



SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Differential Equations

Exercise 9.5

1. $(x^2 + xy)dy = (x^2 + y^2)dx$

Ans. The given differential equation i.e., $(x^2 + xy) dy = (x^2 + y^2) dx$ can be written as:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy} \dots\dots\dots(1)$$

$$\text{Let } F(x, y) = \frac{x^2 + y^2}{x^2 + xy}.$$

$$\text{Now, } F(\lambda x, \lambda y) = \frac{(\lambda x)^2 + (\lambda y)^2}{(\lambda x)^2 + (\lambda x)(\lambda y)} = \frac{x^2 + y^2}{x^2 + xy} = \lambda^0 \cdot F(x, y)$$

This shows that equation (1) is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

Differentiating both sides with respect to x, we get:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of v and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dy}{dx} = \frac{x^2 + (vx)^2}{x^2 + x(vx)}$$

$$\Rightarrow v + x \frac{dy}{dx} = \frac{1 + v^2}{1 + v}$$

$$\Rightarrow x \frac{dy}{dx} = \frac{1 + v^2}{1 + v} - v = \frac{(1 + v)^2 - v(1 + v)}{1 + v}$$

$$\Rightarrow x \frac{dy}{dx} = \frac{1 - v}{1 + v}$$

$$\Rightarrow \left(\frac{1 + v}{1 - v} \right) = dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{2-1+v}{1-v} \right) = dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{2}{1-v} - 1 \right) = dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$-\log(1-v) - v = \log x - \log k$$

$$\Rightarrow v = -2\log(1-v) - \log x + \log k$$

$$\Rightarrow v = \log \left[\frac{k}{x(1-v)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[\frac{k}{x \left(1 - \frac{y}{x} \right)^2} \right]$$

$$\Rightarrow \frac{y}{x} = \log \left[\frac{k}{(x-y)^2} \right]$$

$$\Rightarrow \frac{kx}{(x-y)^2} = e^{\frac{y}{x}}$$

$$\Rightarrow (x-y)^2 = kxe^{-\frac{y}{x}}$$

This is the required solution of the given differential equation.

2. $y' = \frac{x+y}{x}$

Ans. The given differential equation is:

$$y' = \frac{x+y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x} \dots\dots\dots(1)$$

$$\text{Let } F(x,y) = \frac{x+y}{x}.$$

$$\text{Now, } F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x} = \frac{x + y}{x} = \lambda^0 F(x, y)$$

Thus, the given equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

Differentiating both sides with respect to x, we get:

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x + vx}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 + v$$

$$x \frac{dv}{dx} = 1$$

$$\Rightarrow dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$v = \log x + C$$

$$\Rightarrow \frac{y}{x} = \log x + C$$

$$\Rightarrow y = x \log x + Cx$$

This is the required solution of the given differential equation.

3. $(x - y)dy - (x + y)dx = 0$

Ans. The given differential equation is:

$$(x - y)dy - (x + y)dx = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y} \dots\dots\dots(1)$$

$$\text{Let } F(x, y) = \frac{x + y}{x - y}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda x + \lambda y}{\lambda x - \lambda y} = \frac{x + y}{x - y} = \lambda^0 \cdot F(x, y)$$

Thus, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{d}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{x + vx}{x - vx} = \frac{1 + v}{1 - v}$$

$$x \frac{dv}{dx} = \frac{1 + v}{1 - v} - v = \frac{1 + v - v(1 - v)}{1 - v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{1 + v^2}{1 - v}$$

$$\Rightarrow \frac{1 - v}{(1 + v^2)} dv = \frac{dx}{x}$$

$$\Rightarrow \left(\frac{1}{1 + v^2} - \frac{v}{1 - v^2} \right) dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\tan^{-1} v - \frac{1}{2} \log(1 + v^2) = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log \left[1 + \left(\frac{y}{x} \right)^2 \right] = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log \left(\frac{x^2 + y^2}{x^2} \right) = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} [\log(x^2 + y^2) - \log x^2] = \log x + C$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) - \frac{1}{2} \log(x^2 + y^2) + C$$

