Board – CBSE	Class – 12	Topic – Determinants

1. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, Find A^{-1} and hence solve system of equation

   $x + 2y - 3z = -4$

   $2x + 3y + 2z = 2$

   $3x - 3y - 4z = 11$

2. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ Find AB & hence solve

   $x - y = 3,$

   $2x + 3y + 4z = 17$

   $y + 2z = 7.$

3. Check the consistency and inconsistency of the following linear equations.

   i) $3x - y + 2z = 3$
   $2x + y + 3z = 5$
   $x - 2y - z = 1$

   ii) $2x + y - 2z = 4$
   $x - 2y + z = -2$
   $5x - 5y + z = -2$

   iii) $3x + y = 5$
   $-6x - 2y = 9$

   iv) $x + y - z = 0$
   $x - 2y + z = 0$
   $3x + 6y - 5z = 0$

4. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations.

   $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

5. Without expanding, show that the values of each of the following determinants is zero.

   i) $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$

   ii) $\begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

6. Using the properties of determinants evaluate the following:

   i) $\begin{vmatrix} x+y & x & x \\ x & x+y & x \\ x & x & x+y \end{vmatrix}$

ii) $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

iii) $\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

iv) $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

7. Prove the following identities:

i) $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

ii) $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

iii) $\begin{vmatrix} 0 & b^2a & c^2a \\ a^2b & 0 & c^2b \\ a^2c & b^2c & 0 \end{vmatrix} = 2a^3b^3c^3$

iv) $\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & b \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$

v) $\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2+b^2+c^2)$

vi) $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

vii) $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$

viii) $\begin{vmatrix} -a(b^2+c^2-a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2+a^2-b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2+b^2-c^2) \end{vmatrix} = a\,b\,c(b^2+c^2+a^2)^3$

8. Determine the product $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ and use it to solve the system of equations. $x+y+z=0, x+2y-3z=-14, 2x-y+3z=9$

9. Using the properties of determinants, prove the following:

$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz + xy + yz + zx$ (CBSE – 2009)

10. If x, y, z all are different &

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0, \text{show that } xyz = -1$$

11. If A & B are square matrices of order 3 such that $|A| = -1$ & $|B| = 3$, then find the value of $|3AB|$.

12. Without expanding evaluate the determinant

$$\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (a^y + a^{-y})^2 & (a^y - a^{-y})^2 & 1 \\ (a^z + a^{-z})^2 & (a^z - a^{-z})^2 & 1 \end{vmatrix}, \text{where } a > 0 \text{ and } x, y, z \in R.$$

13. Using the properties of determinants, prove the following:

$$\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$$ [CBSE – 2009]

14. Prove $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ac \end{vmatrix} = 0$

15. Show that: $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in A. P.

16. Find the area of triangle using the determinants if three of its vertices are $(5,2), (-3,-1), (6,0)$.

17. If the points $(a, b), (c, d)$, and $(a + c, b + d)$ are collinear, show that $a d = b c$.

18. Find the value of α so that the points $(1, -5), (-4,5)$, and $(\alpha, 7)$ are collinear.

19. If a, b, & c are distinct real no. and the system of equations

    $ax + a^2y + (a^3 + 1)z = 0$

    $bx + b^2y + (b^3 + 1)z = 0$

    $cx + c^2y + (c^3 + 1)z = 0$ has a non trivial solution show that $abc = -1$.

20. Find the minor of element 5, $\begin{vmatrix} -3 & 6 & 5 \\ 2 & 1 & 0 \\ -1 & 6 & 5 \end{vmatrix}$

21. Find the co-factor of element a_{23}, $\begin{vmatrix} -8 & 6 & 0 \\ 6 & 1 & 0 \\ -1 & 6 & 5 \end{vmatrix}$

22. Using the co-factor of the second row of determinant

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \text{ find the value of } \Delta.$$

23. Find A^{-1} where $\begin{bmatrix} 6 & 4 & 2 \\ -12 & 15 & 18 \\ 25 & -20 & 15 \end{bmatrix}$ and solve the following

$6x - 12y + 25z = 4$

$4x + 15y - 20z = 3$

$2x + 18y + 15z = 10$

24. Find A^{-1} where $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ and solve the following

$3x + 4y + 7z = 14$

$2x - y + 3z = 4$

$x + 2y - 3z = 0$

25. Solve the following system of equations using matrices:

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2 \quad x, y, z \neq 0.$

Answer

4. $x = 3, y = -2, z = -1$

8. $x = -1, y = -2, z = 3$

11. -81

23. $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$

24. $x = 1, y = 1, z = 1$

25. $x = 2, y = 3, z = 5$

26. $\alpha = \frac{57}{10}, y = \frac{5}{4}k, x = \frac{-7}{8}k, z = k$