CBSE 10th

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Q.1 Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, − 2) and B(3, 7)

Sol: Let the given line divide the line segment joining the points A(2, −2) and B(3, 7) in a ratio k : 1.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$
$$\Rightarrow \frac{6k+4+7k-4}{k+1}=0$$
$$\Rightarrow 9k-2=0$$
$$\Rightarrow k=\frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, −2) and B(3, 7) is 2:9.

Q.2 Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.
(i) (7, − 2), (5, 1), (3, − k) (ii) (8, 1), (k, − 4), (2, − 5)

Sol: If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area} = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$
$$0=\frac{1}{2}\{x((2-0))+1(-(0-y))+7(y-(2))\}$$
$$0=\frac{1}{2}\{2x-y+7y-14\}$$

$$0 = \frac{1}{2}\{2x + 6y - 14\}$$

$$2x + 6y - 14 = 0$$

$$x + 3y - 7 = 0$$

This is the required relation between x and y.

Q.3 Find the centre of a circle passing through the points (6, − 6), (3, − 7) and (3, 3).

Sol: Let O (x, y) be the centre of the circle. And let the points (6, −6), (3, −7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore OA = \sqrt{(x-6)^2 + (y+6)^2}$$

$$OB = \sqrt{(x-3)^2 + (y+7)^2}$$

$$OC = \sqrt{(x-3)^2 + (y-3)^2}$$

However, $OA = OB$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 49 + 14y$$

$$\Rightarrow -6x - 2y + 14 = 0$$

$$\Rightarrow 3x + y = 7 \quad ...(1)$$

However, $OA = OC$ (Radii of the same circle)

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 36 - 12x + y^2 + 36 + 12y = x^2 + 9 - 6x + y^2 + 9 - 6y$$

$$\Rightarrow -6x + 18y + 54 = 0$$

$$\Rightarrow -3x + 9y = -27 \quad ...(2)$$

On adding equation (1) and (2), we obtain

$$10y = -20$$

$$y = -2$$

From equation (1), we obtain

$$3x - 2 = 7$$

$$3x = 9$$

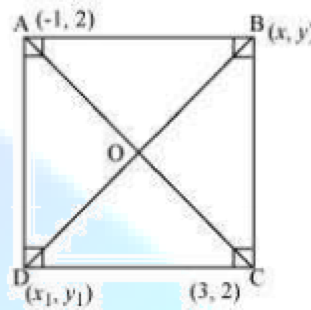
$x = 3$

Therefore, the centre of the circle is $(3, -2)$.

Q.4: The two opposite vertices of a square are $(-1, 2)$ and $(3, 2)$. Find the coordinates of the other two vertices.

Sol: Let ABCD be a square having $(-1, 2)$ and $(3, 2)$ as vertices A and C respectively.

Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.



We know that the sides of a square are equal to each other.

$\therefore AB = BC$

$$\Rightarrow \sqrt{(x+1)^2+(y-2)^2} = \sqrt{(x-3)^2+(y-2)^2}$$

$$\Rightarrow x^2+2x+1+y^2-4y+4 = x^2+9-6x+y^2+4-4y$$

$$\Rightarrow 8x = 8 \Rightarrow x = 1$$

We know that in a square, all interior angles are of 90°

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2 = \left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$$\Rightarrow 4+y^2+4-4y+4+y^2+4-4y = 16$$

$$\Rightarrow 2y^2+16-8y = 16$$

$$\Rightarrow 2y^2-8y = 0$$

$$\Rightarrow y(y-4) = 0$$

$$\Rightarrow y = 0 \text{ or } 4$$

We know that in a square, the diagonals are of equal length and bisect each other at 90°.

Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinates of point O} = \left(\frac{-1+3}{2}, \frac{2+2}{2}\right)$$

$$\left(\frac{1+x_1}{2}\right) + \left(\frac{y+y_1}{2}\right) = (1, 2)$$

$$\Rightarrow \frac{1+x_1}{2} = 1 \Rightarrow 1 + x_1 = 2$$

$$\Rightarrow x_1 = 1$$

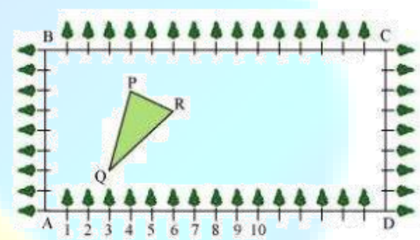
$$\text{and} \frac{y+y_1}{2} = 2 \Rightarrow y + y_1 = 4$$

$$\text{If } y = 0, \Rightarrow y_1 = 4$$

$$\text{If } y = 4, \Rightarrow y_1 = 0$$

Therefore, the required coordinates are (1, 0) and (1, 4).

Q.5 The class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure.



The students are to sow seeds of flowering plants on the remaining area of the plot.