This is the required solution of the given differential equation.

$$4. \Rightarrow \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2} \log\left(\frac{x^2 + y^2}{x^2}\right) = \log x + C$$

Ans. The given differential equation is:

$$(x^2 - y^2)dx + 2xydy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x^2 - y^2)}{2xy} \dots\dots (1)$$

$$\text{Let } F(x, y) = \frac{-(x^2 - y^2)}{2xy}$$

$$\therefore F(\lambda x, \lambda y) = \left[\frac{(\lambda x)^2 - (\lambda y)^2}{2(\lambda x)(\lambda y)} \right] = \frac{-(x^2 - y^2)}{2xy} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = - \left[\frac{x^2 - (vx)^2}{2x(vx)} \right]$$

$$v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\Rightarrow x \frac{dv}{dx} = - \frac{(1 + v^2)}{2v}$$

$$\Rightarrow \frac{2v}{1 + v^2} dv = - \frac{dx}{x}$$

Integrating both sides, we get:

$$\log(1+v^2) = -\log x + \log C = \log \frac{C}{x}$$

$$\Rightarrow 1+v^2 = \frac{C}{x}$$

$$\Rightarrow \left[1 + \frac{y^2}{x^2}\right] = \frac{C}{x}$$

$$\Rightarrow x^2 + y^2 = Cx$$

This is the required solution of the given differential equation.

5. $x^2 \frac{dy}{dx} - x^2 - 2y^2 + xy$

Ans. The given differential equation is:

$$x^2 \frac{dy}{dx} - x^2 - 2y^2 + xy$$

$$\frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2} \dots\dots\dots (1)$$

$$\text{Let } F(x,y) = \frac{x^2 - 2y^2 + xy}{x^2}$$

$$\therefore F(\lambda x, \lambda y) = \frac{(\lambda x)^2 - 2(\lambda y)^2 + (\lambda x)(\lambda y)}{(\lambda x)^2} = \frac{x^2 - 2y^2 + xy}{x^2} = \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get: Integrating both

$$v + x \frac{dv}{dx} = \frac{x^2 - 2(vx)^2 + x \cdot (vx)}{x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = 1 - 2v^2 + v$$

$$\Rightarrow x \frac{dv}{dx} = 1 - 2v^2$$

$$\Rightarrow \frac{dv}{1-2v^2} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \cdot \frac{dv}{\frac{1}{2}-v^2} = \frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \cdot \left[\frac{dv}{\left(\frac{1}{\sqrt{2}}\right)^2 - v^2} \right] = \frac{dx}{x}$$

Integrating both sides, we get:

$$\frac{1}{2} \cdot \frac{1}{2 \times \frac{1}{\sqrt{2}}} \log \left| \frac{\frac{1}{\sqrt{2}} + v}{\frac{1}{\sqrt{2}} - v} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{\frac{1}{\sqrt{2}} + \frac{y}{x}}{\frac{1}{\sqrt{2}} - \frac{y}{x}} \right| = \log|x| + C$$

$$\Rightarrow \frac{1}{2\sqrt{2}} \log \left| \frac{x + \sqrt{2}y}{x - \sqrt{2}y} \right| = \log|x| + C$$

This is the required solution for the given differential equation.

6. $xdy - ydx = \sqrt{x^2 + y^2} dx$

Ans. $xdy - ydx = \sqrt{x^2 + y^2} dx$

$$\Rightarrow xdy = \left[y + \sqrt{x^2 + y^2} \right] dx$$

$$\frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x^2} \dots\dots\dots(1)$$

$$\text{Let } F(x,y) = \frac{y + \sqrt{x^2 + y^2}}{x^2}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda x + \sqrt{(\lambda x)^2 + (\lambda y)^2}}{\lambda x^2} = \frac{y + \sqrt{x^2 + y^2}}{x^2} = \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of v and in equation (1), we get:

$$v + x \frac{dy}{dx} = \frac{vx + \sqrt{x^2 + (vx)^2}}{x}$$

$$\Rightarrow v + x \frac{dy}{dx} = v + \sqrt{1 + v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\log|v + \sqrt{1 + v^2}| = \log|x| + \log C$$

$$\Rightarrow \log\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right| = \log|Cx|$$

$$\Rightarrow \log\left|\frac{y + \sqrt{x^2 + y^2}}{x}\right| = \log|Cx|$$

$$\Rightarrow y + \sqrt{x^2 + y^2} = Cx^2$$

This is the required solution of the given differential equation.

