SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $\dfrac{3x^2}{x^6+1}$

Ans. Let $x^3 = t$

$\therefore 3x^2dx = dt$

$\Rightarrow \int \dfrac{3x^2}{x^6+1}dx = \int \dfrac{dt}{t^2+1}$

$= \tan^1 t + C$

$= \tan^{-1}\left(x^3\right) + C$

2. $\dfrac{1}{\sqrt{1+4x^2}}$

Ans. Let $2x = t$

$\therefore 2dt = dt$

$\Rightarrow \int \dfrac{1}{\sqrt{1+4x^2}}dx = \dfrac{1}{2}\dfrac{dt}{\sqrt{1+t^2}}$

$= \dfrac{1}{2}\left[\log\left|t+\sqrt{t^2+1}\right|\right] + C \qquad \left[\int \dfrac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$

$= \dfrac{1}{2}\log\left|2x+\sqrt{4x^2+1}\right| + C$

3. $\dfrac{1}{\sqrt{(2-x)^2+1}}$

Ans. Let $2-x = t$

$\Rightarrow -dx = dt$

$\Rightarrow \int \dfrac{1}{\sqrt{(2-x)^2+1}}dx = -\int \dfrac{dt}{\sqrt{t^2+1}}dt$

$= -\log\left|t+\sqrt{t^2+1}\right| + C \qquad \left[\int \dfrac{1}{\sqrt{x^2+a^2}}dt = \log\left|x+\sqrt{x^2+a^2}\right|\right]$

$$= -\log\left|2 - x + \sqrt{(2-x)^2 + 1}\right| + C$$

$$= \log\left|\frac{1}{(2-x) + \sqrt{x^2 - 4x + 5}}\right| + C$$

4. $\frac{1}{\sqrt{9 - 25x^2}}$

Ans. Let $5x = t$

$\therefore 5dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{9-25x^2}}\, dt = \frac{1}{5}\int \frac{1}{9 - t^2}\, dt$$

$$= \frac{1}{5}\int \frac{1}{\sqrt{3^3 - t^2}}\, dt$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{1}{5}\sin^{-1}\left(\frac{5x}{3}\right) + C$$

5. $\frac{3x}{1 + 2x^4}$

Ans. Let $\sqrt{2}x^2 = t$

$\therefore \sqrt{2}x\ dx = dt$

$$\Rightarrow \frac{3x}{1+2x^4}\, dx = \frac{3}{2\sqrt{2}}\int \frac{dt}{1+t^2}$$

$$= \frac{3}{2\sqrt{2}}\left[\tan^{-1} t\right] + C$$

$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x\right)^2 + C$$

6. $\frac{x^2}{1 - x^6}$

Ans. Let $x^3 = t$

$\therefore 3x^2 dx = dt$

$$\Rightarrow \int \frac{x^2}{1-x^6}\,dx = \frac{1}{3}\int \frac{dt}{1-t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\log\left|\frac{1+t}{1-t}\right|\right] + C$$

$$= \frac{1}{6}\log\left|\frac{1+x^3}{1-x^3}\right| + C$$

7. $\dfrac{x-1}{\sqrt{x^2-1}}$

Ans. $$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx$$

For $\dfrac{x}{\sqrt{x^2-1}}dx$, let $x^2-1=t \Rightarrow 2x\,dx = dt$

$$\therefore \frac{x}{\sqrt{x^2-1}}\,dx = \frac{1}{2}\int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2}\int t^{-\frac{1}{2}}dt$$

$$= \frac{1}{2}\left[2t^{\frac{1}{2}}\right]$$

$$= \sqrt{t}$$

$$= \sqrt{x^2-1}$$

From (1), we obtain

$$\int \frac{x-1}{\sqrt{x^2-1}}\,dx = \int \frac{x}{\sqrt{x^2-1}}\,dx - \int \frac{1}{\sqrt{x^2-1}}\,dx \quad \left[\int \frac{1}{\sqrt{x^2-a^2}}\,dt = \log\left|x+\sqrt{x^2-a^2}\right|\right]$$

