SpeedLabs

MATHS

CBSE 8th

TEEVRA EDUTECH PVT. LTD.

Q.1 Factorise the following expressions.

(i) $a^2 + 8a + 16$ (ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$ (iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$ (vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 41m$ [Hint: Expand $(l + m)^2$ first]

(viii) $a^4 + 2a^2b^2 + b^4$

Sol:

(i) $a^2 + 8a + 16$

By using identity

$x^2 + (a + b)x + ab = (x + a)(x + b)$

Product of constant terms in the expression is 16.

$ab = 16$

Two factors of product $= 8 \times 2$

Where $a = 8, b = 2$

$a + b = 8 + 2 = 10$

These values are not valid.

So, we take another values of a and b

$b = 4 \times 4 = 16$

Here, $a = 4$, $b = 4$ and $a + b = 4 + 4 = 8$

These values of a and b are valid.

$a^2 + 8a + 16 = a^2 + (4 + 4)a + 4 \times 4$

Hence, factors of $a^2 + 8a + 16 = (a + 4)(a + 4) = (a + 4)^2$.

(ii) $p^2 - 10p + 25$

By using identity

$x^2 + (a + b)x + ab = (x + a)(x + b)$

Product of constant terms is 25

ab =25

Product of two factors = -5×-5

Here a = - 5, b = - 5

Sum = - 5 - 5 = - 10

$p^2 + (-5 - 5)p + (-5)(-5)$

Hence, factors of $p^2 - 10p + 25 = [(p + (-5))(P) + (-5)]$

$= (p - 5)(p - 5) = (p - 5)^2$

(iii) $25m^2 + 30m + 9$

$= (5m)^2 + 2 \times 5m \times 3 + (3)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, a = 5m and b = 3

$= (5m + 3)^2 = (5m + 3)(5m + 3)$

Hence, factors of $25m^2 + 30m + 9 = (5m + 3)(5m + 3)$

$= (5m + 3)^2$

(iv) $49y^2 + 84yz + 36z^2$

$= (7y)^2 + 2 \times 7y \times 6z + (6Z)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, $a = 7y$ and $b = 6z$

$= (7y + 6z)^2 = (7y+ 6z)(7y + 6z)$

Hence, factors of $49y^2 + 84yz + 36z^2 = (7y + 6z)(7y + 6z)$

$= (7y + 6z)^2$.

(v) $4x^2 - 8x + 4$

$= (2x)^2 - 2 \times 2x \times 2 + (2)^2$

By using identity $(a - b)^2 = a^2- 2ab + b^2$

Here, $a= 2x$ and $b = 2$

$= (2x - 2)^2 = (2x - 2)(2x - 2)$

Hence, factors of $4x^2 - 8x + 4 = (2x - 2)(2x - 2)$

$= 2(x - 1)^2(x - 1) = 4(x - 1)^2$.

Alternate method

$4x^2 - 8x + 4 = 4(x^2 - 2x + 1)$

By comparing expression

$x^2 + (a + b)x + ab = (x + a)(x + b)$

$= ab = 1, a = - 1, b = - 1$

$= 4(x^2 + (- 1-1)x + (- 1)(- 1))$

$= 4[x + (- 1)] [x + (- 1)]$

$= 4(x - 1)(x - 1)$.

(vi) $121b^2 - 88bc + 16c^2$

$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$

By using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = 11b$ and $b = 4c$

$= (11b - 4c)^2 = (11b - 4c)(11b - 4c)$

Hence, factors of $121b^2 - 88bc + 16c^2$

$= (11b - 4c)(11b - 4c)$.

(vii) $(l + m)^2 - 41m$

By using identity $(a + b)^2 = a^2 + 2ab + b^2$

$= (l)^2 + 2 \times 1 \times m + (m)^2 - 41m$

$= l^2 + 21m + m^2 - 41m$

$= l^2 - 21m + m^2 = (l)^2 - 2 \times 1 \times m + (m)^2$

By using identity $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = 1$ and $b = m$

$(l - m)^2 = (l - m)(l - m)$

Hence, factors of $(l + m)^2 - 41m = (l - m)(l - m) = (l - m)^2$

(viii) $a^4 + 2a^2b^2 + b^4$

$= (a^2)^2 + 2a^2 \times b^2 + (b^2)^2$

By using identity $(a + b)^2 = a^2 + 2ab + b^2 = (a^2 + b^2)^2 = (a^2 + b^2)(a^2 + b^2)$

Hence, factors of $a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)(a^2 + b^2)$

$= (a^2 + b^2)^2$.

Q.2 Factorise.

(i) $4p^2 - 9q^2$ (ii) $63a^2 - 112b^2$

(iii) $49x^2 - 36$ (iv) $16x^5 - 144x^2$

(v) $(l + m)^2 - (l - m)^2$ (iv) $9x^2y^2 - 16$

(vii) $(x^2 - 2xy + y^2) - z^2$ (viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Sol:

(i) $4p^2 - 9q^2 = (2p)^2 \_ (3q)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$(2p - 3q)(2p + 3q)$.

(ii) $63a^2 - 112b^2 = 7(9a^2 - 16b2)$

∵ 7 is taken common to make all te1"ll1sin perfect square form

$= 7[(3a)^2 - (4b)^2]$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$=7[(3a - 4b)(3a + 4b)]$

(iii) $49x^2 - 36$

$= (7x)^2 - (6)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= (7x - 6)(7x + 6)$.

(iv) $16x^5 - 144x^3 = 16x^3$

$(x - 9) = 16x^3(x^2 - 3^2)$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= 16x^3(x - 3)(x + 3)$.

