SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 Find the second order derivatives of the function.

$x^2 + 3x + 2$

Sol: Let. $y = x^2 + 3x + 2$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dt}(3x) + \frac{d}{dt}(2) = 2x + 3 + 0 = 2x + 3$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(2x+3) = \frac{d}{dx}(2x) + \frac{d}{dx}(3) = 2 + 0 = 2$$

Q.2 Find the second order derivatives of the function

x^{20}

Sol: Let. $y = x^{20}$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x^{20}) = 20x^{19}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(20x^{19}) = 20\frac{d}{dx}(x^{19}) = 20.19.x^{18} = 380x^{18}$$

Q.3 Find the second order derivatives of the function.

$x \,.\cos x$

Sol: Let. $y = x\,.\cos x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x\,.\cos x) = \cos x.\frac{d}{dx}(x) + \frac{d}{dx}(\cos x) = \cos x\,.\,1 + x(-\sin x) = \cos x - \sin x$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[\cos x - x\sin x] = \frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x)$$

$$= -\sin x - \left[\sin x.\frac{d}{dx}(x) + x\frac{d}{dx}(\sin x)\right]$$

$$= -\sin x - (\sin x + x\cos x)$$

$$= -(x\cos x + 2\sin x)$$

Q.4 Find the second order derivatives of the function.

$\log x$

Sol: Let. $y = \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(\log x) = \frac{1}{x}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

Q.5 Find the second order derivatives of the function.

$x^3 \log x$

Sol: Let. $y = x^3 \log x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \log x) = \log x . \frac{d}{dx}(x^3) + x^3 \frac{d}{dx}(\log x) = \log x . 3x^2 + x^3 . \frac{1}{x}$$

$$= \log x . 3x^2 + x^2 = x^2(1 + 3\log x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[x^2(1 + 3\log x)] = (1 + 3\log x). \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(1 + 3\log x)$$

$$= (1 + 3\log x). 2x + x^2 . \frac{3}{x}$$

$$= 2x + 6x\log x + 3x$$

$$= 5x + 6x\log x$$

Q.6 Find the second order derivatives of the function.

$e^x \sin 5x$

Sol: Let. $y = e^x \sin 5x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(e^x \sin 5x) = \sin 5x . \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(\sin 5x)$$

$$= (\sin 5x + e^x . \cos 5x \frac{d}{dx}(5x) = e^x . \sin 5x + e^x . \cos 5x . 5$$

$$= e^x(\sin 5x + 5\cos 5x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[e^x.(\sin 5x + 5\cos 5x)]$$

$$= (\sin 5x + 5\cos 5x). \frac{d}{dx}(e^x) + e^x . \frac{d}{dx}(\sin 5x + 5\cos 5x)$$

$$= (\sin 5x + 5\cos 5x)e^x + e^x\left[\cos 5x \frac{d}{dx}(5x) + 5(-\sin 5x). \frac{d}{dx}(5x)\right]$$

$$= e^x(\sin 5x + 5\cos 5x) + e^x(5\cos 5x - 25\sin 5x)$$

$$= e^x(10\cos 5x - 24\sin 5x) = 2e^x(5\cos 5x - 12\sin 5x)$$

Q.7 Find the second order derivatives of the function.

$e^{6x} \cos 3x$

Sol: Let. $y = e^{6x} \cos 3x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(e^{6x} \cos 3x) = \cos 3x . \frac{d}{dx}(e^{6x}) + e^{6x} \frac{d}{dx}(\cos 3x)$$

$= \cos 3x \,.\, e^{6x} . \frac{d}{dx}(6x) + e^{6x}(-\sin 3x) . \frac{d}{dx}(3x)$

$= 6e^{6x} . \cos 3x - 3e^{6x} . \sin 3x \quad \ldots \ldots (1)$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(6e^{6x} . \cos 3x - 3e^{6x} . \sin 3x) = 6 . \frac{d}{dx}(e^{6x} . \cos 3x) - 3\frac{d}{dx}(e^{6x} . \sin 3x)$

$= 6 . [6e^{6x} . \cos 3x - 3e^{6x} . \sin 3x] - 3 . \left[\sin 3x \frac{d}{dx}(e^{6x}) + e^{6x} . \frac{d}{dx}(\sin 3x)\right]$

$= 36\, e^{6x} . \cos 3x - 18e^{6x} . \sin 3x - 3[\sin 3x \,.\, e^{6x} . 6 + e^{6x} . \cos 3x \,.\, 3]$

$= 36\, e^{6x} . \cos 3x - 18e^{6x} . \sin 3x - 18 \sin 3x \,.\, e^{6x} - 9e^{6x} . \cos 3x$

$= 27\, e^{6x} . \cos 3x - 36e^{6x} . \sin 3x$

$= 9\, e^{6x}(\cos 3x - 4 \sin 3x)$

Q.8 Find the second order derivatives of the function.

