SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $\frac{x}{a}+\frac{y}{b}=1$

Ans. $\frac{x}{a}+\frac{y}{b}=1$

Differentiating both sides of the given equation with respect to x, we get:

$$\frac{1}{a}+\frac{1}{b}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{1}{a}+\frac{1}{b}y'=0$$

Again, differentiating both sides with respect to x, we get:

$$0+\frac{1}{b}y'=0$$

$$\Rightarrow \frac{1}{b}y^{m}=0$$
$$\Rightarrow y^{m}=0$$

Hence, the required differential equation of the given curve is $y^{m}=0$

2. $y^2=a\left(b^2-x^2\right)$

Ans. $y^2=a\left(b^2-x^2\right)$

Differentiating both sides with respect to x, we get:

$$2y^2\frac{dy}{dx}=a\left(-2x\right)$$

$$\Rightarrow 2yy'=-2ax$$
$$\Rightarrow yy'=-ax....(1)$$

Again, differentiating both sides with respect to x, we get:

$$y'.y'+yy^{m}=-a$$
$$\Rightarrow \left(y'\right)^2+yy^{m}=-a.....(2)$$

Dividing equation (2) by equation (1), we get:

$$\frac{\left(y'\right)^2+yy^{m}}{yy'}=\frac{-a}{-ax}$$

$\Rightarrow xyy''' + x(y')^2 - yy''' = 0$

This is the required differential equation of the given curve.

3. $y = ae^{3x} + be^{-2x}$

Ans. $y = ae^{3x} + be^{-2x}$....(1)

Differentiating both sides with respect to x, we get:

$y = 3ae^{3x} - be^{-2x}$....(2)

Again, differentiating both sides with respect to x, we get:

$y''' = 9ae^{3x} - 4be^{-2x}$....(3)

Multiplying equation (1) with (2) and then adding it to equation (2), we get:

$$\left(2ae^{3x} + 2be^{-2x}\right) + \left(3ae^{3x} - 2bc^{-2x}\right) = 2y + y'$$

$$\Rightarrow 5ae^{3x} = 2y + y'$$

$$\Rightarrow ae^{3x} = \frac{2y + y'}{5}$$

Now, multiplying equation (1) with equation (3) and subtracting equation (2) from it, we get:

$$\left(3ae^{3x} + 3be^{-2x}\right) - \left(3ae^{-2x} - 2be^{-2x}\right) = 3y - y'$$

$$\Rightarrow 5be^{-2x} = 3y - y'$$

$$\Rightarrow be^{-2x} = \frac{3y - y'}{5}$$

Substituting the values of ae^{3x} and be^{-2x} in equation (3), we get:

$$y'' = 9.\frac{(2y + y')}{5} + 4\frac{(3y - y')}{5}$$

$$\Rightarrow y'' = \frac{18y + 9y'}{5} + \frac{12y - 4y'}{5}$$

$$\Rightarrow y'' = \frac{30y + 5y'}{5}$$

$$\Rightarrow y'' = 6y + y'$$
$$\Rightarrow y'' - y' - 6y = 0$$

This is the required differential equation of the given curve.

4. $y = e^{2x}(a + bx)$

Ans. $y = e^{2x}(a + bx)$....(1)

Differentiating both sides with respect to x, we get:

$$y = 2e^{2x}(a+bx) + e^{2x}.b$$
$$\Rightarrow y' = e^{2x}(2a+2bx+b).....(2)$$

Multiplying equation (1) with equation (2) and then subtracting it from equation (2), we get:

$$y' - 2y = e^{2x}(2a+2bx+b) - e^{2x}(2a+2bx)$$

$$\Rightarrow y' - 2 = be^{2x}.....(3)$$

Differentiating both sides with respect to x, we get:

$$y''k - 2y' = 2be^{2x}.....(4)$$

Dividing equation (4) by equation (3), we get:

$$\frac{y'' - 2y'}{y' - 2y} = 2$$

$$\Rightarrow y'' - 2y' = 2y' - 4y$$
$$\Rightarrow y'' - 4y' + 4y = 0$$

This is the required differential equation of the given curve.

