SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = 2at^2, \quad y = at^4$

Sol: The given equations are $x = 2at^2,\ and\ \ y = at^4$

Then, $\frac{dx}{dt} = \frac{d}{dt}(2at^2) = 2a.\frac{d}{dt}(t^2) = 2a.2t = 4at$

$\frac{dy}{dt} = \frac{d}{dt}(at^4) = a.\frac{d}{dt}(t^4) = a.4.t^3 = 4at^3$

$\therefore\ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4at^3}{4at} = t^2$

Q.2 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = a\cos\theta, y = b\cos\theta$

Sol: The given equations are x = a cos θ and y = b cos θ

Then, $\frac{dx}{d\theta} = \frac{d}{d\theta}(\mathrm{a}\cos\theta) = a(-\sin\theta) = -a\sin\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(b\cos\theta) = b(-\sin\theta) = -b\sin\theta$

$\therefore\ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-b\sin\theta}{-a\sin\theta} = \frac{b}{a}$

Q.3 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = sin\,t, y = cos\,2t$

Sol: The given equations are x = sin t and y = cos 2t

Then, $\frac{dx}{dt} = \frac{d}{dt}(\sin t) = \cos t$

$\frac{dy}{dt} = \frac{d}{dt}(cos\,2t) = a - sin\,2t\frac{d}{dt}(2t) = -2sin\,2t$

$\therefore\ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-sin\,2t}{\cos t} = \frac{-2.2\sin t\cos t}{\cos t} = -4\sin t$

Q.4 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = 4t, y = \frac{4}{t}$

Sol: The given equations are $x = 4t\ and\ y = \frac{4}{t}$

Then, $\frac{dx}{dt} = \frac{d}{dt}(4t) = 4$

$\frac{dy}{dt} = \frac{d}{dt}\left(\frac{4}{t}\right) = 4\frac{d}{dt}\left(\frac{1}{t}\right) = 4.\left(\frac{-1}{t^2}\right) = \frac{-4}{t^2}$

$\therefore \ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{-4}{t^2}\right)}{4} = \frac{-1}{t^2}$

Q.5 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = cos\,\theta - \cos 2\theta\,, y = sin\,\theta - \sin 2\theta$

Sol: The given equations are $x = cos\,\theta - \cos 2\theta$ and $y = sin\,\theta - \sin 2\theta$

Then, $\frac{dx}{d\theta} = \frac{d}{d\theta}(cos\,\theta - \cos 2\theta) = \frac{d}{d\theta}(\cos\theta) - \frac{d}{d\theta}(\cos 2\theta)$

$= -sin\,\theta - (-2\sin 2\theta) = 2\sin 2\theta - \sin\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(sin\,\theta - \sin 2\theta) = (\sin\theta) - \frac{d}{d\theta}(\sin 2\theta)$

$= cos\,\theta - \cos 2\theta$

$\therefore \ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{cos\,\theta - \cos 2\theta}{2sin\,2\theta - \sin\theta}$

Q.6 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = a(\theta - \sin\theta), y = a(1 + \cos\theta)$

Sol: The given equations are $x = a(\theta - \sin\theta)\ and\ y = a(1 + \cos\theta)$

Then, $\frac{dx}{d\theta} = \left[\frac{d}{d\theta}(\theta) - \frac{d}{d\theta}(\sin\theta)\right] = a(1 - \cos\theta)$

$\frac{dy}{d\theta} = \left[\frac{d}{d\theta}(1) + \frac{d}{d\theta}(\cos\theta)\right] = a[0 + (-\sin\theta)] = -a\sin\theta$

$\therefore \ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{-\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = -\cot\frac{\theta}{2}$

Q.7 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}\ , y = \frac{cos^3\,t}{\sqrt{\cos 2t}}$

Sol: The given equations are $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}\ and\ y = \frac{cos^3\,t}{\sqrt{\cos 2t}}$

Then, $\frac{dx}{dt} = \frac{d}{dt}\left[\frac{\sin^3 t}{\sqrt{\cos 2t}}\right]$

$= \frac{\sqrt{\cos 2t}.\frac{d}{dt}(\sin^3 t) - \sin^3 t\,\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$