$$= \sqrt{x^2-1} - \log\left|x+\sqrt{x^2-1}\right| + C$$

8. $\dfrac{x^2}{\sqrt{x^6+a^6}}$

Ans. Let $x^3 = t$

$$\Rightarrow 3x^2dx = dt$$

$$\therefore \frac{x^2}{\sqrt{x^6+a^6}}\,dt = \frac{1}{3}\frac{dt}{\sqrt{t^2+\left(a^3\right)^2}}$$

$$= \frac{1}{3}\log\left|t+\sqrt{t^2+a^6}\right|+C$$

$$= \frac{1}{3}\log\left|x^3+\sqrt{t^2+a^6}\right|+C$$

9. $\dfrac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

Ans. Let $\tan x = t$

$$\therefore \sec^2 x\, dx = dt$$

$$\Rightarrow \int \frac{\sec^2 x}{\tan^2 x+4}\,dx = \int \frac{dt}{\sqrt{t^2+2^2}}$$

$$= \log\left|t+\sqrt{t^2+4}\right|+C$$

$$= \log\left|\tan x+\sqrt{\tan^2 x+4}\right|+C$$

10. $\dfrac{1}{\sqrt{x^2+2x+2}}$

Ans. $\displaystyle\int \frac{1}{\sqrt{x^2+2x+2}}\,dx = \frac{1}{\sqrt{\left(x+1\right)^2+\left(1\right)^2}}\,dx$

Let $x+1=t$

$$\therefore dx = dt$$

$$\Rightarrow \int \frac{1}{\sqrt{x^2+2x+2}}\,dx = \int \frac{1}{\sqrt{t^2+1}}\,dt$$

$$= \log\left|t+\sqrt{t^2+1}\right|+C$$

$$= \log\left|\left(x+1\right)+\sqrt{\left(x+1\right)^2+1}\right|+C$$

$$= \log\left|\left(x+1\right)+\sqrt{x^2+2x+2}\right|+C$$

11. $\dfrac{1}{\sqrt{9x^2+6x+5}}$

Ans. $\displaystyle\int \frac{1}{\sqrt{9x^2+6x+5}}\,dx = \int \frac{1}{(3x+1)^2+(2)^2}\,dx$

Let $(3x+1)=t$

$\therefore 3dx = dt$

$\displaystyle\int \frac{1}{(3x+1)^2+(2)^2}\,dx = \frac{1}{3}\int \frac{1}{t^2+2^2}\,dt$

$=\dfrac{1}{3}\left[\dfrac{1}{2}\tan^{-1}\left(\dfrac{t}{2}\right)\right]+C$

$=\dfrac{1}{6}\tan^{-1}\left(\dfrac{3x+1}{2}\right)+C$

12. $\dfrac{1}{7-6x-x^2}$

Ans. $7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$=7-\left(x^2+6x+9-9\right)$

$=16-\left(x^2+6x+9\right)$

$=16-(x+3)^2$

$=(4)^2-(x+3)^2$

$\displaystyle\therefore \int \frac{1}{(4)^2-(x+3)^2}\,dx \int \frac{1}{(4)^2-(t)^2}\,dt$

$=\sin^{-1}\left(\dfrac{t}{4}\right)+C$

$-\sin^{-1}\left(\dfrac{x+3}{4}\right)+C$

13. $\dfrac{1}{\sqrt{(x-1)(x-2)}}$

Ans. $(x-1)(x-2)$ can be written as x^2-3x+2.

Therefore,

$$x^2-3x+2$$

$$=x^2-3x+\frac{9}{4}-\frac{9}{4}+2$$

$$=\left(x-\frac{3}{2}\right)^2-\frac{1}{4}$$

$$=\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2$$

$$\therefore \int\frac{1}{\sqrt{(x-1)(x-2)}}dx=\int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$$\therefore dx=dt$$

$$\Rightarrow \int\frac{1}{\sqrt{\left(x-\frac{3}{2}\right)^2-\left(\frac{1}{2}\right)^2}}dx\int\frac{1}{\sqrt{t^2-\left(\frac{1}{2}\right)^2}}dt$$

$$=\log\left|t+\sqrt{t^2-\left(\frac{1}{2}\right)^2}\right|+C$$

$$=\log\left|\left(x-\frac{3}{2}\right)+\sqrt{x^2-3x+2}\right|+C$$

14. $\frac{1}{\sqrt{8+3x-x^2}}$

Ans. $8+3x-x^2$ can be written as $8-\left(x^2-3x\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow\int\frac{1}{\sqrt{8+3x-x^2}}\,dx=\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}\,dx$$