5. $y = e^{x}(a\cos x + b\sin x)$

Ans. $y = e^{x}(a\cos x + b\sin x)....(1)$

Differentiating both sides with respect to x, we get:

$$y' = e^{x}(a\cos x + b\sin x) + e^{x}(-a\sin x + b\cos x)$$

$$\Rightarrow y' = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right]......(2)$$

Again, differentiating with respect to x, we get:

$$y' = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right] + e^{x}\left[-(a+b)\sin x - (a-b)\operatorname{cox}\right]$$

$$y'' = e^{x}\left[2b\cos x - 2a\sin x\right]$$

$$y'' = 2e^{x}\left[b\cos x - a\sin x\right]$$

$$\Rightarrow \frac{y''}{2} = e^{x}\left[b\cos x - a\sin x\right].....(3)$$

Adding equations (1) and (3), we get:

$$y + \frac{y''}{2} = e^{x}\left[(a+b)\cos x - (a-b)\sin x\right]$$

$$\Rightarrow y+\frac{y''}{2}=y'$$

$$\Rightarrow 2y+y''=2y'$$

$$\Rightarrow y''+2y'=2y=0$$

This is the required differential equation of the given curve.

6. Form the differential equation of the family of circles touching the y-axis at the origin.

Ans. The center of the circle touching the y-axis at origin lies on the x-axis.

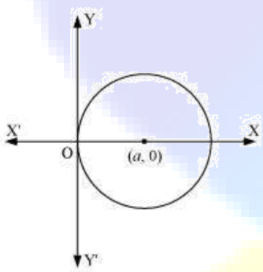
Let (a, 0) be the center of the circle.

Since it touches the y-axis at origin, its radius is a.

Now, the equation of the circle with center (a, 0) and radius (a) is

$$\left(x-a\right)^2+y^2=a^2.$$

$$\Rightarrow x^2+y^2=2ax....(1)$$



Differentiating equation (1) with respect to x, we get:

$$2x+2yy'=2a$$

$$\Rightarrow x+yy'=a$$

Now, on substituting the value of a in equation (1), we get:

$$x^2+y^2=2\left(x+yy'\right)x$$

$$\Rightarrow x^2+y^2=2x^2+2xyy'$$

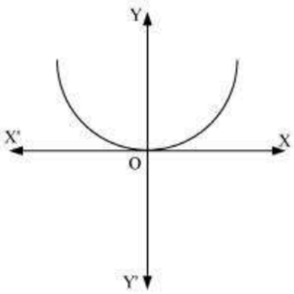
$$\Rightarrow 2xyy'+x^2=y^2$$

This is the required differential equation.

7. Form the differential equation of the family of parabolas having vertex at origin and axis along positive y-axis.

Ans. The equation of the parabola having the vertex at origin and the axis along the positive y-axis is:

$x^2 = 4ay...(1)$



Differentiating equation (1) with respect to x, we get:

$2x = 4ay'...(2)$

Dividing equation (2) by equation (1), we get:

$$\frac{2x}{x^2} = \frac{4ay'}{4ay}$$

$$\Rightarrow \frac{2}{x} = \frac{y'}{y}$$

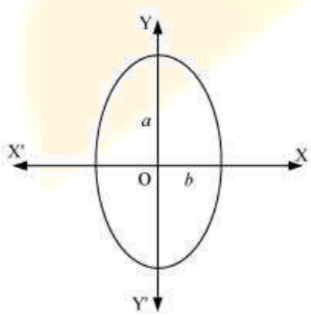
$$\Rightarrow xy' = 2y$$
$$\Rightarrow xy' - 2y = 0$$

This is the required differential equation.

8. Form the differential equation of the family of ellipses having foci on y-axis and center at origin.

Ans The equation of the family of ellipses having foci on the y-axis and the center at origin is as follows:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.........(1)$$