$= \frac{\sqrt{\cos 2t}.\ 3\sin^2 t\ .\ \frac{d}{dt}(\sin t) - \sin^3 t \times \frac{1}{2\sqrt{\cos 2t}}\ .\ \frac{d}{dt}(\cos 2t)}{\cos 2t}$

$= \frac{3\sqrt{\cos 2t}.3\ \sin^2 t\ .\cos t\ - \frac{\sin^3 t}{2\sqrt{\cos 2t}}.\ (-2\sin 2t)}{\cos 2t}$

$= \frac{3\cos 2t\ .\ \sin^2 t\ .\cos t + \sin^3 t\ \sin 2t}{\cos 2t\sqrt{\cos 2t}}$

$\frac{dy}{dt} = \frac{d}{dt}\left[\frac{cos^3\,t}{\sqrt{\cos 2t}}\right]$

$= \frac{\sqrt{\cos 2t}.\frac{d}{dt}(\cos^3 t) - \cos^3 t\,\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t}$

$$= \frac{\sqrt{\cos 2t}.\ 3\ cos\ t\ .\ \frac{d}{dt}(\cos t) - cos^3\ t \times \frac{1}{2\sqrt{\cos 2t}}.\ \frac{d}{dt}(\cos 2t)}{\cos 2t}$$

$$= \frac{3\sqrt{\cos 2t}.\sin^2 t\ .(-\sin t)\ \ -\cos^3 t\ \ \frac{1}{2\sqrt{\cos 2t}}.\ (-2\sin 2t)}{\cos 2t}$$

$$= \frac{-3\cos 2t\ .\ \ cos^2\ t\ .\sin t + \cos^3\ t\ \ \sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\therefore\ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-3\cos 2t\ .\ \ cos^2\ t\ .\sin t + \cos^3\ t\ \ \sin 2t}{3\cos 2t\ .\ \ \sin^2 t\ .\cos t + \sin^3 t\ \ \sin 2t}$$

$$= \frac{-3\cos 2t\ .\ \ cos^2\ t\ .\sin t + \cos^3 t\ \ (2\sin t\cos t)}{3\cos 2t\ .\ \ \sin^2 t\ .\cos t + \sin^3 t\ \ (2\sin t\cos t)}$$

$$= \frac{\left[-3\cos 2t\ .\ \ \cos t + 2\cos^3 t\right](\sin t\cos t)}{[3\cos 2t\ .\ \ \sin t + 2\sin^3 t\ ](\sin t\cos t)}$$

$$= \frac{\left[-3\left(2\cos^2 t - 1\right)\cos t + 2\cos^3 t\ \right]}{[3(1-2\sin^2 t)\sin t + 2\sin^3 t\ ]}$$

$$= \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t}$$

$$= -\cot 3t$$

Q.8 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

$x\ = a\left(\cos t + \log\tan\frac{t}{2}\right), y\ = a\sin t$

Sol: The given equations are $x\ = a\left(\cos t + \log\tan\frac{t}{2}\right)\ \ and\ y\ = a\sin t$

Then, $\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\left(\log\tan\frac{t}{2}\right)\right]$

$$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \cot\frac{t}{2}.\sec^2\frac{t}{2}\frac{d}{dt}\left(\frac{t}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}.\frac{1}{\cos^2\frac{t}{2}} \times \left(\frac{1}{2}\right)\right]$$

$$= a\left[-\sin t + \frac{1}{2\ \sin\frac{t}{2}\cos^2\frac{t}{2}}\right]$$

$$= a\left[-\sin t + \frac{1}{\sin t}\right]$$

$$= a\left[\frac{-\sin^2 t + 1}{\sin t}\right]$$

$$= a\frac{\cos^2 t}{\sin t}$$

$$\frac{dy}{dt} = \frac{d}{dt}(\sin t) = a\cos t$$

$$\therefore\ \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a\cos t}{\left(a\frac{\cos^2 t}{\sin t}\right)} = \frac{\sin t}{\cos t} = \tan t$$

Q.9 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a\sec\theta, y = b\tan\theta$

Sol: The given equations are $x = a\sec\theta$ *and* $y = b\tan\theta$

Then, $\frac{dx}{d\theta} = a.\frac{d}{d\theta}(\sec\theta) = a\sec\theta\tan\theta$

$\frac{dy}{d\theta} = a.\frac{d}{d\theta}(\tan\theta) = b\sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b}{a}a\sec\theta\cot\theta = \frac{b\cos\theta}{a\cos\theta\sin\theta} = \frac{b}{a}\times\frac{1}{\sin\theta} = \frac{b}{a}\ cosec\ \theta$

Q.10 If x and y are connected parametrically by the equation, without eliminating the parameter, find $\frac{dy}{dx}$.

