SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $\frac{x}{(x+1)(x+2)}$

Ans. Let $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$\Rightarrow x = A(x+2) + B(x+1)$

Equating the coefficients of x and constant term , we obtain

$A + B = 1$

$2A + B = 0$

On solving, we obtain

$A = -1$ and $B = 2$

$\therefore \frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{2}{(x+2)}$

$\Rightarrow \int \frac{x}{(x+1)(x+2)} dx = \int \frac{-1}{(x+1)} + \frac{2}{(x+2)} dx$

$= -\log|x+1| + 2\log|x+2| + C$

$= \log(x+2)^2 - \log|x+1| + C$

$= \log\frac{(x+2)^2}{(x+1)} + C$

2. $\frac{1}{x^2-9}$

Ans. Let $\frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)}$

$1 = A(x-3) + B(x+3)$

Equating the coefficients of x and constant term , we obtain

$A + B = 0$

$-3A + 3B = 1$

On solving, we obtain

$$A = -\frac{1}{6} \text{ and } B = \frac{1}{6}$$

$$\therefore \frac{1}{(x+3)(x-3)} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$$

$$\Rightarrow \int \frac{1}{(x^2+9)} dx = \int \left(\frac{-1}{6(x+3)} + \frac{1}{6(x-3)} \right) dx$$

$$= -\frac{1}{6} \log|x+3| + \frac{1}{6} \log|x-3| + C$$

$$= \frac{1}{6} \log\left|\frac{(x-3)}{(x+3)}\right| + C$$

3. $\frac{3x-1}{(x-1)(x-2)(x-3)}$

Ans. Let $\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$3x-1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain A = 1, B = - 5 , and C = 4

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)}$$

$$\Rightarrow \int \frac{3x-1}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{(x-1)} - \frac{5}{(x-2)} + \frac{4}{(x-3)} \right\} dx$$

$$= \log|x-1| - 5\log|x-2| + 4\log|x-3| + C$$

4. $\frac{x}{(x-1)(x-2)(x-3)}$

Ans. Let $\frac{x}{(x-1)(x-2)(x-3)} = \frac{A}{(x-1)} + \frac{B}{(x-2)} + \frac{C}{(x-3)}$

$$x = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2) \quad \ldots(1)$$

Substituting x = 1, 2, and 3 respectively in equation (1), we obtain

$$A = \frac{1}{2}, B = -2 \text{ and } C = \frac{3}{2}$$

$$\therefore \frac{x}{(x-1)(x-2)(x-3)} = \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)}$$

$$\Rightarrow \int \frac{x}{(x-1)(x-2)(x-3)} dx = \int \left\{ \frac{1}{2(x-1)} - \frac{2}{(x-2)} + \frac{3}{2(x-3)} \right\} dx$$

$$= \frac{1}{2}\log|x-1| - 2\log|x-2| + \frac{3}{2}\log|x-3| + C$$

5. $\frac{2x}{x^2+3x+2}$

Ans. Let $\frac{2x}{x^2+3x+2} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$

$$2x = A(x+2) + B(x+1)$$

Substituting x - - 1 and - 2 in equation (1), we obtain A = - 2 and B = 4

$$\therefore \frac{2x}{(x+1)(x+2)} dx = \frac{-2}{(x+1)} + \frac{4}{(x+2)}$$

$$\int \frac{2x}{(x+1)(x+2)} dx = \left\{ \frac{4}{(x+2)} + \frac{2}{(x+1)} \right\} dx$$

$$= 4\log|x+2| - 2\log|x+1| + C$$

6. $\frac{1-x^2}{x(1-2x)}$

Ans. It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(1 - x^2)$ by x (l - 2x), we obtain

$$\frac{1-x^2}{x(1-2x)} = \frac{1}{2} + \frac{1}{2}\left(\frac{2-x}{x(1-2x)} \right)$$