7. $\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x dy$

Ans. The given differential equation is:

$$\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y dx = \left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x dy$$

$$\frac{dy}{dx} = \frac{\left\{x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right)\right\} y}{\left\{y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right)\right\} x} \dots\dots\dots(1)$$

$$\text{Let } F(x,y) = \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\left\{ \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) + \lambda y \sin\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda y}{\left\{ y \sin\left(\frac{\lambda y}{\lambda x}\right) - \lambda x \cos\left(\frac{\lambda y}{\lambda x}\right) \right\} \lambda x}$$

$$= \frac{\left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y}{\left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x}$$

$$= \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x = \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{(x \cos v + vx \sin v) \cdot vx}{(vx \sin v - x \cos v) \cdot x}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v}{v \sin v - \cos v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \cos v + v^2 \sin v - v^2 + v \cos v}{v \sin v - \cos v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$$

$$\Rightarrow \left[\frac{v \sin v - \cos v}{v \cos v} \right] dv = \frac{2dx}{x}$$

$$\Rightarrow \left(\tan v - \frac{1}{v} \right) dv = \frac{2dx}{x}$$

Integrating both sides, we get:

$$\log(\sec v) - \log v = 2 \log x + \log C$$

$$\Rightarrow \log \left(\frac{\sec v}{v} \right) = \log(Cx^2)$$

$$\Rightarrow \left(\frac{\sec v}{v} \right) = Cx^2$$

$$\Rightarrow \sec v = Cx^2 v$$

$$\Rightarrow \sec \left(\frac{y}{x} \right) = C \cdot x^2 \cdot \frac{y}{x}$$

$$\Rightarrow \sec \left(\frac{y}{x} \right) = Cxy$$

$$\Rightarrow \cos \left(\frac{y}{x} \right) = \frac{1}{Cxy} = \frac{1}{C} \cdot \frac{1}{xy}$$

$$\Rightarrow xy \cos \left(\frac{y}{x} \right) = k \left(k = \frac{1}{C} \right)$$

This is the required solution of the given differential equation.

8. $x \frac{dy}{dx} - y + x \sin \left(\frac{y}{x} \right) = 0$

Ans. $x \frac{dy}{dx} - y + x \sin \left(\frac{y}{x} \right) = 0$

$$\Rightarrow x \frac{dy}{dx} = y - x \sin \left(\frac{y}{x} \right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x \sin \left(\frac{y}{x} \right)}{x} \dots \dots (1)$$

$$\text{Let } F(x, y) = \frac{y - x \sin \left(\frac{y}{x} \right)}{x}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y - \lambda x \sin\left(\frac{\lambda y}{\lambda x}\right)}{\lambda x} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{vx - x \sin v}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \sin v$$

$$\Rightarrow -\frac{dv}{\sin v} = \frac{dx}{x}$$

$$\Rightarrow \operatorname{cosec} v dv = -\frac{dx}{x}$$

Integrating both sides, we get:

$$\log|\operatorname{cosec} v - \cot v| = -\log x + \log C = \log \frac{C}{x}$$

$$\Rightarrow \operatorname{cosec}\left(\frac{y}{x}\right) - \cot\left(\frac{y}{x}\right) = \frac{C}{x}$$

$$\Rightarrow \frac{1}{\sin\left(\frac{y}{x}\right)} - \frac{\cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{C}{x}$$

$$\Rightarrow x \left[1 - \cos\left(\frac{y}{x}\right) \right] = C \sin\left(\frac{y}{x}\right)$$

This is the required solution of the given differential equation.

