SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 Find area of the triangle with vertices at the point given in each of the following:

(i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8) (iii) (−2, −3), (3, 2), (−1, −8)

Sol: (i) The area of the triangle with vertices (1, 0), (6, 0), (4, 3) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

$$\Delta = \frac{1}{2}[1(0-3) - 0(6-4) + 1(18-0)]$$

$$\Delta = \frac{1}{2}[-3+18] = \frac{15}{2} \text{ square units}$$

(ii) The area of the triangle with vertices (2, 7), (1, 1), (10, 8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(1-8) - 7(1-10) + 1(8-10)]$$

$$= \frac{1}{2}[2(-7) - 7(-9) + 1(-2)]$$

$$= \frac{1}{2}[-14 + 63 - 2] = \frac{1}{2}[-16 + 63]$$

$$= \frac{47}{2}$$

(iii) The area of the triangle with vertices (−2, −3), (3, 2), (−1, −8) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2+8) + 3(3+1) + 1(-24+2)]$$

$$= \frac{1}{2}[-2(10) + 3(4) + 1(-22)]$$

$$= \frac{1}{2}[-20 + 12 + -22]$$

$$= -\frac{30}{2} = -15$$

Hence, the area of the triangle is $|-15| = 15$ square units.

Q.2 Show that points $A(a, b+c), B(b, c+a), C(c, a+b)$ are collinear

Sol: Area of ΔABC is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b-a & c+a & 0 \\ c-a & a+b & 0 \end{vmatrix} \qquad (\text{Applying } R_2 \to R_2 - R_1 \text{ and } R_3 \to R_3 - R_1)$$

$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix}$

$= \frac{1}{2}(a-b)(c-a)\begin{vmatrix} a & b+c & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}$ (Applying $R_3 \to R_3 + R_2$)

Thus, the area of the triangle formed by points A, B, and C is zero.

Hence, the points A, B, and C are collinear.

Q.3 Find values of k if area of triangle is 4 square units and vertices are

(i) (k, 0), (4, 0), (0, 2) (ii) (−2, 0), (0, 4), (0, k)

Sol: We know that the area of a triangle whose vertices are (x_1, y_1), (x_2, y_2), and (x_3, y_3) is the absolute value of the determinant (Δ), where

$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

It is given that the area of triangle is 4 square units.

$\therefore \Delta = \pm 4$.

(i) The area of the triangle with vertices (k, 0), (4, 0), (0, 2) is given by the relation,

$\Delta = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= \frac{1}{2}[k(0-2) - 0(4-0) + (8-0)]$

$= \frac{1}{2}[-2k+8] = -k+4$

$\therefore -k + 4 = \pm 4$

When $-k + 4 = -4, k = 8$.

When $-k + 4 = 4, k = 0$.

Hence, $k = 0, 8$.

(ii) The area of the triangle with vertices (−2, 0), (0, 4), (0, k) is given by the relation,

$\Delta = \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix}$

$= \frac{1}{2}[-2(4-k)]$

$= k - 4$

$\therefore k - 4 = \pm 4$

When $k - 4 = -4, k = 0$.

When $k - 4 = 4, k = 8$.

Hence, $k = 0, 8$.

Q.4 (i) Find equation of line joining (1, 2) and (3, 6) using determinants

(ii) Find equation of line joining (3, 1) and (9, 3) using determinants

Sol: (i) Let P (x, y) be any point on the line joining points A (1, 2) and B (3, 6). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \ \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[1(6-y) - 2(3-x) + 1(3y-6x)] = 0$$

$$\Rightarrow 6 - y - 6 + 2x + 3y - 6x = 0$$

$$\Rightarrow 2y - 4x = 0$$

$$\Rightarrow y = 2x$$

Hence, the equation of the line joining the given points is $y = 2x$.

(ii) Let P (x, y) be any point on the line joining points A (3, 1) and B (9, 3). Then, the points A, B, and P are collinear. Therefore, the area of triangle ABP will be zero.

$$\therefore \ \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{1}{2}[3(3-y) - 1(9-x) + 1(9y-3x)] = 0$$

$$\Rightarrow 9 - 3y - 9 + x + 9y - 3x = 0$$

$$\Rightarrow 6y - 2x = 0$$

$$\Rightarrow x - 3y = 0$$

Hence, the equation of the line joining the given points is $x - 3y = 0$

Q.5 If area of triangle is 35 square units with vertices (2, −6), (5, 4), and (k, 4). Then k is

A. 12 B. −2 C. −12, −2 D. 12, −2

Sol: The area of the triangle with vertices (2, -6), (5, 4), and (k, 4) is given by the relation,

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[2(4-4) + 6(5-k) + 1(20-4k)]$$

$$= \frac{1}{2}[30 - 6k + 20 - 4k]$$

$$= \frac{1}{2}[50 - 10k]$$

$$= 25 - 5k$$

It is given that the area of the triangle is ± 35.

Therefore, we have:

$$\Rightarrow 25 - 5k = \pm 35$$

$$\Rightarrow 5(5-k) = \pm 35$$

$\Rightarrow 5 - k = \pm 7$

When $5 - k = -7, k = 5 + 7 = 12$.

When $5 - k = 7, k = 5 - 7 = -2$.

Hence, $k = 12, -2$.

The correct answer is D.