SpeedLabs
MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $\sin^2(2x+5)$

Ans. $\sin 2^2 x+5=\frac{1-\cos 2(2x+5)}{2}=\frac{1-\cos(4x+10)}{2}$

$$\Rightarrow \int \sin^2(2x+5)dx = \int \frac{1-\cos(4x+10)}{2}$$

$$=\frac{1}{2}x-\frac{1}{2}\int \cos(4x+10)dx$$

$$=\frac{1}{2}x-\frac{1}{2}\left(\frac{\sin(4x+10)}{4}\right)+C$$

$$=\frac{1}{2}x-\frac{1}{8}\sin(4x+10)+C$$

2. $\sin 3x \cos 4x$

Ans. It is known that, $\sin A\cos B=\frac{1}{2}\{\sin(A+B)+\sin(A-B)\}$

$$\therefore \int \sin 3x\cos 4x\,dx=\frac{1}{2}\int\{\sin(3x+4x)+\sin 3(x-4x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x+\sin(-x)\}dx$$

$$=\frac{1}{2}\int\{\sin 7x-\sin x\}dx$$

$$=\frac{1}{2}\int \sin 7x\,dx-\frac{1}{2}\int \sin x\,dx$$

$$=\frac{1}{2}\left(\frac{-\cos 7x}{7}\right)-\frac{1}{2}(-\cos x)+C$$

3. $\cos 2x \cos 4x \cos 6x$

Ans. It is known that, $\cos A \cos B = \frac{1}{2}\int\left\{\sin(A+B)+\sin(A-B)\right\}$

$$\therefore \int \sin 3x \cos 4x \, dx = \frac{1}{2}\int\left\{\sin(3x+4x)+\sin(3x-4x)\right\} dx$$

$$= \frac{1}{2}\int\left\{\sin 7x + \sin(-x)\right\} dx$$

$$= \frac{1}{2}\int\left\{\sin 7x + \sin x\right\} dx$$

$$= \frac{1}{2}\int \sin 7x \, dx - \frac{1}{2}\int \sin x \, dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}(-\cos x) + C$$

$$= \frac{-\cos 7x}{14} + \frac{\cos x}{2} + C$$

4. $\sin^3(2x + 1)$

Ans. Let $I = \sin^3(2x+1)$

$$\Rightarrow \int \sin^3(2x+1)dx = \int \sin^2(2x+1)\cdot\sin(2x+1)dx$$

$$= \int 1-\cos^2(2x+1)\sin(2x+1)dx$$

Let $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1)dx = dt$$

$$\Rightarrow \sin(2x+1)dx = \frac{-dt}{2}$$

$$\Rightarrow I = \frac{-1}{2}\int(1-t^2)dt$$

$$= \frac{-1}{2}\left\{t - \frac{t^3}{3}\right\}$$

$$= \frac{-1}{2}\left\{\cos(2x+1) - \frac{\cos^3(2x+1)}{3}\right\}$$

$$=\frac{-\cos(2x+1)}{2}+\frac{\cos^3(2x+1)}{6}+C$$

5. $\sin^3 x\cos^3 x$

Ans. Let $I=\int \sin^3 x\cdot dx$

$$=\int \cos^3 x\cdot\sin^2 x\cdot\sin x.dx$$

$$=\int \cos^3 x\left(1-\cos^2 x\right)\sin x.dx$$

Let $\cos x=t$

$\Rightarrow -\sin x\cdot dx=dt$

$\Rightarrow I=-\int t^3\left(1-t^2\right)dt$

$$=-\int\left(t^3-t^5\right)dt$$

$$=-\left\{\frac{t^4}{4}-\frac{t^5}{6}\right\}+C$$

$$=-\left\{\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}\right\}+C$$

$$=\frac{\cos^4 x}{4}-\frac{\cos^6 x}{6}+C$$

6. $\sin x\sin 2x\sin 3x$

Ans. It is known that $\sin A\sin B=\frac{1}{2}\left\{\cos(A-B)-\cos(A+B)\right\}$

$$\therefore \sin x\sin 2x\sin 3xdx=\int \sin x\left[\cdot\frac{1}{2}\left\{\cos(2x-3)x-\cos(2x+3x)\right\}dx\right]$$