9. $y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0$

Ans. $ydx + x \log\left(\frac{y}{x}\right)dy - 2xdy = 0$

$$\Rightarrow ydx \left[2x - x \log\left(\frac{y}{x}\right) \right] dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2x - x \log\left(\frac{y}{x}\right)} \dots\dots (1)$$

$$\text{Let } F(x,y) = \frac{y}{2x - x \log\left(\frac{y}{x}\right)}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y}{2(\lambda x) - (\lambda x) \log\left(\frac{\lambda y}{\lambda x}\right)} = \frac{y}{2x - \log\left(\frac{y}{x}\right)} = \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{vx}{2x - x \log v}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v}{2 - \log v} - v$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - 2v + v \log v}{2 - \log v}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v \log v - v}{2 - \log v}$$

$$\Rightarrow \frac{2 - \log v}{v(\log v - 1)} dv = \frac{dx}{x}$$

$$\Rightarrow \left[\frac{1 + (1 - \log v)}{v(\log v - 1)} \right] dv = \frac{dx}{x}$$

$$\Rightarrow \left[\frac{1}{v(\log v - 1)} - \frac{1}{v} \right] dv = \frac{dx}{x}$$

$$\int \frac{1}{v(\log v - 1)} dv - \int \frac{1}{v} dv = \int \frac{1}{x} dx$$

$$\Rightarrow \int \frac{dv}{v(\log v - 1)} - \log v = \log x + \log C \dots \dots (2)$$

$$\Rightarrow \text{Let } \log v - 1 = t$$

$$\Rightarrow \frac{d}{dv}(\log v - 1) = \frac{dt}{dv}$$

$$\Rightarrow \frac{1}{v} = \frac{dt}{dv}$$

$$\Rightarrow \frac{dv}{v} = dt$$

Therefore, equation (1) becomes:

$$\Rightarrow \int \frac{dt}{t} - \log v = \log x + \log C$$

$$\Rightarrow \log t - \log \left(\frac{y}{x} \right) = \log(Cx)$$

$$\Rightarrow \log \left[\log \left(\frac{y}{x} \right) - 1 \right] - \log \left(\frac{y}{x} \right) = \log(Cx)$$

$$\Rightarrow \log \left[\frac{\log \left(\frac{y}{x} \right) - 1}{\frac{y}{x}} \right] = \log(Cx)$$

$$\Rightarrow \frac{x}{y} \left[\log \left(\frac{y}{x} \right) - 1 \right] = \log Cx$$

$$\Rightarrow \log\left(\frac{y}{x}\right) - 1 = Cy$$

This is the required solution of the given differential equation.

10. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

Ans. $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

$$\Rightarrow \left(1 + e^{\frac{x}{y}}\right) dx = -e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}} \dots\dots\dots (1)$$

$$\text{Let } F(x, y) = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$$

$$\therefore F(\lambda x, \lambda y) = \frac{-e^{\frac{\lambda x}{\lambda y}} \left(1 - \frac{\lambda x}{\lambda y}\right)}{1 + e^{\frac{\lambda x}{\lambda y}}} = \frac{-e^{\frac{\lambda x}{\lambda y}} \left(1 - \frac{\lambda x}{\lambda y}\right)}{1 + e^{\frac{\lambda x}{\lambda y}}} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$x = vy$$

$$\Rightarrow \frac{d}{dy}(x) = \frac{d}{dy}(vy)$$

$$\Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$$

Substituting the values of x and in equation (1), we get:

$$v + y \frac{dy}{dy} = \frac{-e^v (1 - v)}{1 + e^v}$$

$$\Rightarrow y \frac{dy}{dy} = \frac{-e^v (1 - v)}{1 + e^v} - v$$

$$\Rightarrow y \frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1 + e^v}$$

$$\Rightarrow y \frac{dv}{dy} = - \left[\frac{v + e^v}{1 + e^v} \right]$$

$$\Rightarrow \left[\frac{1 + e^v}{v + e} \right] dv = - \frac{dy}{y}$$

Integrating both sides, we get:

$$\Rightarrow \log(v + e^v) = -\log y + \log C = \log \left(\frac{C}{y} \right)$$

$$\Rightarrow \left[\frac{x}{y} + e^{\frac{x}{y}} \right] = \frac{C}{y}$$

$$\Rightarrow x + ye^{\frac{x}{y}} = C$$

This is the required solution of the given differential equation.