Let $\frac{2-x}{x(1-2x)} = \frac{A}{x} + \frac{B}{(1-2x)}$

$$\Rightarrow (2-x) = A(1-2x) + Bx \qquad ....(1)$$

Substituting x = 0 and $\frac{1}{2}$ jn equation (1), we obtain

A = 2 and B = 3

$$\therefore \frac{2-x}{x(1-2x)}=\frac{2}{x}+\frac{3}{1-2x}$$

Substituting in equation (1), we obtain

$$\frac{1-x^2}{x(1-2x)}=\frac{1}{2}+\frac{1}{2}\left\{\frac{2}{x}+\frac{3}{(1-2x)}\right\}$$

$$\Rightarrow \int \frac{1-x^2}{x(1-2x)}\,dx=\int\left\{\frac{1}{2}+\frac{1}{2}\left(\frac{2}{x}+\frac{3}{1-2x}\right)\right\}dx$$

$$=\frac{x}{2}+\log|x|+\frac{3}{2(-2)}\log|1-2x|+C$$

$$=\frac{x}{2}+\log|x|+\frac{3}{4}\log|1-2x|+C$$

7. $\dfrac{x}{(x^2+1)}$

Ans. Let $\dfrac{x}{(x^2+1)(x-1)}=\dfrac{Ax+B}{(x^2+1)}+\dfrac{C}{(x-1)}$

$$x=(Ax+B)(x-1)+C(x^2+1)$$

$$x=Ax^2-Ax+Bx-B+Cx^2+C$$

Equating the coefficients of x^2, x , and constant term, we obtain

A + C = 0

- A + B = 1

- B + C = 0

On solving these equations, we obtain

$$A=-\frac{1}{2},B=\frac{1}{2},\text{ and }C=\frac{1}{2}$$

$$\therefore \int\frac{x}{(x^2+1)(x-1)}=-\frac{1}{2}\int\frac{x}{x^2+1}\,dx+\frac{1}{2}\int\frac{1}{x^2+1}dx+\frac{1}{2}\int\frac{1}{x^2-1}\,dx$$

$$=-\frac{1}{4}\frac{2x}{x^2+1}\,dx+\frac{1}{2}\tan^{-1}x+\frac{1}{2}\log|x-1|+C$$

Equating the coefficients of x 2, x , and constant term, we obtain

A + C = 0

- A + B = 1

- B + C = 0

On solving these equations, we obtain

$$\text{Consider} \int \frac{2x}{x^2+1}dx, \text{let } \left(x^2+1\right) = t \Rightarrow 2x\,dx = dt$$

$$\Rightarrow \int \frac{2x}{x^2+1}dx = \int \frac{dt}{t} = \log|t| = \log\left|x^2+1\right|$$

$$\therefore \int \frac{x}{\left(x^2+1\right)(x+1)}dx = -\frac{1}{4}\log\left|x^2+1\right| + \frac{1}{2}\log|x-1| + C$$

$$= \frac{1}{2}\log|x-1| - \frac{1}{4}\log\left|x^2-1\right| + \frac{1}{2}\tan^{-1}x + C$$

8. $\frac{x}{(x-1)^2(x+2)}$

Ans. $\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$$

Substituting x = 1, we obtain

$$B = \frac{1}{2}$$

Equating the coefficients of x^2 and constant term, we obtain

A + C = 0

-2 A + 2B + C = 0

On solving, we obtain

$$A = \frac{2}{9} \text{ and } C = \frac{-2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x-2)}$$

$$\Rightarrow \int \frac{x}{(x-1)^2(x+2)}dx = \frac{2}{9}\int \frac{1}{(x-1)}dx + \frac{1}{3}\int \frac{1}{(x-1)^2}dx - \frac{2}{9}\int \frac{1}{(x+2)^2}dx$$

$$= \frac{2}{9}\log|x-1| + \frac{1}{3}\left(\frac{-1}{x-1}\right) = \frac{2}{9}\log|x+2| + C$$

$$= \frac{2}{9}\log\frac{|x-1|}{|x+2|} - \frac{1}{3(x-1)} + C$$

9. $\dfrac{3x+5}{x^3-x^2-x+1}$

Ans. $\dfrac{3x+5}{x^3-x^2-x+1} = \dfrac{3x+5}{(x-1)^2(x+1)}$

$$\text{Let } \frac{3x+5}{(x-1)^2(x+1)} = \frac{A}{(x+1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+1)}$$