$$=\frac{1}{2}\int\left(\sin x\cos(-x)-\sin x\cos 5x\right)dx$$

$$=\frac{1}{2}\int\left(\sin x\cos x-\sin x\cos 5x\right)dx$$

$$=\frac{1}{2}\int\frac{\sin 2x}{2}dx-\frac{1}{2}\int \sin x\cos 5xdx$$

$$=\frac{1}{4}\left[\frac{-\cos 2x}{2}\right]-\frac{1}{2}\int\left\{\frac{1}{2}\sin(x+5x)+\sin(x-5x)\right\}dt$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\int\left(\sin 6x+\sin\left(-4x\right)\right)dx$$

$$=\frac{-\cos 2x}{8}-\frac{1}{4}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{4}\right]+C$$

$$=\frac{-\cos 2x}{8}-\frac{1}{8}\left[\frac{-\cos 6x}{3}+\frac{\cos 4x}{4}\right]+C$$

$$=\frac{1}{8}\left[\frac{\cos 6x}{3}-\frac{\cos 4x}{2}-\cos 2x\right]+C$$

7. sin Ax sin 8x

Ans. It is known that, sin A sin B = $\frac{1}{2}\cos(A-B)-\cos(A+B)$

$$\therefore\int\sin 4x\sin 8dx=\int\left\{\frac{1}{2}\cos\left(4x-8x\right)-\cos\left(4x+8x\right)\right\}dx$$

$$=\frac{1}{2}\int\left(\cos\left(-4x\right)-\cos 12x\right)dx$$

$$=\frac{1}{2}\int\left(\cos 4x-\cos 12x\right)dx$$

$$=\frac{1}{2}\left[\frac{\sin 4x}{4}-\frac{\sin 12x}{12}\right]$$

8. $\frac{1-\cos x}{1+\cos x}$

Ans. $\frac{1-\cos x}{1+\cos x}=\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$ $\left[2\sin^2\frac{x}{2}=1-\cos x\text{ and }2\cos^2\frac{x}{2}=1\cos x\right]$

$$=\tan^2\frac{x}{2}$$

$$=\left(\sec^2\frac{x}{2}-1\right)$$

$$\therefore\int\frac{1-\cos x}{1+\cos x}dx=\int\left(\sec^2\frac{x}{2}-1\right)dx$$

$$=\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}-x\right]+C$$

$$=2\tan\frac{x}{2}-x+C$$

9. $\dfrac{\cos x}{1+\cos x}$

Ans. $\dfrac{\cos x}{1+\cos x}=\dfrac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$ $\left[\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}\text{ and }\cos x=2\cos^2\frac{x}{2}-1\right]$

$$=\frac{1}{2}\left[1-\tan^2\frac{x}{2}\right]$$

$$\therefore\int\frac{\cos x}{1+\cos x}dx=\frac{1}{2}\left(1+\tan^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\int\left(1+\sec^2\frac{x}{2}+1\right)dx$$

$$=\frac{1}{2}\int\left(1+\sec^2\frac{x}{2}\right)dx$$

$$=\frac{1}{2}\left[2x-\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]+C$$

$$=x-\tan\frac{x}{2}+C$$

10. $\sin^4 x$

Ans. $\sin^4 x=\sin^2 x\sin^2 x$

$$=\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$$

$$=\frac{1}{4}(1-\cos 2x)^2$$

$$=\frac{1}{4}\left[1+\cos^2 2x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\left(\frac{1+\cos^2 4x}{2}\right)-2\cos 2x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]$$

$$\therefore \int \sin^4 x\ dx=\frac{1}{4}\int\left[\frac{3}{2}+\frac{1}{2}\cos 4x-2\cos 2x\right]dx$$

$$=\frac{1}{4}\int\left[\frac{3}{2}x+\frac{1}{2}\left(\frac{\sin 4x}{4}\right)-\frac{2\sin 2x}{2}\right]+C$$

