Question 1: The volume of a conical tent is 1232 m^3 and the area of the base floor is 154 m^2. Calculate the: (i) radius of the floor (ii) height of the tent (iii) length of the canvas required to cover this conical tent if its width is 2 m. **[2008]**

Answer:

Volume = 1232 m^3

Area of the base = 154 m^2

$$\pi r^2 = 154 \Rightarrow r = \sqrt{\left\{\frac{\{154\}}{\{22\}} \times 7\right\}} = 7 \text{ m}$$

$$\frac{\{1\}}{\{3\}} \times \pi \times (7)^2 \times h = 1232 \Rightarrow h = \frac{\{1232 \times 3\}}{\{\pi \times 7^2\}} = 24 \text{ m}$$

Curved Surface Area = $\pi r l = \pi (7) \sqrt{\{24^2 + 7^2\}} = 550 \text{ m}^2$

$$\text{Length of canvas needed} = \frac{\{550\}}{\{2\}} = 225 \text{ m}$$

Question 2: A solid sphere of radius 15 cm is melted and recast into solid right circular cones of radius 2.5 and height 8 cm. Calculate the number of cones recast.
[2013]

Answer:

Sphere: Radius = 15 cm

Cone: Radius = 2.5 cm and Height = 8 cm

Therefore number of cones re-casted =

$$\frac{\left\{\frac{\{4\}}{\{3\}} \times \pi \times (15)^3\right\}}{\left\{\frac{\{1\}}{\{3\}} \times \pi \times (2.5)^2 \times 8\right\}} = 270$$

Question 3: A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Find the height of the cone. **[2002]**

Answer:

Internal diameter = 4 cm $\Rightarrow$ Internal radius = 2 cm

External diameter = 8 cm ⇒ External radius = 4 cm

Radius of the cone = 4 cm

$$\therefore \frac{\{4\}}{\{3\}} \times \pi \times (3)^3 - \frac{\{4\}}{\{3\}} \times \pi \times (2)^3 = \frac{\{1\}}{\{3\}} \times \pi \times (4)^2 \times h$$

$$\Rightarrow h = \frac{\{4 \times (64 - 8)\}}{\{16\}} = 14\text{cm}$$

Question 4: A hemispherical bowl of diameter 7.2 cm is completely filled with chocolate sauce. This sauce is poured into an inverted cone of radius 4.8 cm. Find the height of the cone if it is completely filled. **[2010]**

Answer:

Hemisphere: Radius = 3.6 cm

Cone: Radius = 4.8 cm, Height = h

$$\therefore \frac{\{1\}}{\{2\}}\frac{\{4\}}{\{3\}} \times \pi \times (3.6)^3 = \frac{\{1\}}{\{3\}} \times \pi \times (4.8)^2 \times h$$

$$\Rightarrow h = \frac{\{2 \times 3.6^3\}}{\{4.8^2\}} = 4.05 \text{ cm}$$

Question 5: A solid cone of radius 5 cm and height 8 cm is melted and made into small spheres of radius 0.5 cm. Find the number of spheres formed. **[2011]**

Answer:

Cone: Radius = 5 cm , Height = 8cm

Sphere: Radius = 0.5 cm

$$n \times \frac{\{4\}}{\{3\}} \times \pi \times (0.5)^3 = \frac{\{1\}}{\{3\}} \times \pi \times (5)^2 \times 8$$

$$\Rightarrow n = \frac{\{5^2 \times 8\}}{\{4 \times 0.5^3\}} = 400$$

Question 6: The total area of a solid metallic sphere is 1256 cm^2. It is melted and recast into solid right circular cones of radius 2.5 cm and height 8 cm. Calculate: (i) the radius of the solid sphere, (ii) the number of cones recast. **[2000]**

Answer:

Surface area = 1256 cm^2

(i) $\therefore\ 4\,\pi\,r^2\ =\ 1256$

$$\Rightarrow\ r^2\ =\ \frac{\{1256\}}{\{4\times\ 3.14\}}\ =\ 100$$

$$\Rightarrow\ r\ =\ 10\ \text{cm}$$

(ii) Cone: Radius = 2.5 cm, Height = 8 cm

$$\frac{\{4\}}{\{3\}}\times\pi\times\ (10)^3\ =\ n\times\frac{\{1\}}{\{3\}}\times\pi\times\ (2.5)^2\times\ 8$$

$$\Rightarrow\ n\ =\ \frac{\{4\times\ 10^3\}}{\{2.5^2\times\ 8\}}\ =\ 80$$

Question 7: A hollow sphere of internal and external 6 cm and 8 cm respectively is melted and recast into small cones of base radius 2 cm and height 8 cm. Find the number of cones. **[2012]**

