



SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 Evaluate the determinants in Exercises 1 and 2.

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$$

Sol: $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5) = -2 + 20 = 18$

Q.2 Evaluate the determinants in Exercises 1 and 2.

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Sol: (i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$
 $= (x^2 - x + 1)(x + 1) - (x - 1)(x + 1)$
 $= x^3 - x^2 + x + x^2 - x + 1 - (x^2 - 1)$
 $= x^3 + 1 - x^2 + 1$
 $= x^3 - x^2 + 2$

Q.3 If, $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then show that $|2A| = 4|A|$

Sol: The given matrix is $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$$\therefore 2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2 \times 4 - 4 \times 8 = 8 - 32 = -24$$

$$\text{Now, } |A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 1 \times 2 - 2 \times 4 = 2 - 8 = -6$$

$$\therefore \text{R.H.S} = 4|A| = 4 \times (-6) = -24$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

Q.4 If, $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$ then show that $|3A| = 27|A|$

Sol: The given matrix is $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

It can be observed that in the first column, two entries are zero. Thus, we expand along the first column (C_1) for easier calculation.

$$|A| = \begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = 1(4 - 0) - 0 + 0 = 4$$

$$\therefore 27|A| = 27(4) = 108 \quad \dots\dots(i)$$

$$\text{Now, } 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\therefore |3A| = \begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0 \begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix} = 3(36 - 0) = 3(36) = 108 \quad \dots\dots(ii)$$

From equations (i) and (ii), we have:

$$|3A| = 27|A|$$

Hence, the given result is proved

Q.5 Evaluate the determinants

$$(i) \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix} \quad (ii) \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$(iii) \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} \quad (iv) \begin{vmatrix} 2 & -1 & -2 \\ 0 & -2 & -1 \\ 3 & -5 & 1 \end{vmatrix}$$

Sol: (i) Let . $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

It can be observed that in the second row, two entries are zero. Thus, we expand along the second row for easier calculation.

$$|A| = -0 \begin{vmatrix} -1 & -2 \\ 5 & 0 \end{vmatrix} + 0 \begin{vmatrix} 3 & -2 \\ 3 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (-15 + 3) = 12$$

(ii) Let . $A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

By expanding along the first row, we have:

$$|A| = 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} + 4 \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix}$$

$$= 3(1 + 6) + 4(1 + 4) + 5(3 - 2) = 3(7) + 4(5) + 5(1) = 46$$

(iii) Let $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

By expanding along the first row, we have:

$$|A| = 0 \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix}$$

$$= 0 - 1(0 - 6) + 2(-3 - 0) = -1(-6) + 2(-3) = 6 - 6 = 0$$

(iv) Let $\begin{vmatrix} 2 & -1 & -2 \\ 0 & -2 & -1 \\ 3 & -5 & 1 \end{vmatrix}$

By expanding along the first column, we have:

$$|A| = 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - 0 \begin{vmatrix} -1 & -2 \\ -5 & 0 \end{vmatrix} + 3 \begin{vmatrix} -1 & -2 \\ 2 & -1 \end{vmatrix}$$

$$= 2(0 - 5) - 0 + 3(1 + 4) = -10 + 15 = 5$$

Q.6 If, $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, Find $|A|$.

Sol: Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

By expanding along the first row, we have:

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} \\ &= 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5) = 1(3) - 1(-3) - 2(3) = 6 - 6 = 0 \end{aligned}$$

Q.7 Find values of x , if

(i) $\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ (ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Sol: (i) $\begin{vmatrix} 2 & 4 \\ 2 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$$\Rightarrow 2 \times 1 - 5 \times 4 = 2x \times x - 6 \times 4$$

$$\Rightarrow 2 - 20 = 2x^2 - 24$$

$$\Rightarrow 2x^2 = 6$$

$$\Rightarrow x^2 = 3$$

$$\Rightarrow x^2 = \pm \sqrt{3}$$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$$\Rightarrow 2 \times 5 - 3 \times 4 = x \times 5 - 3 \times 2x$$

$$\Rightarrow 10 - 12 = 5x - 6x$$

$$\Rightarrow -2 = -x$$

$$\Rightarrow x = 2$$

Q.8 If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6 (B) ± 6 (C) -6 (D) 0

Sol: $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$$\Rightarrow x^2 - 36 = 36 - 36$$

$$\Rightarrow x^2 - 36 = 0$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x^2 = \pm 6$$

Hence, the correct answer is B.