(v) $(l + m)^2 - (l - m)^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= [(l + m) - (l - m)][(l + m) + (l - m)]$

$= (l + m - l + m)(l + m + l - m)$

$= (2m)(21) = 41m.$

(vi) $9x^2y^2 - 16$

$= (3xy)^2 - 4^2$

By using identity, $a^2 - b^2 = (a - b)(a + b)$

$= (3xy - 4)(3xy + 4).$

(vii) $(x^2 - 2xy + y^2) - z^2$

By using identity, $a^2 - 2ab + b^2 = (a - b)^2$

$= (x - y)^2 - z^2$

Now, by using identity, $a^2 - b^2 = (a - b)(a + b)$

$= [(x - y) - z][(x - y) + z]$

$= (x - y - z)(x - y + z).$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

$= 25a^2 - (4b^2 - 28bc + 49c^2)$

$= 25a^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2]$

$= (5a)^2 - (2b - 7c)^2$

Now, by using identity, $a^2 - b^2 = (a - b)(a + b)$ $(\because a^2 - 2ab + b^2 = (a^2 - b^2))]$

$= [5a - (2b - 7c)][5a + (2b - 7c)]$

$= (5a - 2b + 7c)(5a + 2b - 7c).$

Q.3 Factorise the expressions.

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y\,(y + z) + 9\,(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

Sol:

(i) $ax^2 + bx$

$= x(ax + b)$.

(ii) $7p^2 + 21q^2$

$= 7(p^2 + 3q^2)$.

(iii) $2x^3 + 2xy^2 + 2xz^2$

$= 2x(x^2 + y^2 + z^2)$.

(iv) $am^2 + bm^2 + bn^2 + an^2$

$= m^2(a + b) + n^2(b + a)$ [$\because$(a + b) is common in both terms]

$= (a + b)(m^2 + n^2)$.

(v) $(lm + l) + m + 1$

$= l(m + 1) + l(m + 1)$

$= (m + 1)(l + 1)$.

(vi) $y(y + z) + 9(y + z)$

$= (y + z)(y + 9)$.

(vii) $5y^2 - 20y - 8z + 2yz$

$= 5y^2 - 20y + 2yz - 8z$

$= 5y(y - 4) + 2z(y - 4)$

$= (y - 4)(5y + 2z)$

(viii) $10ab + 4a + 5b + 2$

$= 2 \times 5ab + 2 \times 2a + 5b + 2$

$= 2a(5b + 2) + 1(5b + 2)$ [∵ 1 is multiple to both terms]

$= (5b + 2)(2a + 1)$

(ix) $6xy - 4y + 6 - 9x$

$= 6xy - 9x - 4y + 6$

$= 3x(2y - 3) - 2(2y - 3)$

$= (2y - 3)(3x - 2)$

Q.4 Factorise

(i) $a^4 - b^4$ (ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$ (iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Sol:

(i) $a^4 - b^4 = (a^2)^2 - (b^2)^2$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= (a^2 - b^2)(a^2 + b^2)$

$= (a - b)(a + b)(a^2 + b^2)$. [Again using the same identity]

(ii) $p^4 - 81$

$= (p^2)^2 - (9)^2 \quad [\because a^2 - b^2 = (a - b)(a + b)]$

$= (p^2 - 3^2)(p^2 + 9)$

$= (p - 3)(p + 3)(p^2 + 9)$

(iii) $x^4 - (y + z)^4$

$= (x^2)^2 - [(y + z)^2]^2$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= [x^2 - (y + z)^2][x^2 + (y + z)^2]$

Now again using the same identity for first factor

$= [x - (y + z)][x + (y + z)][x^2 + (y + z)^2]$

$= (x - y - z)(x + y + z)[x^2 + (y + z)^2]$.

(iv) $x^4 - (x - z)^4$

$= (x^2)^2 - [(x - z)^2]^2$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= [x^2 - (x - z)^2][x^2 + (x - z)^2]$

$= [x - (x - z)][x + (x - z)][x^2 + (x - z)^2]$

(Again using same identity)

$= (x - x + z)(x + x - Z)[x^2 + (x - Z)^2]$

$= z(2x - z)[x^2 + (x - z)^2] = z(2x - z)(x^2 + x^2 - 2xz + z^2)$

$= z(2x - z)(2x^2 - 2xz + Z^2)$.

(v) $a^4 - 2a^2b^2 + b^4$

$= (a^2)^2 - 2 \times a^2 \times b^2 + (b^2)^2$ [Using identity $(a - b)^2 = a^2 - 2ab + b^2$]

$= (a^2 - b^2)^2$ [Using identity $a^2 - b^2 = (a - b)(a + b)$]

$= [(a - b)(a + b)]^2$

$= (a - b)^2 (a + b)^2 \quad \because (xy)^m = x^m . y^m$

Q.5 Factorise the following expressions.

(i) $p^2 + 6p + 8$ (ii) $q^2 - 10q + 21$ (iii) $p^2 + 6p - 16$

Sol:

(i) $p^2 + 6p + 8$

$= p^2 + (4 + 2)p + 4 \times 2$ (as $8 = 4 \times 2$)

$= p^2 + 4p + 2p + 4 \times 2$

$= p(p + 4) + 2(p + 4)$

$= (p + 4)(P + 2)$.

(ii) $q^2 - 10q + 21$

$= q^2 - (7 + 3)q + 7 \times 3$ as $21 = 7 \times 3$

$= q^2 - 7q - 3q + 7 \times 3$

$= q(q - 7) - 3(q - 7)$

$= (q - 7)(q - 3)$.

(iii) $p^2 + 6p - 16$

$= p^2 + (8 - 2)p + 8 \times (-2)$ as $-16 = 8 \times -2$

$= p^2 + 8p - 2p + 8 \times (-2)$

$= p(p + 8) - 2(p + 8)$

$= (p + 8)(p - 2)$