$x = a(\cos\theta + \theta\sin\theta), y = a(\sin\theta - \theta\cos\theta)$

Sol: The given equations are $x = a(\cos\theta + \theta\sin\theta)$ *and* $y = a(\sin\theta - \theta\cos\theta)$

Then, $\frac{dx}{d\theta} = a\left[\frac{d}{d\theta}\cos\theta + \frac{d}{d\theta}(\theta\sin\theta)\right] = a\left[-\sin\theta + \theta\frac{d}{d\theta}(\sin\theta) + \sin\theta\frac{d}{d\theta}(\theta)\right]$

$= a[-\sin\theta + \cos\theta + \sin\theta] = a\theta\cos\theta$

$\frac{dy}{d\theta} = a\left[\frac{d}{d\theta}\sin\theta + \frac{d}{d\theta}(\theta\cos\theta)\right] = a\left[\cos\theta - \left\{\theta\frac{d}{d\theta}(\cos\theta) + \cos\theta\frac{d}{d\theta}(\theta)\right\}\right]$

$= a[\cos\theta + \theta\sin\theta - \cos\theta] = a\theta\sin\theta$

$\therefore \frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$

Q.11 If $x = \sqrt{a^{\sin^{-1}t}}, y = \sqrt{a^{cos^{-1}t}}$

Sol: The given equations are $x = \sqrt{a^{\sin^{-1}t}}$ *and* $y = \sqrt{a^{cos^{-1}t}}$

$x = \sqrt{a^{\sin^{-1}t}}$ *and* $y = \sqrt{a^{cos^{-1}t}}$

$x = \left(a^{\sin^{-1}t}\right)^{\frac{1}{2}}$ *and* $y = \left(a^{\cos^{-1}t}\right)^{\frac{1}{2}}$

Taking logarithm on both sides, we obtain

$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{x}\log x\right) + \frac{d}{dx}\left[\frac{1}{x}\ \log(\sin x)\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x.\frac{d}{dx}\left(\frac{1}{x}\right)\frac{1}{x}\frac{d}{dx}(\log x)\right] + \left[\log(\sin x)\frac{d}{dx}\left(\frac{1}{x}\right) + \frac{1}{x}.\frac{d}{dx}\{\log(\sin x)\}\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x.\frac{d}{dx}\left(-\frac{1}{x^2}\right)\frac{1}{x}.\frac{1}{x}\right] + \left[\log(\sin x)\left(-\frac{1}{x^2}\right) + \frac{1}{x}.\frac{1}{\sin x}.\frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \left[\log x.\frac{d}{dx}\left(-\frac{1}{x^2}\right)\frac{1}{x}.\frac{1}{x}\right] + \left[\log(\sin x)\left(-\frac{1}{x^2}\right) + \frac{1}{x}.\frac{1}{\sin x}.\frac{d}{dx}(\sin x)\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x}{x^2} + \frac{-\log(\sin x)+x\cot x}{x^2}\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log x-\log(\sin x)+x\cot x}{x^2}\right]$

$\Rightarrow \frac{dv}{dx} = (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right]$ …….. (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow \frac{dy}{dx} = (x\cos x)^x[1 - x\tan x + \log(x\cos x)] + (x\sin x)^{\frac{1}{x}}\left[\frac{1-\log(x\sin x)+x\cot x}{x^2}\right]$

Q.12 Find $\frac{dy}{dx}$ of function.