Let $x-\frac{3}{2}=t$

$\therefore dx=dt$

$$\Rightarrow\frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}\,dx=\frac{1}{\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2-t^2}}\,dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

15. $\frac{1}{\sqrt{(x-a)(x-b)}}$

Ans. $(x-a)(x-b)$ can be writeen as $x^2-(a+b)x+ab.$

Therefore,

$x^2-(a+b)x+ab$

$$=x^2-(a+b)x+\frac{(a+b)^2}{4}-\frac{(a+b)^2}{4}+ab$$

$$=\left[x-\left(\frac{a+b}{2}\right)\right]^2-\frac{(a-b)^2}{4}$$

$$\Rightarrow \int \frac{1}{\sqrt{(x-a)(x-b)}}\,dx = \int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left(\frac{a+b}{2}\right)^2}}\,dx$$

$$\text{Let } x-\left(\frac{a+b}{2}\right) = t$$

$$\therefore dx = dt$$

$$\int \frac{1}{\sqrt{\left\{x-\left(\frac{a+b}{2}\right)\right\}^2 - \left(\frac{a+b}{2}\right)^2}}\,dx = \int \frac{1}{\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}}\,dt$$

$$= \log\left|t+\sqrt{t^2-\left(\frac{a-b}{2}\right)^2}\right| + C$$

$$= \log\left|\left\{x-\left(\frac{a-b}{2}\right)\right\} + \sqrt{(x-a)(x-b)}\right| + C$$

16. $\frac{4x+1}{\sqrt{2x^2+x-3}}$

Ans. $\text{Let } 4x+1 = A\frac{d}{dx}\left(2x^2+x-3\right)+B$

$$\Rightarrow 4x+1 = A(4x+1)+B$$

$$\Rightarrow 4x+1 = 4Ax+A+B$$

Equating the coefficients of x and constant term on both sides, we obtain

$$4A = 4 \Rightarrow A = 1$$

$$A+B = 1 \Rightarrow B = 0$$

$$\text{Let } 2x^2 + x - 3 = t$$

$$\therefore (4x + 1)dx = dt$$

$$\Rightarrow \int \frac{4x+1}{\sqrt{2x^2+x-3}}\,dx = \int \frac{1}{\sqrt{t}}dt$$

$$= 2\sqrt{t}+C$$

$$= 2\sqrt{2x^2+x-3}+C$$

17. $\dfrac{x+2}{\sqrt{x^2-1}}$

Ans. Let $x+2 = A\dfrac{d}{dx}\left(x^2-1\right)+B$ ...(1)

$\Rightarrow x+2 = A(2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$2A = 1 \Rightarrow A = \dfrac{1}{2}$

$B = 2$

From (1), we obtain

$(x+2) = \dfrac{1}{2}(2x)+2$

Then, $\int \dfrac{x+2}{\sqrt{x^2-1}}\,dx = \int \dfrac{\frac{1}{2}(2x)+2}{\sqrt{x^2-1}}\,dx$

$= \dfrac{1}{2}\int \dfrac{2x}{\sqrt{x^2-1}}\,dx + \int \dfrac{2}{\sqrt{x^2-1}}\,dx$ ...(2)

In $\dfrac{1}{2}\int \dfrac{2x}{\sqrt{x^2-1}}\,dx$, let $x^2-1 = t \Rightarrow 2x\,dx = dt$

$\dfrac{1}{2}\int \dfrac{2x}{\sqrt{x^2-1}}\,dx = \dfrac{1}{2}\int \dfrac{dt}{\sqrt{t}}$