11. $(x + y)dy + (x - y)dx = 0; y = 1 \text{ when } x = 1$

Ans. $(x + y)dy + (x - y)dx = 0$

$$\Rightarrow (x + y)dy = -(x - y)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(x - y)}{x + y} \dots\dots(1)$$

$$\text{let } F(x, y) = \frac{-(x - y)}{x + y}.$$

$$\therefore F(x, y) = \frac{-(\lambda x - \lambda y)}{\lambda x + \lambda y} = \frac{-(x - y)}{x + y} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{-(x - vx)}{x + vx}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{v - 1}{v + 1}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - 1}{v + 1} - v = \frac{v - 1 - v(v + 1)}{v + 1}$$

$$\Rightarrow x \frac{dv}{dx} = \frac{v - 1 - v^2 - v}{v + 1} = \frac{-(1 + v^2)}{v + 1}$$

$$\Rightarrow \frac{(v + 1)}{1 + v^2} dv = -\frac{dx}{x}$$

$$\Rightarrow \left[\frac{v}{1 + v^2} + \frac{1}{1 + v^2} \right] dv = -\frac{dx}{x}$$

Integrating both sides, we get:

$$\frac{1}{2} \log(1 + v^2) + \tan^{-1} v = -\log x + k$$

$$\Rightarrow \log(1 + v^2) + 2 \tan^{-1} v = -2 \log x + 2k$$

$$\Rightarrow \log[(1 + v^2) \cdot x^2] + 2 \tan^{-1} v = 2k$$

$$\Rightarrow \log \left[\left(1 + \frac{y^2}{x^2} \right) \cdot x^2 \right] + 2 \tan^{-1} \frac{y}{x} = 2k$$

$$\Rightarrow \log(x^2 + y^2) + 2 \tan^{-1} \frac{y}{x} = 2k \dots \dots (2)$$

Now, $y = 1$ at $x = 1$.

$$\Rightarrow \log 2 + 2 \tan^{-1} 1 = 2k$$

$$\Rightarrow \log 2 + 2 \times \frac{\pi}{4} = 2k$$

$$\Rightarrow \frac{\pi}{2} + \log 2 = 2k$$

Substituting the value of $2k$ in equation (2), we get:

$$\log(x^2 + y^2) + 2 \tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{2} + \log 2$$

This is the required solution of the given differential equation.

12. $x^2 dy + (xy + y^2) dx = 0; y = 1 \text{ when } x = 1$

Ans. $x^2 dy + (xy + y^2) dx = 0$

$$\Rightarrow x^2 dy = -(xy + y^2) dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(xy + y^2)}{x^2} \dots \dots \dots (1)$$

$$\text{Let } F(x, y) = \frac{-(xy + y^2)}{x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{-(\lambda x \cdot \lambda y + (\lambda y)^2)}{(\lambda x)^2} = \frac{-(xy + y^2)}{x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{-[x \cdot vx + (vx)^2]}{x^2} = -v - v^2$$

$$\Rightarrow x \frac{dv}{dx} = -v^2 - 2v = -v(v + 2)$$

$$\Rightarrow \frac{dv}{v(v + 2)} = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \left[\frac{(v + 2) - v}{v(v + 2)} \right] dv = -\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2} \left[\frac{1}{v} - \frac{1}{v+2} \right] dv = -\frac{dx}{x}$$