$x^y + y^x = 1$

Sol: The given function is $x^y + y^x = 1$

Let $u = x^y$ and $v = y^x$

then, the function becomes $u + v = 1$

$\therefore \frac{du}{dx} + \frac{dv}{dx}$ .... (1)

Then, $u = x^y$

$\Rightarrow \log u = \log(x^y)$

$\Rightarrow \log u = y\log x$

Differentiating both sides with respect to x, we obtain

$\frac{1}{u}\frac{du}{dx} = \log x\frac{dy}{dx} + y.\frac{d}{dx}(\log x)$

$\Rightarrow \frac{du}{dx} = u\left[\log x\frac{dy}{dx} + y.\frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = x^y\left(\log x\frac{dy}{dx} + \frac{y}{x}\right)$ .... (2)

$v = y^x$

$\Rightarrow \log v = \log(y^x)$

$\Rightarrow \log v = x\log y$

Differentiating both sides with respect to x, we obtain

$\frac{1}{v}\frac{dv}{dx} = \log y.\frac{d}{dx}(x) + \frac{d}{dx}(\log y)$

$\Rightarrow \frac{dv}{dx} = v\left(\log y.1 + x.\frac{1}{y}.\frac{dy}{dx}\right)$

$\Rightarrow \frac{dv}{dx} = y^x\left(\log y + \frac{x}{y}.\frac{dy}{dx}\right)$ .... (3)

Therefore, from (1), (2), and (3), we obtain

$\Rightarrow x^y\left(\log x\frac{dy}{dx} + \frac{y}{x}\right)\ y^x\left(\log y + \frac{x}{y}.\frac{dy}{dx}\right) = 0$

$= (x^y\ \log x + x\,y^{x-1})\frac{dy}{dx} = -(y^x\log y + y\,x^{y-1})$

$\therefore \frac{dy}{dx} = -\frac{(y^x\log y + y\,x^{y-1})}{(x^y\ \log x + x\,y^{x-1})}$

Q.13 Find $\frac{dy}{dx}$ of function.

$y^x = x^y$

Sol: The given function is $y^x = x^y$

Taking logarithm on both the sides, we obtain

$x\log y = y\log x$

Differentiating both sides with respect to x, we obtain

$\log y \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log y) = \log x \cdot \frac{d}{dx}(y) + y \cdot \frac{d}{dx}(\log x)$

$\Rightarrow \log y \cdot 1 + x \cdot \frac{1}{y} \cdot \frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + y \cdot \frac{1}{x}$

$\Rightarrow \log y + \frac{x}{y} \cdot \frac{dy}{dx} = \log x \cdot \frac{dy}{dx} + \frac{y}{x}$

$\Rightarrow \left(\frac{x}{y} - \log y\right)\frac{dy}{dx} = \frac{y}{x} - \log y$

$\Rightarrow \left(\frac{x - \log y}{y}\right)\frac{dy}{dx} = \frac{y - \log y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{y - \log y}{x - \log y}\right)$

Q.14 Find $\frac{dy}{dx}$ of function.

$(\cos x)^y = (\cos y)^x$

Sol: The given function is $(\cos x)^y = (\cos y)^x$

Taking logarithm on both the sides, we obtain

$y \log \cos x = x \log \cos y$

Differentiating both sides with respect to x, we obtain

$\log \cos x \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}(\log \cos x) = \log y \cdot \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(\log \cos y)$

$\Rightarrow \log \cos x \frac{dy}{dx} + y \cdot \frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x) = \log \cos y \cdot 1 + x \cdot \frac{1}{\cos y}\frac{d}{dx}(\cos y)$

$\Rightarrow \log \cos x \frac{dy}{dx} + \frac{y}{\cos x} \cdot (-\sin x) = \log \cos y + \frac{x}{\cos y}(-\sin y)\frac{dy}{dx}$

$\Rightarrow \log \cos x \frac{dy}{dx} - y \tan x = \log \cos y - x \tan y \frac{dy}{dx}$

$\Rightarrow (\log \cos x + x \tan y)\frac{dy}{dx} = y \text{ yan } x + \log \cos y$

$\therefore \frac{dy}{dx} = \frac{y \text{ yan } x + \log \cos y}{\log \cos x + x \tan y}$

Q.15 Find $\frac{dy}{dx}$ of function.