$$3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$

$$3x+5 = A(x^2-1) + B(x+1) + C(x^2+1-2x) \;.....(1)$$

Substituting x = 1 in equation (1), we obtain

B = 4

Equating the coefficients of x 2 and x, we obtain

A + C = 0

B - 2C = 3

On solving, we obtain

$$A = -\frac{1}{2} \text{ and } C = \frac{1}{2} \;\; \therefore \frac{3x+5}{(x-1)^2(x+1)} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\Rightarrow \int \frac{3x+5}{(x-1)^2(x+1)}dx = -\frac{1}{2}\int \frac{1}{x-1}dx + 4\int \frac{1}{(x-1)^2}dx + \frac{1}{2}\int \frac{1}{(x-1)}dx$$

$$= -\frac{1}{2}\log|x-1| + 4\left(\frac{-1}{x-1}\right) + \frac{1}{2}\log|x+1| + C$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| - \frac{4}{(x-1)} + C$$

10. $\dfrac{2x-3}{(x^2-1)(2x+3)}$

Ans. $\dfrac{2x-3}{(x^2-1)(2x+3)}=\dfrac{2x-3}{(x+1)(x-1)(2x+3)}$

Let $\dfrac{2x-3}{(x+1)(x-1)(2x+3)}=\dfrac{A}{(x+1)}+\dfrac{B}{(x-1)}+\dfrac{C}{(2x+3)}$

$\Rightarrow(2x-3)=A(x+1)(2x+3)+B(x+1)(2x+3)+C(x+1)(x+1)$

$\Rightarrow(2x-3)=A(2x^2+x-3)+B(2x^2+5x+3)+C(x^2-1)$

$\Rightarrow(2x-3)=(2A+2B+C)x^2+(A+5B)x+(-3A+3B-C)$

Equating the coefficients of$x^2$and x, we obtain

$B=-\dfrac{1}{10},A=\dfrac{5}{2},\text{and } C=-\dfrac{24}{5}$

$\therefore\dfrac{2x-3}{(x+1)(x-1)(2x+3)}\ \dfrac{5}{2(x+1)}-\dfrac{1}{10(x-1)}-\dfrac{5}{2(x+1)}-\dfrac{24}{5(2x+3)}$

$\Rightarrow\int\dfrac{2x-3}{(x^2-1)(2x+3)}dx=\dfrac{5}{2}\int\dfrac{1}{(x+1)}dx-\dfrac{1}{10}\int\dfrac{1}{x-1}dx-\dfrac{24}{5}\int\dfrac{1}{2x+3}dx$

$=\dfrac{5}{2}\log|x+1|-\dfrac{1}{10}\log|x-1|-\dfrac{24}{5\times2}\log|2x+3|$

$=\dfrac{5}{2}\log|x+1|-\dfrac{1}{10}\log|x-1|-\dfrac{12}{5}\log|2x+3|+C$

11. $\dfrac{5x}{(x+1)(x^2-4)}$

Ans. $\dfrac{5x}{(x+1)(x^2-4)}=\dfrac{5x}{(x+1)(x+2)(x-2)}$

Let $\dfrac{5x}{(x+1)(x+2)(x-2)}=\dfrac{A}{(x+1)}+\dfrac{B}{(x+2)}+\dfrac{C}{(x-2)}$

$5x=A(x+2)(x-2)+B(x+1)(x-2)+C(x+1)(x+2)$ .....(1)

$A=\dfrac{5}{3},B=-\dfrac{5}{2},\text{and } C=\dfrac{5}{6}$

Substituting x = - 1 , - 2 , and 2 respectively in equation (1), we obtain

$$\therefore \frac{5x}{(x+1)(x+2)(x-2)} = \frac{5}{3(x+1)} - \frac{5}{2(x+2)} + \frac{5}{6(x-2)}$$

$$\Rightarrow \int \frac{5x}{(x+1)(x^2-4)} dx = \frac{5}{3}\int \frac{1}{(x+1)} dx - \frac{5}{2}\int \frac{1}{(x+2)} dx + \frac{5}{6}\int \frac{1}{(x-2)} dx$$

$$= \frac{5}{3}\log|x+1| - \frac{5}{2}\log|x+2| + \frac{5}{6}\log|x-2| + C$$

12. $\frac{x^3+x+1}{x^2-1}$

Ans. It can be seen that the given integrand is not a proper fraction.