Answer:

Sphere: Internal radius = 6 cm, External radius = 8 cm

Cone: Radius = 2 cm, Height = 8 cm

$$\frac{\{4\}}{\{3\}}\times\pi\times\ (8)^3\ -\frac{\{4\}}{\{3\}}\times\pi\times\ (6)^3\ =\ n\times\frac{\{1\}}{\{3\}}\times\pi\times\ (2)^2\times\ 8$$

$$\Rightarrow\ n\ =\ \frac{\{4\times\ (8^3-6^3)\}}{\{2^2\times\ 8\}}\ =\ 37$$

Question 8: The surface area of a solid metallic sphere is 2464 cm^2. It is melted and recast into solid right circular cones of radius 3.5 cm and height 7 cm. Calculate: (i) the radius of the sphere (ii) the number of cones recast. **[2014]**

Answer:

Surface area of sphere = 2464 cm^2

Cone: Radius = 3.5 cm, Height = 7 cm

$$4\,\pi\,r^2\ =\ 2464\ \Rightarrow\ r^2\ =\ \frac{\{2464\times 7\}}{\{4\times\ 22\}}\ =\ 196$$

Hence R = 14 cm

$$\frac{\{4\}}{\{3\}}\times\pi\times\ (14)^3\ =\ n\times\frac{\{1\}}{\{3\}}\times\pi\times\ (3.5)^2\times\ 7$$

$$\Rightarrow n = \frac{\{4 \times 14^3\}}{\{3.5^2 \times 7\}} = 128$$

Question 9: A vessel in the form of an inverted cone is filled with water to the brim. Its height is 20 cm and diameter is 16.8 cm. Two equal solid cone are dropped in it so that they are fully submerged. As a result, one third of the water in original cone overflows. What is the volume of each of the solid cones submerged? [2006]

Answer:

Conical Vessel: Height = 20 cm , Radius = 8.4 cm

The amount of water overflow $= \frac{\{1\}}{\{3\}} \times \frac{\{1\}}{\{3\}} \pi (8.4)^2 \times 20$

Therefore the volume of one sphere $= \frac{\{1\}}{\{2\}} \times \frac{\{1\}}{\{3\}} \times \frac{\{1\}}{\{3\}} \pi (8.4)^2 \times 20 = 246.4 \text{ cm}^3$

Question 10: A metallic sphere of radius 10.5 cm is melted and then re-casted into small cones each of radius 3.5 cm and height 3 cm. Find the number of cones thus formed. **[2005]**

Answer:

Volume of the sphere melted $= \frac{\{4\}}{\{3\}} \pi r^3 = \frac{\{4\}}{\{3\}} \pi (10.5)^3 \text{ cm}^3$

Volume of the cones formed $= \frac{\{1\}}{\{3\}} \pi r^2 h = \frac{\{1\}}{\{3\}} \pi (3.5)^2 \times 3 \text{ cm}^3$

Therefore the number of cones formed $= \frac{\left\{\frac{\{4\}}{\{3\}} \pi (10.5)^3\right\}}{\left\{\frac{\{1\}}{\{3\}} \pi (3.5)^2 \times 3\right\}} = 126$

Question 11: A girl fills a cylindrical bucket 32 cm in height and 18 cm in radius with sand. She empties the bucket on the ground and makes a conical heap of the sand. If the height of the conical heap is 24 cm, find: (i) the radius and (ii) the slant height of the heap. Give your answer correct to one place of decimal. **[2004]**

Answer:

Volume of the conical heap = Volume of the bucket

(i) $\Rightarrow \frac{\{1\}}{\{3\}} \pi r^2 \times 24 = \pi \times (18)^2 \times (32)$

$\Rightarrow r^2 = 1296 \Rightarrow r = 36$ cm

(ii) $l^2 = 24^2 + 36^2$

$\Rightarrow l^2 = 1872 \Rightarrow l = 43.3$ cm

Question 12: A vessel is in the form of inverted cone. Its height is 11 cm and the radius of its top, which is open, is 2.5 cm . It is filled with water to the rim. When lead shots, each of which is a sphere of radius 0.25 cm , are dropped into the vessel, {2}/{5} of the water flows out. Find the number of lead shots dropped into the vessel. [2003]

Answer:

Volume of water in the cone $= \frac{\{1\}}{\{3\}} \pi r^2 h = \frac{\{1\}}{\{3\}} \pi . (2.5)^2 . 11 = \frac{\{68.75\}}{\{3\}} \pi$