(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Sol: (i) Taking A as origin, we will take AD as x-axis and AB as y-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\text{Area of } \Delta PQR = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}\{(4)(2-5) + 3(5-6) + 6(6-2)\}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}\{-12 - 3 + 24\}$$

$$\text{Area of } \Delta PQR = \frac{9}{2} \text{square units}$$

(ii) Taking C as origin, CB as x-axis, and CD as y-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

$$\text{Area of } \Delta PQR = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta PQR = \frac{1}{2}\{12(6-3) + 13(3-2) + 10(2-6)\}$$

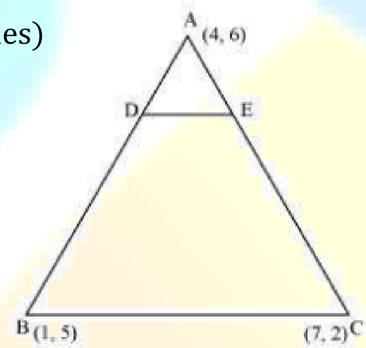
$$\text{Area of } \Delta PQR = \frac{1}{2}\{36 + 13 - 40\}$$

$$\text{Area of } \Delta PQR = \frac{9}{2} \text{ square units}$$

It can be observed that the area of the triangle is same in both the cases.

Q.6 The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Sol:



Given that, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$

$$\frac{AD}{AD + DB} = \frac{AE}{AE + EC} = \frac{1}{4}$$

$$\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of point } D = \left(\frac{1\times1 + 3\times4}{1+3}, \frac{1\times5 + 3\times6}{1+3}\right)$$

$$\text{Coordinates of point } D = \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$\text{Coordinates of point } E = \left(\frac{1\times7 + 3\times4}{1+3}, \frac{1\times2 + 3\times6}{1+3}\right)$$

$$\text{Coordinates of point E} = \left(\frac{19}{4}, \frac{20}{4}\right)$$

$$\text{Area of } \Delta ADE = \frac{1}{2}\left\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right\}$$

$$\text{Area of } \Delta ADE = \frac{1}{2}\left\{4\left(\frac{23}{4} - \frac{20}{4}\right) + \frac{13}{4}\left(\frac{20}{4} - 6\right) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right\}$$

$$\text{Area of } \Delta ADE = \frac{1}{2}\left\{3 - \frac{13}{4} + \frac{19}{16}\right\} = \frac{1}{2}\left\{\frac{48 - 52 + 19}{16}\right\} = \frac{15}{32}\text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left\{4(5-2) + 1(2-6) + 7(6-5)\right\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left\{12 - 4 + 7\right\}\text{ square units} = \frac{15}{2}\text{ squre units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Q.7 Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D.

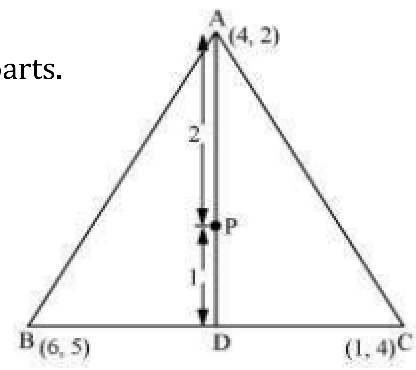
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x1, y1), B(x2, y2), and C(x3, y3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Sol:



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of point D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of point P} = \left(\frac{2\times\frac{7}{2}+1\times 4}{2+1}, \frac{2\times\frac{9}{2}+1\times 2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of point E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of point Q} = \left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts.

Therefore, F is the mid-point of side AB.

$$\text{Coordinates of point F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of point R} = \left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.
Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

Coordinates of point $D = \left(\frac{x_1 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

Let the centroid of this triangle be O. and Point O divides the side AD in a ratio 2:1.

$$\text{Coordinates of point } O = \left(\frac{2 \times \frac{x_2 + x_3}{2} + 1 \times x_1}{2+1}, \frac{2 \times \frac{y_2 + y_3}{2} + 1 \times y_1}{2+1}\right)$$

$$\text{Coordinates of point } O = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

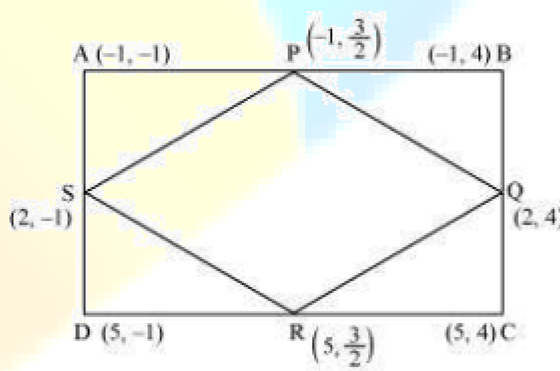
Q.8 ABCD is a rectangle formed by the points A (− 1, − 1), B (− 1, 4), C (5, 4) and D (5,− 1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.

Sol:

P is the mid point of side AB.

Therefore, the cordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

similarly, the coordinates of Q, R and S are $(2,4), \left(5, \frac{3}{2}\right)$, and $(2,-1)$ respectively.



$$\text{Length of } PQ = \sqrt{(-1-2)^2 + \left(\frac{3}{2} - 4\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of } QR = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of } RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of } SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of } PR = \sqrt{(-1-5)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = 6$$

$$\text{Length of } QSP = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure.

However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.