SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Q.1 Examine the consistency of the system of equations.

$x + 2y = 2$

$2x + 3y = 3$

Sol: The given system of equations is:

$x + 2y = 2$

$2x + 3y = 3$

The given system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

Now,

$|A| = 1(3) - 2(2) = 3 - 4 = -1 \neq 0$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q.2 Examine the consistency of the system of equations.

$2x - y = 5$

$x + y = 4$

Sol: The given system of equations is:

$2x - y = 5$

$x + y = 4$

The given system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$

Now,

$|A| = 2(1) - (-1)(1) = 2 + 1 = 3 \neq 0$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q.3 Examine the consistency of the system of equations.

$x + 3y = 5$

$2x + 6y = 8$

Sol: The given system of equations is:

$x + 3y = 5$

$2x + 6y = 8$

The given system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$

Now,

$|A| = 1(6) - 3(2) = 6 - 6 = 0$

$\therefore$ A is a singular matrix.

Now,

$(\text{adj } A) = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$(\text{adj } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq 0$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Q.4 Examine the consistency of the system of equations.

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

Sol: The given system of equations is:

$x + y + z = 1$

$2x + 3y + 2z = 2$

$ax + ay + 2az = 4$

This system of equations can be written in the form $AX = B$, where

$A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Now,

$|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a)$

$= 4a - 2a - a = 4a - 3a = a \neq 0$

$\therefore$ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q.5 Examine the consistency of the system of equations.

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

Sol: The given system of equations is:

$3x - y - 2z = 2$

$2y - z = -1$

$3x - 5y = 3$

This system of equations can be written in the form of AX = B, where

$$A=\begin{bmatrix}3 & -1 & -2\\0 & 2 & -1\\3 & -5 & 0\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}2\\-1\\3\end{bmatrix}$$

Now,

$|A| = 3(0-5) - 0 + 3(1+4) = -15 + 15 = 0$

∴ A is singular matrix.

Now,

$$(\text{adj } A) = \begin{bmatrix}-5 & 10 & 5\\-3 & 6 & 3\\-6 & 12 & 6\end{bmatrix}$$

$$(\text{adj } A)B = \begin{bmatrix}-5 & 10 & 5\\-3 & 6 & 3\\-6 & 12 & 6\end{bmatrix}\begin{bmatrix}2\\-1\\3\end{bmatrix} = \begin{bmatrix}-10-10+15\\-6-6+9\\-12-12+18\end{bmatrix} = \begin{bmatrix}-5\\-3\\-6\end{bmatrix} \neq 0$$

Thus, the solution of the given system of equations does not exist. Hence, the system of equations is inconsistent.

Q.6 Examine the consistency of the system of equations.

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

Sol: The given system of equations is:

$5x - y + 4z = 5$

$2x + 3y + 5z = 2$

$5x - 2y + 6z = -1$

This system of equations can be written in the form of AX = B, where

$$A=\begin{bmatrix}5 & -1 & 4\\2 & -3 & 5\\5 & -2 & 6\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix} \text{ and } B=\begin{bmatrix}5\\2\\-1\end{bmatrix}$$

Now,

$|A| = 5(18+10) + 1(12-25) + 4(-4-15)$

$= 5(28) + 1(-13) + 4(-19)$

$= 140 - 13 - 76$

$= 51 \neq 0$

∴ A is non-singular.

Therefore, A^{-1} exists.

Hence, the given system of equations is consistent.

Q.7 Solve system of linear equations, using matrix method.

$5x + 2y = 4$

$7x + 3y = 5$

Sol: The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

Now,

$|A| = 15 - 14 = 1 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A^{-1} = \frac{1}{|A|}$ (adj A)

$\therefore\ A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$

$\therefore\ X = A^{-1}B = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 - 10 \\ -28 + 25 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Hence, x = 2 nad y = −3

Q.8 Solve system of linear equations, using matrix method.

$2x - y = -2$

$3x + 4y = 3$

Sol: The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

Now,

$|A| = 8 + 3 = 11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A^{-1} = \frac{1}{|A|}$ (adj A) $= \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$

$\therefore\ X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8 + 3 \\ 6 + 6 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -5 \\ 12 \end{bmatrix} = \begin{bmatrix} -\frac{5}{11} \\ \frac{12}{11} \end{bmatrix}$

Hence, $x = -\frac{5}{11}$ and $y = \frac{12}{11}$

Q.9 Solve system of linear equations, using matrix method.

$4x - 3y = 3$

$3x - 5y = 7$

Sol: The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

Now,

$|A| = -20 + 9 = -11 \neq 0$

Thus, A is non-singular. Therefore, its inverse exists.

Now,

$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}$

$\therefore\ X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 15 - 21 \\ 9 - 28 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -6 \\ -19 \end{bmatrix} = \begin{bmatrix} -\frac{6}{11} \\ -\frac{19}{11} \end{bmatrix}$

Hence, $x = \frac{-6}{11}$ and $y = \frac{-19}{11}$

Q.10 $5x + 2y = 3$

$3x + 2y = 5$

Sol: The given system of equations is:

$5x + 2y = 3$

$3x + 2y = 5$

The given system of equations can be written in the form of AX = B, where

$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Now,

$|A| = 10 - 6 = 4 \neq 0$

Thus, A is non-singular. Therefore, A^{-1} exists.