$\tan^{-1} x$

Sol: Let. $y = \tan^{-1} x$

Then,

$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right) = \frac{d}{dx}(1+x^2)^{-1} = (-1)(1+x^2)^{-2} . \frac{d}{dx}(1+x^2)$

$= \frac{-1}{(1+x^2)^2} \times 2x = \frac{-2x}{(1+x^2)^2}$

Q.9 Find the second order derivatives of the function.

$\log(\log x)$

Sol: Let. $y = \log(\log x)$

Then,

$\frac{dy}{dx} = \frac{d}{dx}[\log(\log x)] = \frac{1}{\log x} . \frac{d}{dx}(\log x) = \frac{1}{x \log x} = (x \log x)^{-1}$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[(x \log x)^{-1}] = (-1) . (x \log x)^{-2} . \frac{d}{dx}(x \log x)$

$= \frac{-1}{(x \log x)^2} . \left[\log x \,.\, 1 + x . \frac{1}{x}\right] = \frac{-(1 + \log x)}{(x \log x)^2}$

Q.10 Find the second order derivatives of the function.

$\sin(\log x)$

Sol: Let. $y = \sin(\log x)$

Then,

$\frac{dy}{dx} = \frac{d}{dx}[\sin(\log x)] = \cos(\log x) . \frac{d}{dx}(\log x) = \frac{\cos(\log x)}{x}$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{\cos(\log x)}{x}\right]$$

$$= \frac{x.\frac{d}{dx}[\cos(\log x)] - \cos(\log x).\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x.\frac{d}{dx}\left[-\sin(\log x).\frac{d}{dx}(\log x)\right] - \cos(\log x).1}{x^2}$$

$$= \frac{-\sin(\log x).\frac{1}{x} - \cos(\log x)}{x^2}$$

$$= \frac{-[\sin(\log x) + \cos(\log x)]}{x^2}$$

Q.11 If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$

Sol: It is given that, $y = 5\cos x - 3\sin x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(5\cos x) - \frac{d}{dx}(3\sin x) = 5\frac{d}{dx}(\cos x) - 3\frac{d}{dx}(\sin x)$$

$$= 5(-\sin x) - 3\cos x = -(5\sin x + 3\cos x)$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}[-(5\sin x + 3\cos x)]$$

$$= -\left[5\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)\right]$$

$$= -[5\cos x + 3(-\sin x)]$$

$$= -[5\cos x - 3\sin x] = -y$$

$$\therefore \frac{d^2y}{dx^2} + y = 0$$

Q.12 If $y = \cos^{-1} x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

Sol: It is given that, $y = \cos^{-1} x$

Then,

$$\frac{dy}{dx} = \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} = -(1-x^2)^{-\frac{1}{2}}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}\left[(1-x^2)^{-\frac{1}{2}}\right]$$

$$= -\left(-\frac{1}{2}\right).(1-x^2)^{-\frac{3}{2}}.\frac{d}{dx}(1-x^2)$$

$$= \frac{1}{2\sqrt{(1-x^2)^3}} \times (-2x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-x}{\sqrt{(1-x^2)^3}} \qquad \ldots\ldots.(i)$$

$y = \cos^{-1} x \quad \Rightarrow x = \cos y$

Putting $x = \cos y$ in equation (i), we obtain

$$\therefore \frac{d^2y}{dx^2} = \frac{-\cos y}{\sqrt{(1-\cos^2 y)^3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\cos y}{\sqrt{(\sin^2 y)^3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\cos y}{\sin^3 y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-\cos y}{\sin y} \times \frac{1}{\sin^2 y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\cot y\ .\operatorname{cosec}^2 y$$

Q.13 If $y = 3\cos(\log x) + 4\sin(\log x)$, Show that $x^2y_2 + xy_1 + y = 0$

Sol: It is given that, $y = 3\cos(\log x) + 4\sin(\log x)$

Then,

$$y_1 = 3.\frac{d}{dx}[\cos(\log x)] + 4.\frac{d}{dx}[\sin(\log x)]$$

$$= 3\left[-\sin(\log x).\frac{d}{dx}(\log x)\right] + 4\left[\cos(\log x).\frac{d}{dx}(\log x)\right]$$

$$\therefore y_1 = \frac{-3\ \sin(\log x)}{x} + \frac{4\ \cos(\log x)}{x} = \frac{4\ \cos(\log x) - 3\ \sin(\log x)}{x}$$

$$\therefore y_2 = \frac{d}{dx}\left(\frac{4\ \cos(\log x) - 3\ \sin(\log x)}{x}\right)$$

$$= \frac{x.\{4\cos(\log x) - 3\sin(\log x)\}' - \{4\cos(\log x) - 3\sin(\log x)\}(x)'}{x^2}$$

$$= \frac{x.[4\{\cos(\log x)\}' - 3\{\sin(\log x)\}'] - \{4\cos(\log x) - 3\sin(\log x)\}.1}{x^2}$$