$xy = e^{(x-y)}$

Sol: The given function is $xy = e^{(x-y)}$

Taking logarithm on both the sides, we obtain

$\log(xy) = \log e^{(x-y)}$

$\Rightarrow \log x + \log y = (x - y)\log e$

$\Rightarrow \log x + \log y = (x - y) \times 1$

$\Rightarrow \log x + \log y = (x - y)$

Differentiating both sides with respect to x, we obtain

$\frac{d}{dx}(\log x) + \frac{d}{dx}(\log y) + x \cdot \frac{d}{dx}(x) - \frac{dy}{dx}$

$\Rightarrow \frac{1}{x}+\frac{1}{y}\frac{dy}{dx}=1-\frac{dy}{dx}$

$\Rightarrow \left(1+\frac{1}{y}\right)\frac{dy}{dx}=1-\frac{1}{x}$

$\Rightarrow \left(\frac{y+1}{y}\right)\frac{dy}{dx}=\frac{x-1}{x}$

$\therefore \frac{dy}{dx}=\frac{y\,(x-1)}{x\,(y+1)}$

Q.16 Find the derivative of the function given by $f(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)$ and hence find $f'(1)$.

Sol: The given function is $f(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)$

Taking logarithm on both the sides, we obtain

$\log f(x)=\log(1+x)+\log(1+x^2)+\log(1+x^4)+\log(1+x^8)$

Differentiating both sides with respect to x, we obtain

$\frac{1}{f(x)}.\frac{d}{dx}[f(x)]=\frac{d}{dx}\log(1+x)+\frac{d}{dx}\log(1+x^2)+\frac{d}{dx}\log(1+x^4)+\frac{d}{dx}\log(1+x^8)$

$\Rightarrow \frac{1}{f(x)}.f'(x)=\frac{1}{1+x}\frac{d}{dx}(1+x)+\frac{1}{1+x^2}\frac{d}{dx}(1+x^2)+\frac{1}{1+x^4}\frac{d}{dx}(1+x^4)+\frac{1}{1+x^8}\frac{d}{dx}(1+x^8)$

$\Rightarrow f'(x)=f(x)\left[\frac{1}{1+x}+\frac{1}{1+x^2}\,2x+\frac{1}{1+x^4}.4x^3+\frac{1}{1+x^8}\,.8x^7\right]$

$\therefore\ f'(x)=(1+x)(1+x^2)(1+x^4)(1+x^8)\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+\frac{4x^3}{1+x^4}+\frac{8x^7}{1+x^8}\right]$

Hence, $f'(1)=(1+1)(1+1^2)(1+1^4)(1+1^8)\left[\frac{1}{1+1}+\frac{2\times1}{1+1^2}+\frac{4\times1^3}{1+1^4}+\frac{8\times1^7}{1+1^8}\right]$

$=2\times2\times2\times2\left[\frac{1}{2}+\frac{2}{2}+\frac{4}{2}+\frac{8}{2}\right]$

$=16\times\left[\frac{1+2+4+8}{2}\right]$

$=16\times\frac{15}{2}=20$

Q.17 Differentiate $(x^2-5x+8)(x^3+7x+9)$ in three ways mentioned below

(i) By using product rule.

(ii) By expanding the product to obtain a single polynomial.

(iii) By logarithmic differentiation.

Do they all give the same answer?

Sol: Let $x^2-5x+8=u$ and $x^3+7x+9=v$

(i) Let $x^2-5x+8=u$ and $x^3+7x+9=v$

$\therefore y=uv$

$\Rightarrow \frac{dy}{dx}=\frac{du}{dx}.v+u.\frac{dv}{dx}$ (By using product rule)

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(x^2-5x+8).(x^3+7x+9)+(x^2-5x+8).\frac{d}{dx}(x^3+7x+9)$

$\Rightarrow \frac{dy}{dx}=\frac{d}{dx}(2x-5).(x^3+7x+9)+(x^2-5x+8).\frac{d}{dx}(3x^2+7)$

$\Rightarrow \frac{dy}{dx}=2x(x^3+7x+9)-5(x^3+7x+9)+x^2(3x^2+7)-5x(3x^2+7)+8(3x^2+7)$

