CBSE 10th

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Q.1 Find the area of the triangle whose vertices are:

(i) (2, 3), (− 1, 0), (2, − 4) (ii) (− 5, − 1), (3, − 5), (5, 2)

Sol: (i) (2, 3), (− 1, 0), (2, − 4)

Area of a triangle is given by

$$\text{Area of triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of triangle} = \frac{1}{2}\{2(0-(-4)) + 1((-4)-(-3)) + 2(3-0)\}$$

$$= \frac{1}{2}\{8+7+6\}$$

$$= \frac{21}{2}\text{Square units}$$

(ii) (− 5, − 1), (3, − 5), (5, 2)

Area of a triangle is given by

$$\text{Area of triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of triangle} = \frac{1}{2}\{(-5)((-5)-(-2)) + 3((2)-(-1)) + 5(-1-(-5))\}$$

$$= \frac{1}{2}\{35+9+20\}$$

$$= 32 \text{ Square units}$$

Q.2 In each of the following find the value of 'k', for which the points are collinear.

(i) (7, − 2), (5, 1), (3, − k) (ii) (8, 1), (k, − 4), (2, − 5)

Sol:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, −2) (5, 1), and (3, k), area = 0

$$\frac{1}{2}\left\{7(1-k)+5\left(k-(-2)\right)+3\left(-2-(-1)\right)\right\}=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$k=4$$

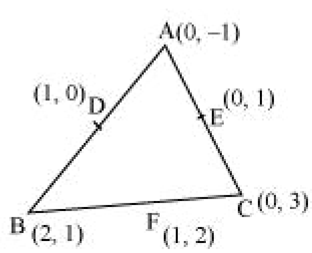
(ii) For collinear points, area of triangle formed by them is zero.

Therefore, for points (8, 1), (k, -4), and (2, -5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\}+k\{(-5)-(-1)\}+2\{1-(-4)\}\right]=0$$
$$8-6k+10=0$$
$$6k=18$$
$$k=3$$

Q.3 Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Sol:



Let the vertices of the triangle be A (0, −1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$\text{Area of triangle} = \frac{1}{2}\left\{x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right\}$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\left\{1\left(2-1\right) + 1\left(1-0\right) + 0\left(0-2\right)\right\}$$

$$= \frac{1}{2}\left(1+1\right) = 1 \text{ square units}$$

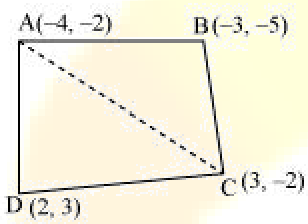
$$\text{Area of } \Delta ABC = \frac{1}{2}\left\{0\left(1-3\right) + 2\left(3-\left(-1\right)\right) + 0\left(-1-1\right)\right\}$$

$$= \frac{1}{2}\left(8\right) = 4 \text{ square unit}$$

Therefore, required ratio $= 1 : 4$

Q.4 Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)

Sol:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

Area of a triangle $= \frac{1}{2}\{x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)\}$

Area of Δ ABC $= \frac{1}{2}\ [(-4)\{(-5) — 2)\} + (-3)\{(-2) — 2)\} + 3\{(-2) — 5)\}]$

$= \frac{1}{2}\ (12 + 0 + 9) = \frac{21}{2}$ square units

Area of Δ ACD $= \frac{1}{2}\ [(-4)\{(-2) - (3)\} + 3\{(3) - (-2)\} + 2\{(-2) - (-2)\}]$

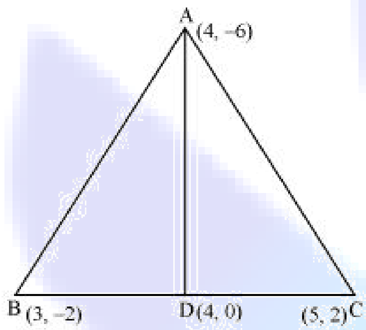
$= \frac{1}{2}\{20 + 15 + 0\} = \frac{53}{2}$ square units

Area of □ ABCD = Area of Δ ABC + Area of ΔACD

$= \left(\frac{21}{2} + \frac{35}{2}\right)$Square units = 28 square units.

Q.5 You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, − 6), B (3, − 2) and C (5, 2)

Sol:



Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

Coordinates of points D $= \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$

Area of triangle $= \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of △ ABD $= \frac{1}{2}[(4)\{(-2) - (0)\} + (3)\{(0) - (-6)\} + (4)\{(-6) - (-2)\}]$

$= \frac{1}{2}(-8 + 18 - 16) = -3$ square units

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

Area of △ ADC $= \frac{1}{2}[(4)\{0 - (2)\} + (4)\{(2) - (-6)\} + (5)\{(-6) - (0)\}]$

$= \frac{1}{2}\{-8 + 32 - 30\} = -3$ square units

However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.