$$=\frac{1}{8}\left[3x+\frac{\sin 4x}{4}-2\sin 2x\right]+C$$

$$=\frac{3x}{8}-\frac{1}{4}\sin 2x+\frac{1}{32}\sin 4x+C$$

11. $\cos^4 2x$

Ans. $\cos^4 2x=\left(\cos^2 2x\right)^2$

$$=\left(\frac{1+\cos 4x}{2}\right)^2$$

$$=\frac{1}{4}\left[1+\cos^2 4x+2\cos 4x\right]$$

$$=\frac{1}{4}\left[1+=\left(\frac{1+\cos 8\ x}{2}\right)+2\cos 4x\right]$$

$$=\frac{1}{4}\left[1+\frac{1}{2}+\frac{\cos 8x}{2}2\cos 4x\right]$$

$$=\frac{1}{4}\left[\frac{3}{2}+\frac{\cos 8x}{2}+2\cos 4x\right]$$

$$\therefore \int \cos^4 2x\ dx=\int\left(\frac{3}{8}+\frac{\cos 8x}{8}+\frac{\cos 4x}{2}\right)dx$$

$$=\frac{3}{8}x+\frac{\sin 8x}{64}+\frac{\sin 4x}{8}+C$$

12. $\dfrac{\sin^2 x}{1+\cos x}$

Ans. $\dfrac{\sin^2 x}{1+\cos x} = \dfrac{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}{2\cos^2\frac{x}{2}}$ $\left[\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2};\cos x = 2\cos^2\frac{x}{2} - 1\right]$

$$= \frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}{2\cos^2\frac{x}{2}}$$

$$= 2\sin^2\frac{x}{2}$$

$$= 1 - \cos x$$

$$\therefore \int \frac{\sin^2 x}{1+\cos x}\,dx = \int (1-\cos x)\,dx$$

$$= x - \sin x + C$$

13. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}$

Ans. $\dfrac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha} = \dfrac{-2\sin\frac{2x+2\alpha}{2}\sin\frac{2x-2\alpha}{2}}{-2\sin\frac{x+\alpha}{2}\sin\frac{x-\alpha}{2}}$ $\left[\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$

$$= \frac{\sin(x+\alpha)\sin(x-\alpha)}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= \frac{\left[2\sin\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x+\alpha}{2}\right)\right]\left[2\sin\left(\frac{x-\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\right]}{\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}$$

$$= 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)$$

$$= 2\cos\left[\left(\frac{x+\alpha}{2}+\frac{x-\alpha}{2}\right) + \cos\frac{x+\alpha}{2} - \frac{x-\alpha}{2}\right]$$

$$= 2\left[\cos(x) + \cos\alpha\right]$$

$= 2\cos x + 2\cos\alpha$

$\therefore \int \frac{2\cos x - 2\cos\alpha}{\cos x - \cos\alpha}\, dx = \int 2\cos x + 2\cos\alpha$

$= 2\left[\sin x + \cos\alpha\right] + C$

14. $\frac{\cos x - \sin x}{1 + \sin 2x}$

Ans. $\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{\left(\sin^2 x + \cos^2 x\right) + 2\sin x\cos x}$ $\left[\sin^2 x + \cos^2 x = 1; \sin 2x = 2\sin x\cos x\right]$

$= \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2}$

$\text{Let} \sin x + \cos x = 1$

$\therefore \left(\cos x - \sin x\right)dx = dt$

$\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x}\, dx = \int \frac{\cos x - \sin x}{\left(\sin x + \cos x\right)^2}\, dt$

$= \int \frac{dt}{t^2}$

$= \int t^2 dt$

$= -t^{-1} + C$

$= -\frac{1}{t} + C$

$= \frac{-1}{\sin x + \cos x} + C$

15. $\tan^3 2x \sec 2x$

Ans. $\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$

$= \left(\sec^2 2x - 1\right)\tan 2x \sec 2x$

$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$

$\therefore \int \tan^3 2x \sec 2x\, dx = \int \sec^2 2x \tan 2x \sec 2x dx - \int \tan 2x \sec 2x\, dx$

$= \int \sec^2 2x\, \tan 2x \sec 2x\, dx - \frac{\sec 2x}{2}$

$$\text{Let } \sec 2x = t$$

$$\therefore 2\sec 2x \tan 2x dx = dt$$

$$\therefore \int \tan^3 2x \sec 2x = \frac{1}{2}\int t^2 dt - \frac{\sec 2x}{2} + C$$