$\Rightarrow \frac{dy}{dx} = (2x^4 + 14x^2 + 18x) - (5x^3 + 35x + 45) + (3x^4 + 7x^2) - 15x^3 - 35x + 24x^2 + 56$

$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(ii) $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

$= x^2(x^3 + 7x + 9) - 5x(x^3 + 7x + 9) + 8(x^3 + 7x + 9)$

$= x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(x^5 - 5x^4 + 15x^3 - 26x^2 + 11x + 72)$

$= \frac{d}{dx}(x^5) - \frac{d}{dx}(5x^4) + \frac{d}{dx}(15x^3) - \frac{d}{dx}(26x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(72)$

$= 5x^4 - 5 \times 4x^3 + 15 \times 3x^2 - 26 \times 2x + 11 \times 1 + 0$

$= 5x^4 - 20x^3 + 45x^2 - 52x + 11$

(iii) $y = (x^2 - 5x + 8)(x^3 + 7x + 9)$

Taking logarithm on both the sides, we obtain

$\log y = \log(x^2 - 5x + 8) + \log(x^3 + 7x + 9)$

Differentiating both sides with respect to x, we obtain

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\log(x^2 - 5x + 8) + \frac{d}{dx}\log(x^3 + 7x + 9)$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2-5x+8}.\frac{d}{dx}(x^2 - 5x + 8) + \frac{1}{x^3+7x+9}.\frac{d}{dx}(x^3 + 7x + 9)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x^2-5x+8}.\times (2x - 5) + \frac{1}{x^3+7x+9} \times (3x^2 + 7)\right]$

$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\frac{2x-5}{x^2-5x+8} + \frac{3x^2+7}{x^3+7x+9}\right]$

$\Rightarrow \frac{dy}{dx} = (x^2 - 5x + 8)(x^3 + 7x + 9)\left[\frac{(2x-5)(x^3+7x+9)}{x^2-5x+8} + \frac{(3x^2+7)(x^2-5x+8)}{x^3+7x+9}\right]$

$\Rightarrow \frac{dy}{dx} = 2x(x^3 + 7x + 9) - 5(x^3 + 7x + 9) + x^2(3x^2 + 7) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = (2x^4 + 14x^2 + 18x) - (5x^3 + 35x + 45) + (3x^4 + 7x^2) - 5x(3x^2 + 7) + 8(3x^2 + 7)$

$\Rightarrow \frac{dy}{dx} = 5x^4 - 20x^3 + 45x^2 - 52x + 11$

From the above three observations, it can be concluded that all the results of $\frac{dy}{dx}$ are same.

Q.18 If u, v and w are functions of x, then show that $\frac{d}{dx}(u.v.w) = \frac{du}{dx}v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$ in two ways-first by repeated application of product rule, second by logarithmic differentiation.

Sol: Let $y = u.v.w = u.(v.w)$

By applying product rule, we obtain

$\frac{dy}{dx} = \frac{du}{dx}.v.w = u.\frac{d}{dx}(v.w)$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}.v.w = u.\left[\frac{dv}{dx}.w + v.\frac{dw}{dx}\right]$

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx}.v.w = u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$

By taking logarithm on both sides of the equation $y = u.v.w$, we obtain

$\log y = \log u . \log v . \log w$

Differentiating both sides with respect to x, we obtain

$\log y = \log u . \log v . \log w$

$\frac{1}{y}.\frac{dy}{dx} = \frac{d}{dx}(\log u).\frac{d}{dx}(\log v).\frac{d}{dx}(\log w)$

$\Rightarrow \frac{1}{y}.\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}$

$\Rightarrow \frac{dy}{dx} = y\left(\frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}\right)$

$\Rightarrow \frac{dy}{dx} = u.v.w\left(\frac{1}{u}\frac{du}{dx}.\frac{1}{v}\frac{dv}{dx}.\frac{1}{w}\frac{dw}{dx}\right)$

$\therefore \frac{dy}{dx} = \frac{du}{dx}.v.w + u.\frac{dv}{dx}.w + u.v.\frac{dw}{dx}$