Integrating both sides, we get:

$$\frac{1}{2} [\log v - \log(v+2)] = -\log x + \log C$$

$$\Rightarrow \frac{1}{2} \log \left(\frac{v}{v+2} \right) = \log \frac{C}{x}$$

$$\Rightarrow \frac{v}{v+2} = \left(\frac{C}{x} \right)^2$$

$$\Rightarrow \frac{\frac{y}{x}}{\frac{y}{x} + 2} = \left(\frac{C^2}{x} \right)$$

$$\Rightarrow \frac{y}{y+2x} = \frac{C^2}{x^2}$$

$$\Rightarrow \frac{x^2 y}{y+2x} = C^2 \dots\dots (2)$$

Now, $y = 1$ at $x = 1$.

$$\Rightarrow \frac{1}{1+2} = C^2$$

$$\Rightarrow C^2 = \frac{1}{3}$$

Substituting $C^2 = \frac{1}{3}$ in equation (2), we get:

$$\frac{x^2 y}{y+2x} = \frac{1}{3}$$

$$\Rightarrow y+2x = 3x^2 y$$

This is the required solution of the given differential equation.

13. $\left[x \sin^2 \left(\frac{x}{y} - y \right) \right] dx + x dy = 0; y = \frac{\pi}{4} \text{ when } x = 1$

Ans. $\left[x \sin^2 \left(\frac{x}{y} \right) - y \right] dx + x dy = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{\left[x \sin^2\left(\frac{y}{x}\right) - y \right]}{x} \dots\dots(1)$$

$$\text{Let } F(x,y) = -\frac{\left[x \sin^2\left(\frac{y}{x}\right) - y \right]}{x}.$$

$$\therefore F(\lambda x, \lambda y) = -\frac{\left[\lambda x \sin^2\left(\frac{\lambda y}{\lambda x}\right) - \lambda y \right]}{\lambda x} = -\frac{\left[x \sin^2\left(\frac{y}{x}\right) - y \right]}{x} = \lambda^0 \cdot F(x,y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve this differential equation, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = -\frac{\left[x \sin^2 v - vx \right]}{x}$$

$$\Rightarrow v + x \frac{dv}{dx} = -\left[\sin^2 v - v \right] = v - \sin^2 v$$

$$\Rightarrow x \frac{dv}{dx} = -\sin^2 v$$

$$\Rightarrow \frac{dv}{\sin^2 v} = -\frac{dx}{x}$$

$$\Rightarrow \operatorname{cosec}^2 v dv = -\frac{dx}{x}$$

Integrating both sides, we get:

$$-\cot v = -\log|x| - C$$

$$\Rightarrow \cot v = \log|x| + C$$

$$\Rightarrow \cot\left(\frac{y}{x}\right) = \log|x| + \log C$$

$$\Rightarrow \cot\left(\frac{y}{x}\right) = \log|Cx| \dots (2)$$

$$\text{Now, } y = \frac{\pi}{4} \text{ at } x = 1$$

$$\Rightarrow \cot\left(\frac{\pi}{4}\right) = \log|C|$$

$$\Rightarrow 1 = \log C$$

$$\Rightarrow C = e^1 = e$$

Substituting $C = e$ in equation (2), we get:

$$\cot\left(\frac{y}{x}\right) = \log|ex|$$

This is the required solution of the given differential equation.

14. $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0; y = 0 \text{ when } x = 1$

Ans. $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$

$$\Rightarrow \frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0 \dots (1)$$

$$\text{Let } F(x, y) = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right).$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} - \operatorname{cosec}\left(\frac{\lambda y}{\lambda x}\right)$$

$$F(\lambda x, \lambda y) = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right) = F(x, y) = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{d}{dx}$$

Substituting the values of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = v - \operatorname{cosec} v$$

$$\Rightarrow -\frac{dv}{\operatorname{cosec} v} = \frac{dx}{x}$$

$$\Rightarrow -\sin v dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$\cos v = \log x + \log C \log |Cx|$$

$$\cos\left(\frac{y}{x}\right) = \log |Cx| \dots \dots (2)$$

This is the required solution of the given differential equation.