$= \dfrac{1}{2}\left[2\sqrt{t}\right]$

$= \sqrt{t}$

$= \sqrt{x^2-1}$

Then, $\int \dfrac{2}{\sqrt{x^2-1}}\,dx = 2\int \dfrac{1}{\sqrt{x^2-1}}\,dx = 2\log\left|x+\sqrt{x^2-1}\right|$

From equation (2), we obtain

$\int \dfrac{x+2}{\sqrt{x^2-1}}\,dx = \sqrt{x^2-1} + 2\log\left|x+\sqrt{x^2-1}\right| + C$

18. $\dfrac{5x-2}{1+2x+3x^2}$

Ans. Let $5x-2 = A\dfrac{d}{dt}\left(1+2x+3x^2\right)+B$

$\Rightarrow 5x-2 = A\left(2+6x\right)+B$

Equating the coefficient of x and constant term on both sides, we obtain

$5 = 6A \Rightarrow A = \dfrac{5}{6}$

$2A+B = -2 \Rightarrow B = -\dfrac{11}{3}$

$\therefore 5x-2 = \dfrac{5}{6}\left(2+6x\right)+\left(-\dfrac{11}{3}\right)$

$\Rightarrow \int \dfrac{5x-2}{1+2x+3x^2}\,dx = \dfrac{\frac{5}{6}\left(2+6x\right)-\frac{11}{3}}{1+2x+3x^2}\,dx$

$= \dfrac{5}{6}\int \dfrac{2+6x}{1+2x+3x^2}\,dx \; \dfrac{11}{3}\int \dfrac{1}{1+2x+3x^2}\,dx$

Let $I_1 = \int \dfrac{2+6x}{1+2x+3x^2}\,dx$ and $I_2 = \int \dfrac{1}{1+2x+3x^2}\,dx$

$\therefore \int \dfrac{5x-2}{1+2x+3x^2}\,dx = \dfrac{5}{6}I_1 - \dfrac{11}{3}I_2$

$I_1 = \int \dfrac{2+6x}{1+2x+3x^2}\,dx$

Let $1+2x+3x^2 = t$

$\Rightarrow \left(2+6x\right)dx = dt$

$\therefore I_1 = \int \dfrac{dt}{t}$

$I_1 = \log\left|t\right|$

$I_1 = \log\left|\,1+2x+3x^2\right| \qquad ...(2)$

$I_2 = \int \dfrac{1}{1+2x+3x^2}\,dx$

$1+2x+3x^2$ can be written as $1+3\left(x^2+\frac{2}{3}x\right)$.

Therefore,

$$1+3\left(x^2+\frac{2}{3}x\right)$$

$$=1+3\left(x^2+\frac{2}{3}x+\frac{1}{9}-\frac{1}{9}\right)$$

$$=1+3\left(x+\frac{1}{3}\right)^2-\frac{1}{3}$$

$$=\frac{2}{3}+3\left(x+\frac{1}{3}\right)^2$$

$$=3\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]$$

$$=3\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{9}\right)^2\right]$$

$$I_2=\frac{1}{3}\int\frac{1}{\left[\left(x+\frac{1}{3}\right)^2+\left(\frac{\sqrt{2}}{9}\right)^2\right]}dx$$

$$=\frac{1}{3}\left[\frac{1}{\frac{\sqrt{2}}{3}}\tan^{-1}\left(\frac{x\frac{1}{3}}{\frac{\sqrt{2}}{3}}\right)\right]$$

$$=\frac{1}{3}\left[\frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]$$

$$=\frac{\sqrt{2}}{3}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) \qquad ....(3)$$

Substituting equations (2) and (3) in equation (1), we obtain

$$\int\frac{5x-2}{1+2x+3x^2}dx=\frac{5}{6}\left[\log\left|1+2x+3x^2\right|\right]-\frac{11}{3}\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)\right]+C$$

$$=\frac{5}{6}\log\left|1+2x+3x^2\right|-\frac{11}{3\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right)+C$$