Differentiating equation (1) with respect to x, we get:

$$\frac{2x}{b^2} + \frac{2yy'}{b^2} = 0$$

$$\Rightarrow \frac{x}{b^2} + \frac{yy'}{a^2} = 0.......(2)$$

Again, differentiating with respect to x, we get:

$$\frac{1}{b^2}+\frac{y'.y'+y.y''}{a^2}=0$$

$$\Rightarrow \frac{1}{b^2}+\frac{1}{a^2}\left(y'^2+yy''\right)=0$$

$$\Rightarrow \frac{1}{b^2}=-\frac{1}{a^2}\left(y'^2+yy''\right)$$

Substituting this value in equation (2), we get:

$$x\left[-\frac{1}{a^2}\left(\left(y'\right)^2+yy'\right)\right]+\frac{yy'}{a^2}=0$$

$$\Rightarrow -x\left(y'\right)^2-xyy''+yy'=0$$

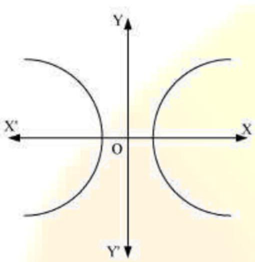
$$\Rightarrow xyy''+\left(y'\right)^2-yy'=0$$

This is the required differential equation.

9. Form the differential equation of the family of hyperbolas having foci on x-axis and centre at origin.

Ans. The equation of the family of hyperbolas with the centre at origin and foci along the x axis is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 ....(1)$$



Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{1}{a^2}-\frac{y'.y'+yy''}{b^2}=0$$

$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left(\left(y'\right)^2+yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left(\left(y'\right)^2+yy''\right)-\frac{yy'}{b^2}=0$$

$$\Rightarrow x\left(y'\right)^2+xyy''-yy'=0$$

$$\Rightarrow xyy''+x\left(y'\right)^2-yy'=0$$

This is the required differential equation.

10. Form the differential equation of the family of circles having centre on y-axis and radius 3 units.

Ans. Let the centre of the circle on y-axis be (0, b). The differential equation of the family of circles with centre at (0, b) and radius 3 is as follows:

$$x^2 + (y-b)^2 = 3^2$$
$$\Rightarrow x^2 + (y-b)^2 = 9 ....(1)$$

Differentiating equation (1) with respect to x, we get:

$$2x + 2(y-b).y' = 0$$
$$\Rightarrow (y-b).y' = -x$$
$$\Rightarrow y - b = \frac{-x}{y'}$$

Substituting the value of (y – b) in equation (1), we get:

$$x^2 + \left(\frac{-x}{y}\right)^2 = 9$$

$$\Rightarrow x^2\left[1 + \frac{1}{(y')}\right] = 9$$

$$\Rightarrow x^2\left((y')^2 + 1\right) = 9(y')^2$$

$$\Rightarrow (x^2 - 9)(y')^2 + x^2 = 0$$

This is the required differential equation.

11. Which of the following differential equations has $y = c_1e^x + c_2e^{-x}$ as the general solution?

(A) $\frac{d^2y}{dx^2} + y = 0$ (B) $\frac{d^2y}{dx^2} - y = 0$ (C) $\frac{d^2y}{dx^2} + 1 = 0$ (D) $\frac{d^2y}{dx^2} - 1 = 0$

Ans. The given equation is:

$$y = c_1e^x + c_2e^{-x} ....(1)$$

Differentiating with respect to x, we get:

$$\frac{dy}{dx} = c_1e^x - c_2e^{-x}$$

Again, differentiating with respect to x, we get:

$$\frac{dy}{dx^2} = c_1e^x + c_2e^{-x}$$

$$\Rightarrow \frac{dy}{dx^2} = y$$

$$\Rightarrow \frac{d^2y}{dx^2} - y = 0$$

This is the required differential equation of the given equation of curve. Hence, the correct answer is B.

12. Which of the following differential equation has $y = x$ as one of its particular solution?

A. $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = x$

B. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = x$

C. $\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0$

D. $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + xy = 0$

Ans. The given equation of curve is y = x. Differentiating with respect to x, we get:

$$\frac{dy}{dx} = 1 \ldots\ldots\ldots (1)$$

Again, differentiating with respect to x, we get:

$$\frac{d^2y}{dx^2} = 0 \ldots\ldots\ldots (2)$$

Now, on substituting the values of y, $\frac{d^2y}{dx^2}$, and $\frac{dy}{dx}$ from equation (1) and (2) in each of the given alternatives, we find that only the differential equation given in alternative C is correct.

$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 - x^2.1 + x.x$$

$$= -x^2 + x^2$$

$$= 0$$

Hence, the correct answer is C