\log y$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \log y.\frac{d}{dx}(x) + \frac{d}{dx}(\log y)$

$\Rightarrow \frac{dv}{dx} = v\left(\log y.1 + x.\frac{1}{y}.\frac{dy}{dx}\right)$

$\Rightarrow \frac{dv}{dx} = y^x\left(\log y + \frac{x}{y}.\frac{dy}{dx}\right)$ …….. (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow x^y\left(\log x\frac{dy}{dx} + \frac{y}{x}\right)\ y^x\left(\log y + \frac{x}{y}.\frac{dy}{dx}\right) = 0$

$= (x^y\ \log x + x\,y^{x-1})\frac{dy}{dx} = -(y^x\log y + y\,x^{y-1})$

$\therefore \frac{dy}{dx} = -\frac{(y^x\log y + y\,x^{y-1})}{(x^y\ \log x + x\,y^{x-1})}$

Q.13 Find $\frac{dy}{dx}$ of function.

$y^x = x^y$

Sol: The given function is $y^x = x^y$

Taking logarithm on both the sides, we obtain

$x\log y = y\log x$

Differentiating both sides with respect to x, we obtain

$\log y \cdot \frac{d}{dx}(x) + x.\frac{d}{dx}(\log y) = \log x \cdot \frac{d}{dx}(y) + y.\frac{d}{dx}(\log x)$

$\Rightarrow \log y \cdot 1 + x.\frac{1}{y}.\frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + y.\frac{1}{x}$

$\Rightarrow \log y + \frac{x}{y}.\frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + \frac{y}{x}$

$\Rightarrow \left(\frac{x}{y} - \log y\right)\frac{dy}{dx} = \frac{y}{x} - \log y$

$\Rightarrow \left(\frac{x-\log y}{y}\right)\frac{dy}{dx} = \frac{y-\log y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y-\log y}{x-\log y}\right)$

Q.14 Find $\frac{dy}{dx}$ of function.

$(\cos x)^y = (\cos y)^x$

Sol: The given function is $(\cos x)^y = (\cos y)^x$

Taking logarithm on both the sides, we obtain

$y \ \log \cos x = x \log \cos y$

Differentiating both sides with respect to x, we obtain

$\log \cos x \cdot \frac{dy}{dx} + y.\frac{d}{dx}(\log \cos x) = \log y \cdot \frac{d}{dx}(x) + x.\frac{d}{dx}(\log \cos y)$

$\Rightarrow \log \cos x \ \frac{dy}{dx} + y.\frac{1}{\cos x}.\frac{d}{dx}(\cos x) = \log \cos y \cdot 1 + x.\frac{1}{\cos y}\frac{d}{dx}(\cos y)$

$\Rightarrow \log \cos x \ \frac{dy}{dx} + \frac{y}{\cos x}.(-\sin x) = \log \cos y + \frac{x}{\cos y}(-\sin y)\frac{dy}{dx}$

$\Rightarrow \log \cos x \ \frac{dy}{dx} - y \tan x = \log \cos y - x \tan y \frac{dy}{dx}$

$\Rightarrow (\log \cos x + x \tan y)\frac{dy}{dx} = y \text{ yan } x + \log \cos y$

$\therefore \ \frac{dy}{dx} = \frac{y \text{ yan } x + \log \cos y}{\log \cos x + x \tan y}$

Q.15 Find $\frac{dy}{dx}$ of function.

$xy = e^{(x-y)}$

Sol: The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$\log(xy) = \log e^{(x-y)}$

$\Rightarrow \log x + \log y = (x-y)\log e$

$\Rightarrow \log x + \log y = (x-y) \times 1$

$\Rightarrow \log x + \log y = (x-y)$

Differentiating both sides with respect to x, we obtain

$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) + x.\frac{d}{dx}(x) - \frac{dy}{dx}$

$\Rightarrow \frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \left(1 + \frac{1}{y}\right)\frac{dy}{dx} = 1 - \frac{1}{x}$

$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$

$\therefore \frac{dy}{dx} = \frac{y\,(x-1)}{x\,(y+1)}$

Q.16 Find the derivative of the function given by $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Sol: The given function is $f(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking logarithm on both the sides, we obtain

$\log f(x) = \log(1+x) + \log(1+x^2) + \log(1+x^4) + \log(1+x^8)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{f(x)}.\frac{d}{dx}[f(x)] = \frac{d}{dx}\log(1+x) + \frac{d}{dx}\log(1+x^2) + \frac{d}{dx}\log(1+x^4) + \frac{d}{dx}\log(1+x^8)$

$\Rightarrow \frac{1}{f(x)}.f'(x) = \frac{1}{1+x}\frac{d}{dx}(1+x) + \frac{1}{1+x^2}\frac{d}{dx}(1+x^2) + \frac{1}{1+x^4}\frac{d}{dx}(1+x^4) + \frac{1}{1+x^8}\frac{d}{dx}(1+x^8)$