$$= \frac{x.[-4\sin(\log x)\ (\log x)' - 3\cos(\log x)\ (\log x)'] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{x.\left[-4\sin(\log x)\frac{1}{x} - 3\cos(\log x).\frac{1}{x}\right] - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{-4\sin(\log x) - 3\cos(\log x) - 4\cos(\log x) + 3\sin(\log x)}{x^2}$$

$$= \frac{-[\sin(\log x) + 7\cos(\log x)]}{x^2}$$

$\therefore\ x^2y_2 + xy_1 + y$

$= x^2\left(\frac{-\sin(\log x) - 7\cos(\log x)}{x^2}\right) + x\left(\frac{4\cos(\log x) - 3\sin(\log x)}{x^2}\right) + 3\cos(\log x) + 4\sin(\log x)$

$= -\sin(\log x) - 7\cos(\log x) + 4\cos(\log x) - 3\sin(\log x) + 3\cos(\log x) + 4\sin(\log x)$

$= 0$

Hence, proved.

Q. 14 If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Sol: It is given that, If $y = Ae^{mx} + Be^{nx}$

Then,

$\frac{dy}{dx} = A.\frac{d}{dx}(e^{mx}) = B.\frac{d}{dx}(e^{nx}) = A.e^{mx}\frac{d}{dx}(mx) + B.e^{nx}\frac{d}{dx}(nx) = Ame^{mx} + Bne^{nx}$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dx}(Ame^{mx} + Bne^{nx}) = Am.\frac{d}{dx}(e^{mx}) + Bn.\frac{d}{dx}(e^{nx})$

$= Ame^{mx}.\frac{d}{dx}(mx) + Bne^{nx}.\frac{d}{dx}(nx) = Am^2(e^{mx}) + Bn^2(e^{nx})$

$\therefore \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$

$= Am^2e^{mx} + Bn^2e^{nx} - (m+n).(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

$= Am^2e^{mx} + Bn^2e^{nx} - Am^2e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2e^{nx} + Amne^{mx} + Bmne^{nx}$

$= 0$

Hence, proved.

Q. 15 If $y = 500e^{7x} + 600e^{-7x}$, show that $\frac{d^2y}{dx^2} = 49y$

Sol: It is given that, If $y = 500e^{7x} + 600e^{-7x}$

Then,

$\frac{dy}{dx} = 500\frac{d}{dx}(e^{7x}) + 600\frac{d}{dx}(e^{-7x})$

$\frac{dy}{dx} = 500.e^{7x}.\frac{d}{dx}(7x) + 600.e^{-7x}\frac{d}{dx}(-7x)$

$= 3500.e^{7x} - 4200.e^{-7x}$

$\therefore \frac{d^2y}{dx^2} = 3500\frac{d}{dx}(e^{7x}) - 4200.\frac{d}{dx}(e^{-7x})$

$= 3500.e^{7x}.\frac{d}{dx}(7x) - 4200.e^{-7x}.\frac{d}{dx}(-7x)$

$= 7 \times 3500.e^{7x} + 7 \times 4200.e^{-7x}$

$= 49 \times 500.e^{7x} + 49 \times 600.e^{-7x}$

$= 49(500.e^{7x} + 600.e^{-7x})$

$= 49y$

Hence, proved.

Q. 16 If $e^y(x+1)=1$, show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

Sol: The given relationship is $e^y(x+1)=1$

$e^y(x+1)=1$

$\Rightarrow e^y=\frac{1}{(x+1)}$

Taking logarithm on both the sides, we obtain

$= y = \log\frac{1}{(x+1)}$

Differentiating both sides with respect to x, we obtain

$\frac{dy}{dx}=(x+1)\frac{d}{dx}\left(\frac{1}{x+1}\right)=(x+1).\frac{-1}{(x+1)^2}=\frac{-1}{x+1}$

$\therefore \frac{d^2y}{dx^2}=-\frac{d}{dx}\left(\frac{1}{x+1}\right)=-\left(\frac{-1}{(x+1)^2}\right)=\frac{1}{(x+1)^2}$

$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{-1}{x+1}\right)^2$

$\Rightarrow \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$

Hence, proved.

Q. 17 If $y=(\tan^{-1}x)^2$, show that $(x^2+1)^2y_2+2x(x^2+1)y_1=2$

Sol: The given relationship is $y=(\tan^{-1}x)^2$

$y_1=2\tan^{-1}x\frac{d}{dx}(\tan^{-1}x)$

$\Rightarrow y_1=2\tan^{-1}x.\frac{1}{1+x^2}$

$\Rightarrow (1+x^2)y_1=2\tan^{-1}x$

Again, differentiating both sides with respect to x, we obtain

$(1+x^2)y_2+2xy_1=2\left(\frac{1}{1+x^2}\right)$

$\Rightarrow (1+x^2)^2y_2+2x(1+x^2)y_1=2$

Hence, proved.