Therefore, on dividing $(x^3 + x + 1)$ by $x^2 - 1$, we obtain

$$\frac{x^3+x+1}{x^2-1} = x + \frac{2x+1}{x^2-1}$$

$$\text{Let } \frac{2x+1}{x^2-1} = \frac{A}{(x+1)} + \frac{B}{(x-1)}$$

$$2x+1 = A(x-1) + B(x+1) \qquad \ldots(1)$$

Substituting x = 1 and - 1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = \frac{3}{2}$$

$$\therefore \frac{x^3+x+1}{x^2-1} = x + \frac{1}{2(x+1)} + \frac{1}{2(x-1)}$$

$$\Rightarrow \int \frac{x^3+x+1}{x^2-1} dx = \int x\, dx + \frac{1}{2}\int \frac{1}{(x+1)} dx + \frac{3}{2}\int \frac{1}{(x-1)} dx$$

$$= \frac{x^2}{2} + \frac{1}{2}\log|x+1| + \frac{3}{2}\log|x-1| + C$$

13. $\frac{2}{(1-x)(1+x^2)}$

Ans. Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{(1-x)} + \frac{Bx+C}{(1+x^2)}$

$$2 = A(1+x^2) + (Bx+C)(1-x)$$

$$2 = A + Ax^2 + Bx - Bx^2 + C - Cx$$

Equating the coefficient of x 2, x, and constant term, we obtain

A - B = 0

B - C = 0

A 4 C = 2

On solving these equations, we obtain

A = 1, B = 1, and C = 1

$$\therefore \frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

$$\Rightarrow \int \frac{2}{(1-x)(1+x^2)}\, dx = \int \frac{1}{1-x}\, dx + \int \frac{x}{1+x^2}\, dx + \int \frac{1}{1+x^2}\, dx$$

$$= -\int \frac{1}{1-x}\, dx + \frac{1}{2}\int \frac{2x}{1+x^2}\, dx + \int \frac{1}{1+x^2}\, dx$$

$$= -\log|x-1| + \frac{1}{2}\log|1+x^2| + \tan^{-1} x + C$$

14. $\dfrac{3x-1}{(x+2)^2}$

Ans. Let $\dfrac{3x-1}{(x+2)^2} = \dfrac{A}{(x+2)} + \dfrac{B}{(x+2)^2}$

$$\Rightarrow 3x-1 = A(x+2) + B$$

Equating the coefficient of x and constant term, we obtain

A = 3

2A + B = —1 ⟹ B = —7

$$\therefore \frac{3x-1}{(x+2)^2} = \frac{3}{(x+2)} - \frac{7}{(x+2)^2}$$

$$\Rightarrow \int \frac{3x-1}{(x+2)^2}\, dx = 3\int \frac{1}{(x+2)}\, dx - 7\int \frac{x}{(x+2)^2} dx$$

$$= 3\log|x+2| - 7\left(\frac{-1}{(x+2)}\right) + C$$

$$= 3\log|x+2| + \frac{1}{(x+2)} + C$$

15. $\frac{1}{x^4-1}$

Ans. $\frac{1}{\left(x^4-1\right)}=\frac{1}{\left(x^2-1\right)\left(x^2+1\right)}=\frac{1}{(x+1)(x-1)\left(1+x^2\right)}$

$$\text{Let } \frac{1}{(x+1)(x-1)\left(1+x^2\right)}=\frac{A}{\left(x^2+1\right)}+\frac{B}{(x-1)}+\frac{Cx+D}{\left(x^2+1\right)}$$

$$1=A(x-1)\left(x^2+1\right)+B(x+1)\left(x^2+1\right)+(Cx+D)\left(x^2-1\right)$$

$$l=A\left(x^3+x-x^2-1\right)+B\left(x^3+x+x^2+1\right)+Cx^3+Dx^2-Cx-D$$

$$l=(A+B+C)x^2+(-A+B+D)x^2+(A+B-C)x+(-A+B+D)$$

Equating the coefficient of x 3, x 2, x, and constant term, we obtain

$A+B+C$

$-A+B+D$

$A+B-C$

$-A+B+D=1$

$A=-\frac{1}{4}, B=\frac{1}{4}, C=0, \text{ and } D=-\frac{1}{2}$

$$\therefore \frac{1}{x^4-1}=\frac{-1}{4(x+1)}+\frac{1}{4(x-1)}-\frac{1}{2\left(x^2+1\right)}$$

$$\Rightarrow \int \frac{1}{x^4-1}dx=-\frac{1}{4}\log|x-1|+\frac{1}{4}\log|x-1|-\frac{1}{2}\tan^{-1}x+C$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\tan^{-1}x+C$$

