SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. sin 2 x

Ans. The anti-derivative of sin 2x is a function of x whose derivative is sin 2x. It is known that,

$$\frac{d}{dx}(\cos 2x) = -2\sin 2x$$

$$\Rightarrow \sin 2x = \frac{1}{2}\frac{d}{dx}(\cos 2x)$$

$$\therefore \sin 2x = \frac{d}{dx}\left(-\frac{1}{2}\cos 2x\right)$$

Therefore, the anti-derivative of $\sin 2x$ is $-\frac{1}{2}\cos 2x$

2. sin 3 x

Ans. The anti-derivative of sin 3x is a function of x whose derivative is sin 3x. It is known that,

$$\frac{d}{dx}(\sin 3x) = 3\cos 3x$$

$$\Rightarrow \cos 3x = \frac{1}{3}\frac{d}{dx}(\sin 3x)$$

$$\therefore \cos 3x = \frac{d}{dx}\left(-\frac{1}{3}\sin 3x\right)$$

Therefore, the anti-derivative of $\cos 3x$ is $-\frac{1}{3}\sin 3x$

3. e^{zx}

Ans. The anti-derivative of e^{zx} is a function of x whose derivative is sin e^{zx}. It is known that,

$$\frac{d}{dx}\left(e^{zx}\right) = 2e^{zx}$$

$$\Rightarrow e^{zx} = \frac{1}{2}\frac{d}{dx}\left(e^{zx}\right)$$

$$\therefore e^{zx} = \frac{d}{dx}\left(-\frac{1}{2}e^{zx}\right)$$

Therefore, the anti-derivative of e^{zx} is $-\frac{1}{2}e^{zx}$

4. $(ax+b)^2$

Ans. The anti-derivative of $(ax+b)^2$ is the function of x whose derivative is $(ax+b)^2$. It is known that,

$$\frac{d}{dx}(ax+b)^3 = 3a(ax+b)^2$$

$$\Rightarrow (ax+b)^2 = \frac{1}{3a}\frac{d}{dx}(ax+b)^3$$

$$\therefore (ax+b)^2 = \frac{d}{dx}\left(\frac{1}{3a}(ax+b)^3\right)$$

Therefore, the anti-derivative of $(ax+b)^2$ is $\frac{1}{3a}(ax+b)^3$

5. $\sin 2x - 4e^{3x}$

Ans. The anti-derivative of $\left(\sin 2x - 4e^{3x}\right)$ is the function of x whose derivative is $\left(\sin 2x - 4e^{3x}\right)$.

It is known that,

$$\frac{d}{dx}\left(-\frac{1}{2}\cos 2x - \frac{4}{3}4e^{3x}\right)\sin 2x - 4e^{3x}$$

Therefore, the anti derivative of $\left(\sin 2x - 4e^{3x}\right)$ is $\left(-\frac{1}{2}\cos 2x - \frac{4}{3}4e^{3x}\right)$

6. $\int \left(4e^{3x}+1\right)dx$

Ans. $\int \left(4e^{3x}+1\right)dx$

$$= 4\int e^{3x}dx + \int 1dx$$

$$= 4\left(\frac{e^{3x}}{3}\right) + x + c$$

$$= \frac{4}{3}e^{3x} + x + C$$

7. $\int x^2\left(1-\frac{1}{x^2}\right)dx$

Ans. $\int x^2\left(1-\frac{1}{x^2}\right)dx$

$$= \int \left(x^2 - 1\right)dx$$

$$=\int x^2dx-\int 1dx$$

$$=\frac{x^3}{3}-x+C$$

8. $\int\left(ax^2+bx+c\right)dx$

Ans. $\int\left(ax^2+bx+c\right)dx$

$$=a\int x^2dx+b\int xdx+c\int 1.dx$$

$$=a\left(\frac{x^3}{3}\right)+b\left(\frac{x^2}{2}\right)+cx+C$$

$$=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+C$$

9. $\int\left(2x^2+e^x\right)dx$

Ans. $\int\left(2x^2+e^x\right)dx$

$$=2\int x^2dx+\int e^xdx$$

$$=2\left(\frac{x^3}{3}\right)+e^x+C$$

$$=\frac{2}{3}x^3+e^x+C$$

10. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$

Ans. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int xdx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^2}{2}+\log|x|-2x+C$$

11. $\int \frac{x^3+5x^2-4}{x^2}dx$

Ans. $\int \frac{x^3+5x^2-4}{x^2}dx$

$= \int \left(x+5-4x^{-2}\right)dx$

$= \int xdx + 5\int 1.dx - 4\int x^{-2}dx$

$= \frac{x2}{2} + 5x - 4\left(\frac{x^{-1}}{-1}\right) + C$

$= \frac{x^2}{2} + 5x + \frac{4}{x} + C$

12. $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

Ans. $\int \frac{x^3+3x+4}{\sqrt{x}}dx$

$= \int \left(x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 4x^{\frac{1}{2}}\right)dx$

$= \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + \frac{3\left(x^{\frac{3}{2}}\right)}{\frac{3}{2}} + \frac{4\left(x^{\frac{1}{2}}\right)}{\frac{1}{2}} + C$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8x^{\frac{3}{2}} + C$