Now, $y = 0$ at $x = 1$.

$$\Rightarrow \cos(0) = \log C$$

$$\Rightarrow 1 = \log C$$

$$\Rightarrow C = e^1 = e$$

Substituting $C = e$ in equation (2), we get:

$$\cos\left(\frac{y}{x}\right) = \log |ex|$$

This is the required solution of the given differential equation.

15. $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0; y = 2 \text{ when } x = 1$

Ans. $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$

$$\Rightarrow 2x^2 \frac{dy}{dx} = 2xy + y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy + y^2}{2x^2} \dots \dots (1)$$

$$\text{Let } F(x, y) = \frac{2xy + y^2}{2x^2}.$$

$$\therefore F(\lambda x, \lambda y) = \frac{2(\lambda x)(\lambda y) + (\lambda y)^2}{2(\lambda x)^2} = \frac{2xy + y^2}{2x^2} = \lambda^0 \cdot F(x, y)$$

Therefore, the given differential equation is a homogeneous equation.

To solve it, we make the substitution as:

$$y = vx$$

$$\Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(vx)$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the value of y and $\frac{dy}{dx}$ in equation (1), we get:

$$v + x \frac{dv}{dx} = \frac{2x(vx) + (vx)^2}{2x^2}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{2v + v^2}{2}$$

$$\Rightarrow v + x \frac{dv}{dx} = v + \frac{v^2}{2}$$

$$\Rightarrow \frac{2}{v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get:

$$2 \cdot \frac{v^{-2+1}}{-2+1} = \log|x| + C$$

$$\Rightarrow -\frac{2}{v} = \log|x| + C$$

$$\Rightarrow -\frac{2}{\frac{y}{x}} = \log|x| + C$$

$$\Rightarrow -\frac{2x}{y} = \log|x| + C \dots (2)$$

Now, $y = 2$ at $x = 1$.

$$\Rightarrow -1 = \log(1) + C$$

$$\Rightarrow C = -1$$

Substituting $C = -1$ in equation (2), we get:

$$-\frac{2x}{y} = \log|x| - 1$$

$$\Rightarrow -\frac{2x}{y} = 1 - |x|x$$

$$\Rightarrow y = \frac{2x}{1 - \log|x|}, (x \neq 0, x \neq e)$$

This is the required solution of the given differential equation.

16. A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution

A. $y = vx$

B. $v = yx$

C. $x = vy$

D. $x = v$

Ans. For solving the homogeneous equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$, we need to make the substitution as $x =$

vy . Hence, the correct answer is C.

17. Which of the following is a homogeneous differential equation?

A. $(4x + 6y + 5)dy - (3y + 2x + 4)dx = 0$

B. $(xy)dx - (x^3 + y^3)dy = 0$

C. $(x^3 + 2y^2)dx + 2xydy = 0$

D. $y^2dx + (x^2 - xy^2 - y^2)dy = 0$

Ans. Function $F(x, y)$ is said to be the homogenous function of degree n , if

$$F(\lambda x, \lambda y) = \lambda^n F(x, y) \text{ for any non-zero constant } (\lambda).$$

Consider the equation given in alternative D:

$$y^2dx + (x^2 - xy - y^2)dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{x^2 - xy - x^2} = \frac{y^2}{y^2 + xy - x^2}$$

$$\text{Let } F(x, y) = \frac{y^2}{y^2 + xy - x^2}.$$

$$\Rightarrow F(\lambda x, \lambda y) = \frac{(\lambda y)^2}{(\lambda y)^2 + (\lambda x)(\lambda y) - (\lambda x)^2}.$$

$$= \frac{\lambda^2 y^2}{\lambda^2 (y^2 + xy - x^2)}$$

$$= \lambda^0 \frac{y^2}{(y^2 + xy - x^2)}$$

$$= \lambda^0 \cdot F(x, y)$$

Hence, the differential equation given in alternative D is a homogenous equation.