$\Rightarrow f'(x) = f(x)\left[\frac{1}{1+x} + \frac{1}{1+x^2}\,2x + \frac{1}{1+x^4}.4x^3 + \frac{1}{1+x^8}\,.8x^7\right]$

$\therefore\ f'(x) = (1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x} + \frac{2x}{1+x^2} + \frac{4x^3}{1+x^4} + \frac{8x^7}{1+x^8}\right]$

Hence, $f'(1) = (1+1)(1+1^2)(1+1^4)(1+1^8)\left[\frac{1}{1+1} + \frac{2\times1}{1+1^2} + \frac{4\times1^3}{1+1^4} + \frac{8\times1^7}{1+1^8}\right]$

$= 2\times2\times2\times2\left[\frac{1}{2} + \frac{2}{2} + \frac{4}{2} + \frac{8}{2}\right]$

$= 16\times\left[\frac{1+2+4+8}{2}\right]$

$= 16\times\frac{15}{2} = 20$

Q.17 Differentiate $(x^2 - 5x + 8)(x^3 + 7x + 9)$ in three ways mentioned below

(i) By using product rule.

(ii) By expanding the product to obtain a single polynomial.

(iii) By logarithmic differentiation.

Do they all give the same answer?

Sol: Let $x^2 - 5x + 8 = u$ and $x^3 + 7x + 9 = v$

(i) Let $x^2 - 5x + 8 = u$ and $x^3 + 7x + 9 = v$

$\therefore y = uv$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}.v + u.\frac{dv}{dx}$ (By using product rule)

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^2 - 5x + 8).(x^3 + 7x + 9) + (x^2 - 5x + 8).\frac{d}{dx}(x^3 + 7x + 9)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(2x - 5).(x^3 + 7x + 9) + (x^2 - 5x + 8).\frac{d}{dx}(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = 2x(x^3 + 7x + 9) - 5(x^3 + 7x + 9) + x^2(3x^2 + 7) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = (2x^4 + 14x^2 + 18x) - (5x^3 + 35x + 45) + (3x^4 + 7x^2) - 15x^3 - 35x + 24x^2 + 56$

$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(ii) $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

$= x^2(x^3 + 7x + 9) - 5x(x^3 + 7x + 9) + 8(x^3 + 7x + 9)$

$= x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72)$

$= \frac{d}{dx}(x^5) - \frac{d}{dx}(5x^4) + \frac{d}{dx}(15x^3) - \frac{d}{dx}(26x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(72)$

$= 5x^4 - 5 \times 4x^3 + 15 \times 3x^2 - 26 \times 2x + 11 \times 1 + 0$

$= 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(iii) $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

Taking logarithm on both the sides, we obtain

$\log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log(x^2 - 5x + 8) + \frac{d}{dx}\log(x^3 + 7x + 9)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2-5x+8}.\frac{d}{dx}(x^2 - 5x + 8) + \frac{1}{x^3+7x+9}.\frac{d}{dx}(x^3 + 7x + 9)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x^2-5x+8}.\times (2x - 5) + \frac{1}{x^3+7x+9} \times (3x^2 + 7)\right]$

$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9}\right]$

$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\frac{(2x-5)(x^3+7x+9)}{x^2-5x+8} + \frac{(3x^2+7)(x^2-5x+8)}{x^3+7x+9}\right]$

$\Rightarrow \frac{dy}{dx} = 2x(x^3 + 7x + 9) - 5(x^3 + 7x + 9) + x^2(3x^2 + 7) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = (2x^4 + 14x^2 + 18x) - (5x^3 + 35x + 45) + (3x^4 + 7x^2) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

From the above three observations, it can be concluded that all the results of $\frac{dy}{dx}$ are same.

Q.18 If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}\ v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$ in two ways-first by repeated application of product rule, second by logarithmic differentiation.

Sol: Let $y = u.v.w = u.(v.w)$

By applying product rule, we obtain

$\frac{dy}{dx} = \frac{du}{dx}.v.w = u.\frac{d}{dx}(v.w)$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}.v.w = u.\left[\frac{dv}{dx}.w + v.\frac{dw}{dx}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}.v.w = u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

By taking logarithm on both sides of the equation $y = u.v.w$, we obtain

$\log y = \log u . \log v . \log w$

Differentiating both sides with respect to x, we obtain

$\log y = \log u . \log v . \log w$

$\frac{1}{y}.\frac{dy}{dx} = \frac{d}{dx}(\log u).\frac{d}{dx}(\log v).\frac{d}{dx}(\log w)$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = u.v.w\left(\frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}\right)$

$\therefore \frac{dy}{dx} = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$