19. $\frac{x+2}{\sqrt{4x-x^2}}$

Ans. Let $x+2=A\frac{d}{dt}\left(4x-x^2\right)+B$

$\Rightarrow x+2=A(4-2x)+B$

Equating the coefficients of x and constant term on both sides, we obtain

$-2A=1\Rightarrow A=-\frac{1}{2}$

$4A+B=2\Rightarrow B=4$

$\Rightarrow(x+2)=-\frac{1}{2}(4-2x)+4$

$\therefore\int\frac{x+2}{\sqrt{4x-x^2}}\,dx=\frac{-\frac{1}{2}(4-2x)+4}{\sqrt{4x-x^2}}\,dx$

$=-\frac{1}{2}\int\frac{4x-x}{\sqrt{4x-x^2}}\,dx+4\int\frac{1}{\sqrt{4x-x^2}}\,dx$

Let $I_1\int\frac{4-2x}{\sqrt{4x-x^2}}\,dx$ and $I_2\int\frac{1}{\sqrt{4x-x^2}}\,dx$

$\therefore\int\frac{4+2}{\sqrt{4x-x^2}}\,dx=-\frac{1}{2}I_1+4I_2\qquad...(1)$

Then, $I_1=\int\frac{4-2x}{\sqrt{4x-x^2}}\,dx$

Let $4x-x^2=t\Rightarrow(4-2x)dx=dt$

$\Rightarrow I_1=\int\frac{dt}{\sqrt{t}}=2\sqrt{t}=2\sqrt{4x-x^2}\qquad....(2)$

$I_2=\int\frac{1}{\sqrt{4x-x^2}}\,dx$

$\Rightarrow 4x-x^2=-\left(-4x+x^2\right)$

$=\left(4x-x^2+4-4\right)$

$$=4-(x-2)^2$$

$$\therefore I_2=\frac{1}{\sqrt{(2)^2-(x-2)^2}}dx=\sin^{-1}\left(\frac{x-2}{2}\right)$$

Using equations (2) and (3) in (1), we obtain

$$\int\frac{x+2}{\sqrt{4x-x^2}}dx=-\frac{1}{2}\left(2\sqrt{4x-x^2}\right)+\sin^{-1}\left(\frac{x-2}{2}\right)+C$$

$$=-\sqrt{4x-x^2}+4\sin^{-1}\left(\frac{x-2}{2}\right)+C$$

20. $\int\frac{dt}{x^2+2x+2}$ equals

A. $x\tan^{-1}(x+1)+C$ **B.** $\tan^{-1}(x+1)+C$ **C.** $(x+1)\tan^{-1}x+C$ **D.** $\tan^{-1}x+C$

Ans. $\int\frac{dt}{x^2+2x+2}=\int\frac{dt}{\left(x^2+2x+2\right)+1}$

$$=\int\frac{1}{(x+1)^2+(1)^2}dx$$

$$=\left[\tan^{-1}(x+1)\right]+C$$

Hence, the correct Answer is B.

21. $\frac{dx}{\sqrt{9x-4x^2}}$

(A) $\frac{1}{9}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$

(C) $\frac{1}{3}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$ (D) $\frac{1}{2}\sin^{-1}\left(\frac{9x-8}{8}\right)+C$

Ans. $\int\frac{dx}{9x-4x^2}$

$$=\frac{1}{\sqrt{-4\left(x^2-\frac{9}{4}x\right)}}dx$$

$$=\int\frac{1}{-4\left(x^2-\frac{9}{4}x+\frac{81}{64}-\frac{81}{64}\right)}dx$$

$$=\frac{1}{2}\int \frac{1}{\sqrt{\left(\frac{9}{8}\right)^2-\left(x-\frac{9}{8}\right)^2}}\,dx$$

$$=\frac{1}{2}\left[\sin^{-1}\left(\frac{x-\frac{9}{8}}{\frac{9}{8}}\right)\right]+C \qquad \left(\int \frac{dy}{\sqrt{a^2-y^2}}=\sin^{-1}\frac{y}{a}+C\right)$$

$$=\frac{1}{2}\sin^{-1}\left(\frac{8x-9}{9}\right)+C$$

Hence, the correct Answer is B.