Q.11 By using properties of determinants, show that:

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

Sol: The given system of equations is:

$2x + y + z = 1$

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9$

The given system of equations can be written in the form of AX = B, where

$A= \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

Now,

$|A| = 2(10+3) - 1(-5-3) + 0 = 2(13) - 1(-8) = 26 + 8 = 34 \neq 0$

$\therefore$ A is non-singular. Therefore, A^{-1} exists.

Now, Now,

$A_{11} = 13, A_{12} = 5, A_{13} = 3$

$A_{21} = 8, A_{22} = -10, A_{23} = -6$

$A_{31} = 1, A_{32} = 3, A_{33} = -5$

$A^{-1} = \frac{1}{|A|}$ (adj A) $= \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$

$\therefore\ X = A^{-1}B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15+27 \\ 3-9-45 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$

Hence, $x = 1$, $y = \frac{1}{2}$ and $z = -\frac{3}{2}$

Hence, the given system of equations is consistent.

Q.12 By using properties of determinants, show that:

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Sol: The given system of equations is:

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

The given system of equations can be written in the form of $AX = B$, where

$A= \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$

Now,

$|A| = 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10 \neq 0$

$\therefore$ A is non-singular. Therefore, A^{-1} exists.

Now,

$A_{11} = 4,\ A_{12} = -5,\ A_{13} = 1$

$A_{21} = 2, A_{22} = 0, A_{23} = -2$

$A_{31} = 2, A_{32} = 5, A_{33} = 3$

$$A^{-1} = \frac{1}{|A|}\ (\text{adj } A) = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\therefore\ X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -1 \\ 1 \end{bmatrix}$$

Hence, $x = 2$, $y = -1$ and $z = 1$.

Q.13 Solve system of linear equations, using matrix method.

$2x + 3y + 3z = 5$

$x - 2y + z = -4$

$3x - y - 2z = 3$

Sol: The given system of equations can be written in the form of $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

Now,

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 2(5) - 3(5) + 3(5) = 40 \neq 0$

$\therefore$ A is non-singular. Therefore, A^{-1} exists.

Now,

$A_{11} = 5,\ A_{12} = 5,\ A_{13} = 5$

$A_{21} = 3, A_{22} = -13, A_{23} = 11$

$A_{31} = 9, A_{32} = 1, A_{33} = -7$

$$A^{-1} = \frac{1}{|A|}\ (\text{adj } A) = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\therefore\ X = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Hence, $x = 1$, $y = 2$ and $z = -1$.

Q.14 Solve system of linear equations, using matrix method.

$x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Sol: The given system of equations can be written in the form of AX = B, where

$A=\begin{bmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$ and $B = \begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$

Now,

$|A| = 1(12 - 5) + 1(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4 \neq 0$

$\therefore$ A is non-singular. Therefore, A^{-1} exists.

Now,

$A_{11} = 7,\ A_{12} = -19,\ A_{13} = -11$

$A_{21} = 1, A_{22} = -1, A_{23} = -1$

$A_{31} = -3, A_{32} = 11, A_{33} = 7$

$$A^{-1} = \frac{1}{|A|}\ (\text{adj } A) = \frac{1}{4}\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}$$

$$\therefore\ X = A^{-1}B = \frac{1}{4}\begin{bmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{bmatrix}\begin{bmatrix}7\\ -5\\ 12\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \frac{1}{4}\begin{bmatrix}49 - 5 - 36\\ -133 + 5 + 132\\ -77 + 5 + 84\end{bmatrix} = \frac{1}{4}\begin{bmatrix}8\\ 4\\ 12\end{bmatrix} = \begin{bmatrix}2\\ 1\\ 3\end{bmatrix}$$

Hence, $x = 2$, $y = 1$ and $z = 3$.

Q.15 If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, , find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

Sol: $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$,

$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = 0 - 6 + 5 = -1 \neq 0$

Now,

$A_{11} = 0,\ A_{12} = -2,\ A_{13} = 1$

$A_{21} = -1, A_{22} = -9, A_{23} = -5$

$A_{31} = 2, A_{32} = 23, A_{33} = 13$

$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ .... (1)

Now, the given system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, , $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

The solution of the system of equation is given by $X = A^{-1}B$.

$X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ [Using (1)]

$= \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Hence, $x = 1$, $y = 2$ and $z = 3$.

Q.16 The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

Sol: Let the cost of onions, wheat, and rice per kg be Rs x, Rs y, and Rs z respectively. Then, the given situation can be represented by a system of equations as:

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

This system of equations can be written in the form of $AX = B$, where

$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, , $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$|A| = 4(12 - 12) - 3(6 - 36) + 2(4 - 24) = 0 + 90 - 40 = 50 \neq 0$

Now,

$A_{11} = 0,\ A_{12} = 30,\ A_{13} = -20$

$A_{21} = -5, A_{22} = 0, A_{23} = 10$

$A_{31} = 10, A_{32} = -20, A_{33} = 10$

$\therefore \text{adj } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|}$ (adj A) $= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

Now,

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix}$

$= \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$

$= \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$

$\therefore x = 5$, $y = 8$ and $z = 8$.

Hence, the cost of onions is Rs 5 per kg, the cost of wheat is Rs 8 per kg, and the cost of rice is Rs 8 per kg.