$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$

$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

16. $\tan^4 x$

Ans. $\tan^4 x$

$= \tan^2 x \tan^2 x$

$= (\sec^2 x - 1) \tan^2 x$

$= \sec^2 x \tan^2 x — \tan^2 x$

$= \sec^2 x \tan^2 x — (\sec^2 x - 1)$

$= \sec^2 x \tan^2 x — \sec^2 x + 1$

$$\therefore \int \tan^4 x dx = \int \sec^2 x \tan^2 x \, dx - \int \sec^2 x dx + \int 1.dx$$

$$= \sec^2 x \tan^2 x \, dx - \tan x + x + C \qquad ...(1)$$

$$\text{consider } \int \sec^2 x \tan^2 x \, dx$$

$$\text{Let } \tan x = 1 \Rightarrow \sec^2 x \, dx = dt$$

$$\Rightarrow \int \sec^2 x \tan^2 x dx = \int t^2 dt = \frac{t^3}{3} = \frac{\tan^3 x}{3}$$

From equation (1), we obtain

$$\tan^4 x dx = \frac{1}{3}\tan^3 x - \tan + x + x + C$$

17. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}$

Ans. $\dfrac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} = \dfrac{\sin^3 x}{\sin^2 x \cos^2 x} + \dfrac{\cos^3 x}{\sin^2 x \cos^2 x}$

$$= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x}$$

$$= \tan x \sec x + \cot x \operatorname{cosec} x$$

$$\therefore \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\tan x \sec x + \cot x \, \text{cosec}x) dx$$

$$= \sec x - \text{coses } x + C$$

18. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

Ans. $\dfrac{\cos 2x + 2\sin^2 x}{\cos^2 x}$

$$\frac{\cos 2x + (1 - \cos 2x)}{\cos^2 x} \qquad [\cos 2x = 1 - 2\sin^2 x]$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

$$\therefore \int \frac{\cos 2x + 2\sin 2x}{\cos^2 x} dx = \int \sec^2 x dx = \tan x + C$$

19. $\dfrac{1}{\sin x \cos^3 x}$

Ans. $\dfrac{1}{\sin x \cos^3 x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos^3 x}$

$$= \frac{\sin x}{\cos^3 x} + \frac{1}{\sin x \cos x}$$

$$= \tan x \sec^2 x + \frac{\frac{1 \cos^2 x}{\sin x \cos x}}{\cos^2 x}$$

$$= \tan x \sec^2 x + \frac{\sec^2 x}{\tan x}$$

$$\therefore \int \frac{1}{\sin x \cos^3 x} dx = \int \tan x \sec^2 x \, dx + \int \frac{\sec^2 x}{\tan x} dx$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$$\Rightarrow \int \frac{1}{\sin x \cos^3 x} dx = \int t dt + \int \frac{1}{t} dt$$

$$= \frac{t^2}{2} + \log |t| + C$$

$$=\frac{1}{2}\tan^2 x+\log|\tan x|+C$$

20. $$\frac{\cos 2x}{(\cos x+\sin x)^2}$$

Ans. $$\frac{\cos 2x}{(\cos x+\sin x)^2}=\frac{\cos 2x}{\cos^2 x+\sin^2 x+2\sin x\cos x}=\frac{\cos 2x}{1+\sin 2x}$$

$$\therefore\int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\frac{\cos 2x}{(1+\sin 2x)}dx$$

Let $1+\sin 2x=t$

$$\Rightarrow 2\cos 2x dx=dt$$

$$\therefore\int\frac{\cos 2x}{(\cos x+\sin x)^2}\,dx=\frac{1}{2}\int\frac{1}{t}\,dt$$

$$=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log|1+\sin 2x|+C$$

$$=\frac{1}{2}\log\left|(\sin x+\cos x)^2\right|+C$$

$$=\log\left|\sin x+\cos x\right|+C$$

21. $\sin^{-2}(\cos x)$

Ans. $\sin^{-1}(\cos x)$

Let $\cos x=t$

Then, $\sin x=\sqrt{1-t^2}$

$$\Rightarrow(-\sin x)dx=dt$$

$$dx=\frac{-dt}{\sin x}$$

$$dx=\frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore\int\sin^{-1}\cos x dx=\int\sin^{-1}t\left(\frac{-dt}{1-t^2}\right)$$