$= \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + 8\sqrt{x} + C$

13. $\int \frac{x3-x2+x-1}{x-1}dx$

Ans. $\int \frac{x3-x2+x-1}{x-1}dx$

On dividing, we obtain

$= \int \left(x^2+1\right)dx$

$= \int x2dx + \int ldx$

$$= \frac{x^3}{3} + x + C$$

14. $\int (1-x)\sqrt{x}dx$

Ans. $\int (1-x)\sqrt{x}dx$

$$= \int \left(\sqrt{x} - x^{\frac{3}{2}} \right) dx$$

$$= \int x\frac{1}{2} dx - \int x^{\frac{3}{2}} dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

$$= \frac{2}{3}x^{\frac{2}{3}} - \frac{2}{3}x^{\frac{5}{2}} + C$$

15. $\int \sqrt{x}\left(3x^2 + 2x + 3\right)dx$

Ans. $\int \sqrt{x}\left(3x^2 + 2x + 3\right)dx$

$$= \int \left(3x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 3x^{\frac{1}{2}} \right) dx$$

$$= 3\int x^{\frac{5}{2}} dx + 2\int x^{\frac{3}{2}} dx + 3\int x^{\frac{1}{2}} dx$$

$$= 3\left(\frac{x^{\frac{7}{2}}}{\frac{7}{2}} \right) + 2\left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}} \right) + 3\frac{\left(x^{\frac{3}{2}} \right)}{\frac{3}{2}} + C$$

$$= \frac{6}{7}x^{\frac{7}{2}} + \frac{4}{5}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + C$$

16. $\int \left(2x - 3\cos x + e^x\right)$

Ans. $\int \left(2x - 3\cos x + e^x\right)$

$$= 2\int x dx - 3\int \cos x dx + \int e^x dx$$

$$=\frac{2x^2}{2}-3(\sin x)+e^x+C$$

$$=x^2-3\sin x+e^x+C$$

17. $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

Ans. $\int\left(2x^2-3\sin x+5\sqrt{x}\right)dx$

$$=2\int x^2dx-3\int \sin\ xdx+5\int x^{\frac{1}{2}}$$

$$=\frac{2x^3}{3}-3(-\cos x)+5\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)+C$$

$$=\frac{2}{3}x^3+3\cos x+\frac{10}{3}x^{\frac{3}{2}}+C$$

18. $\int \sec x(\sec x+\text{ten}x)dx$

Ans. $\int \sec x(\sec x+\text{ten}x)dx$

$$=\int\left(\sec^2 x+\sec x\tan x\right)dx$$

$$=\int \sec^2 xdx+\int \sec x\tan xdx$$

$$=\tan x+\sec x+C$$

19. $\int\frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

Ans. $\int\frac{\sec^2 x}{\operatorname{cosec}^2 x}dx$

$$=\int\frac{\frac{1}{\cos^2 x}}{\frac{1}{\sin^2 x}}dx$$

$$=\int\frac{\sin^2 x}{\cos^2 x}dx$$

$$=\int \tan^2 xdx$$

$$=\int\left(\sec^2 x-1\right)dx$$

$= \int \sec^2 x dx - \int 1 dx$

$= \tan x - x + C$

20. $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

Ans. $\int \frac{2 - 3\sin x}{\cos^2 x} dx$

$= \int \left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} \right) dx$

$= \int 2\sec 2x dx - 3\int \tan x \sec x dx$

$= 2\tan x - 3\sec x + c$

21. The anti derivative of $\left(\sqrt{x}\right) + \frac{1}{\sqrt{x}}$ equals

(A) $\frac{1}{3}x^{\frac{1}{3}} + 2x^{\frac{1}{2}} + C$ (B) $\frac{2}{3}x^{\frac{2}{3}} + \frac{1}{2}x^2 + C$ (C) $\frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$ (D) $\frac{3}{2}x^{\frac{3}{2}} + \frac{1}{2}x^2 + C$

Ans. $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$

$= \int x^{\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + C$

22. If $\frac{d}{dx} f(x) = 4x^3 - \frac{3}{x^4}$ such that f(2)= 0, then f(x) is

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (B) $x^3 + \frac{1}{x^4} + \frac{129}{8}$ (C) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ (D) $x^3 + \frac{1}{x^4} - \frac{129}{8}$

Ans. It is given that,

$$\frac{d}{df}f(x)=4x^3-\frac{3}{x^4}$$

$\therefore$ Anti derivative of $4x^2-\frac{3}{x^4}=f(x)$

$$\therefore\ f(x)=\int 4x^3-\frac{3}{x^4}dx$$

$$f(x)=4\int x^3dx-3\int\left(x^{-4}\right)dx$$

$$f(x)=4\left(\frac{x^4}{4}\right)-3\left(\frac{x^{-3}}{-3}\right)+C$$

$$\therefore\ f(x)=x^4+\frac{1}{x^3}+C$$

Also,

$$f(2)=0$$

$$\therefore f(2)=(2)^4+\frac{1}{(2)^3}+C=0$$

$$\Rightarrow 16+\frac{1}{8}+C=0$$

$$\Rightarrow C=\left(16+\frac{1}{8}\right)$$

$$\Rightarrow C=\frac{-129}{8}$$

$$\therefore\ f(x)=x^4+\frac{1}{x^3}-\frac{129}{8}$$

Hence, the correct answer is A.