Volume of the lead shots = volume of the water that over flows

$= \frac{\{2\}}{\{5\}} \times \frac{\{68.75\}}{\{3\}} \pi$

Volume of one lead shot $= \frac{\{4\}}{\{3\}} \pi (0.25)^3 = \frac{\{0.625\}}{\{3\}} \pi$

Therefore the number of lead shots $= \frac{\{\text{Volume of water displaced by lead shots}\}}{\{\text{Volume of one lead shot}\}}$

$= \frac{\left\{\frac{\{2\}}{\{5\}} \times \frac{\{68.75\}}{\{3\}} \pi\right\}}{\left\{\frac{\{0.625\}}{\{3\}} \pi\right\}} = 440$

Question 13: A solid, consisting of a right circular cone standing on a hemisphere, is placed upright in a right circular cylinder, full of water, and touches the bottom. Find the volume of water left in the cylinder, having given that the radius of the cylinder is 3 cm and its height is 6 cm; the radius of the hemisphere is 2 cm and the height of cone is 4 cm. Give your answer to the nearest cubic centimeter. **[1998]**

Answer:

Cylinder: Radius = 3 cm , Height = 6 cm

Cone: Radius = 2 cm , Height = 4 cm

Hemisphere: Radius = 2 cm

Volume of cylinder $= \pi r^2 h = \pi(3)^2 \times 6 = 54\pi$

Volume of hemisphere $= \frac{\{1\}}{\{2\}} \times \frac{\{4\}}{\{3\}} \pi r^3 = \frac{\{2\}}{\{3\}} \pi (2)^3 = \frac{\{16\}}{\{3\}} \pi$

Volume of cone $= \frac{\{1\}}{\{3\}}\pi r^2 h = \frac{\{1\}}{\{3\}}\pi (2)^2 \times 4 = \frac{\{16\}}{\{3\}}\pi$

Therefore the volume of water left in the cylinder $= 54\pi - \frac{\{16\}}{\{3\}}\pi - \frac{\{16\}}{\{3\}}\pi = \frac{\{130\}}{\{3\}}\pi$

$= 136.19\ cm^2 \approx 136\ cm^2$

Question 14: A metal container in the form of a cylinder is surmounted by a hemisphere of the same radius. The internal height of the cylinder is 7 m and the internal radius is 3.5 m. Calculate: (i) the total area of the internal surface, excluding the base; (ii) the internal volume of the container in m^3. **[1999]**

Answer:

Cylinder: Radius $= 3.5$ m, Height $= 7$ m

Hemisphere: Radius $= 3.5$ cm

(i)Internal surface area of cylinder $= 2\pi r h = 2 \times \frac{\{22\}}{\{7\}} \times 3.5 \times 7 = 154\ m^2$

Internal surface are of hemisphere $= \frac{\{1\}}{\{2\}} \times 4\pi r^2 = \frac{\{1\}}{\{2\}} \times 4 \times \frac{\{22\}}{\{7\}} \times 3.5^2 = 77\ m^2$

Therefore the total surface area $= 154 + 77 = 231\ m^2$

(ii) Internal volume $=$ Volume of the cylinder $+$ volume of the hemisphere

$= \frac{\{22\}}{\{7\}} \times 3.5^2 \times 7 + \frac{\{1\}}{\{2\}} \times \frac{\{4\}}{\{3\}} \times \frac{\{22\}}{\{7\}} \times (3.5)^3 = 269.5 + 89.33 = 359.33\ m^3$

Question 15: An exhibition tent is in the form of a cylinder surmounted by a cone. The height of the tent above the ground is 85 m and the height of the cylindrical part is 50 m. If the diameter of the base is 168 m, find the quantity of the canvas required to make the tent. Allow 20% extra for folds and for stitching. Give your answer to the nearest m^2. **[2001]**

Answer:

Cylinder: Radius $= 84$m, Height $= 50$m Cone: Radius $= 84$ m, Height $= 35$m

Surface area of the cylinder $= 2\pi r h = 2 \times \frac{\{22\}}{\{7\}} \times 84 \times 50 = 26400\ m^2$

Curved surface area of the cone $= \pi r l = \frac{\{22\}}{\{7\}} \times 84 \times \sqrt{\{84^2 + 35^2\}} = 24024\ m^2$

Total surface area $= 26400 + 24024 = 50424\ m^2$

Total cloth required (including 20%) $= 50420 \times 1.20 = 60508.8\ m^2 \approx 60509\ m^2$