21. $\sin^{-1}(\cos x)$

Ans. $\text{Let} \cos x = 1$

$\text{Then}, \sin x = \sqrt{1-t^2}$

$\Rightarrow (-\sin x)dx = dt$

$$dt = \frac{-dt}{\sin x}$$

$$dt = \frac{-dt}{\sqrt{1-t^2}}$$

$$\therefore \int \sin^{-1}(\cos x)dx = \int \sin^{-1} t\left(\frac{-dt}{\sqrt{1-t^2}}\right)$$

$$= -\int \frac{\sin^{-1} t}{\sqrt{1-t^2}} dt$$

$\text{Let } \sin^{-1} t = u$

$$\Rightarrow \frac{1}{\sqrt{1-t^2}} dt = du$$

$$\therefore \int \sin^{-1}(\cos x)dx = du$$

$$= \frac{-u^2}{2} + C$$

$$= \frac{-(\sin^1 t)^2}{2} + C$$

$$= \frac{-\left[\sin^{-1}(\cos x)\right]^2}{2} + C \quad \text{......(1)}$$

It is known that,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\therefore \sin^{-1}(\cos x) = \frac{\pi}{2} - \cos^{-1}(\cos x) = \left(\frac{\pi}{2} - x\right)$$

Substituting in equation (1), we obtain

$$\int \sin^{-1}(\cos x)dx = \frac{-\left[\frac{\pi}{2}-2\right]^2}{2}+C$$

$$= -\frac{1}{2}\left(\frac{\pi^2}{2}+x^2-\pi x\right)+C$$

$$= -\frac{\pi^2}{8}-\frac{x^2}{2}+\frac{1}{2}\pi x+C$$

$$= \frac{\pi x}{2}-\frac{x^2}{2}+\left(C-\frac{\pi^2}{8}\right)$$

$$= \frac{\pi x}{2} = \frac{x^2}{2}+C_1$$

22. $$\frac{1}{\cos(x-a)\cos(x-b)}$$

Ans. $$\frac{1}{\cos(x-a)\cos(x-b)} = \frac{1}{\sin(a-b)}\left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\left[\frac{\cos\left[(x-b)\cos(x-a)\right]}{\cos(x-a)\cos(x-b)}\right]$$

$$= \frac{1}{\sin(a-b)}\frac{\left[\cos(x-b)\cos(x-a)-\cos(x-b)\cos(x-a)\right]}{\cos(x-a)\cos(x-b)}$$

$$= \frac{1}{\sin(a-b)}\left[\tan(x-b)\cos(x-a)\right]$$

$$\Rightarrow \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(a-b)}\left[\tan(x-b)\cos(x-a)\right]dx$$

$$= \frac{1}{\sin(a-b)}\left[-\log|\cos(x-b)|+\log|\cos(x-a)|\right]$$

$$= \frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos x-a}{\cos x-b}\right|\right]+C$$

23. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$ is equal to

A. tan x + cot x + C B. tan x + cosec x + C

C. - tan x + cot x + C D. tan x + sec x + C

Ans. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^2 x}{\sin^2 x \cos^2 x} - \frac{\cos^2 x}{\sin^2 x \cos^2 x} \right) dx$

$= \int \left(\sec^2 x + \operatorname{cosec}^2 x \right) dx$

$= \tan x + \cot x + C$

Hence, the correct Answer is A.

24. $\frac{e^x (1+x)}{\cos^2 (e^x x)} dx$ equals

A. $-\cot\left(ex^x\right)+C$ B. $\tan\left(ex^x\right)+C$

C. $-\tan\left(e^x\right)+C$ D. $\cot\left(e^x\right)+C$

Ans. $\frac{e^x (1+x)}{\cos^2 (e^x x)} dx$

Let $ex^2 = t$

$\Rightarrow \left(e^x \cdot x + e^x \cdot 1 \right) dx = dt$

$e^x (x+1) dx = dt$

$\therefore \int \frac{e^x (x+1)}{\cos^2 (e^x x)} dx = \int \frac{dt}{\cos^2 t}$

$= \cos^2 t \, dt$

$= \tan t + C$

$= \tan\left(e^x \cdot x\right) + C$