16. $\frac{1}{x\left(x^n+1\right)}$ [Hint: multiply numerator and denominator by x^{n-1} and put $x^n = t$]

Ans. $\frac{1}{x\left(x^n+1\right)}$ Multiplying numerator and denominator by x^{n-1}, we obtain

$$\frac{1}{x\left(x^n+1\right)}=\frac{x^{n-1}}{x^{n-1}x\left(x^n+1\right)}=\frac{x^{n-1}}{x^n\left(x^n+1\right)}$$

$$\text{Let } x^n=t \Rightarrow x^{n-1}dx=dt$$

$$\therefore \int \frac{1}{x\left(x^n+1\right)}dx = \int \frac{x^{n-1}}{x^n\left(x^n+1\right)}dx = \frac{1}{n}\int \frac{1}{t\left(t+1\right)}dt$$

$$\text{Let } \frac{1}{t\left(t+1\right)} = \frac{A}{t} + \frac{B}{\left(t+1\right)}$$

$$1 = A\left(1+t\right) + Bt \qquad \text{....}(1)$$

Substituting t = 0, - 1 in equation (1), we obtain

A = 1 and B = - 1

$$\therefore \frac{1}{t\left(t+1\right)} = \frac{1}{t} + \frac{1}{\left(t+1\right)}$$

$$\Rightarrow \int \frac{1}{x\left(x^n+1\right)}dx = \frac{1}{n}\int \left\{\frac{1}{t} - \frac{1}{t\left(t+1\right)}\right\}dx$$

$$= \frac{1}{n}\left[\log|t| - \log|t+1|\right] + C$$

$$= \frac{1}{n}\left[\log\left|x^n\right| - \log\left|x^n+1\right|\right] + C$$

$$= \frac{1}{n}\log\left|\frac{x^n}{x^n+1}\right| + C$$

17. $\dfrac{\cos x}{\left(1-\sin x\right)\left(2-\sin x\right)}$ [Hint: Put sin x = t]

Ans. $\dfrac{\cos x}{\left(1-\sin x\right)\left(2-\sin x\right)}$

$$\text{Let } \sin x = t \Rightarrow \cos x\, dx = dt$$

$$\therefore \int \frac{\cos x}{\left(1-\sin x\right)\left(2-\sin x\right)}dx = \int \frac{dt}{\left(1-t\right)\left(2-t\right)}$$

$$\text{Let } \frac{1}{\left(1-t\right)\left(2-t\right)} = \frac{A}{\left(1-t\right)} + \frac{B}{\left(2-t\right)}$$

$$1 = A\left(2-t\right) + B\left(1-t\right) \qquad \text{....}(1)$$

Substituting t = 2 and then t = 1 in equation (1), we obtain

$$\therefore \frac{1}{\left(1-t\right)\left(2-t\right)} = \frac{1}{\left(1-t\right)} + \frac{1}{\left(2-t\right)}$$

$$\Rightarrow \int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx = \int \left\{ \frac{1}{(1-t)} + \frac{1}{(2-t)} \right\}$$

$$= -\log|1-t| + \log|2-t| + C$$

$$= \log\left|\frac{2-t}{1-t}\right| + C$$

$$= \log\left|\frac{2-\sin x}{1-\sin x}\right| + C$$

18. $$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$$

Ans. $$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \frac{(4x^2+10)}{(x^2+3)(x^2+4)}$$

$$\text{Let } \frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+4)}$$

$$4x^2+10 = (Ax+B)(x^2+4) + Cx + D(x^2+4)$$

$$4x^2+10 = Ax^3 + 4Ax + Bx^3 + 4B + Cx^3 + 3Cx + Dx^2 + 3D$$

$$4x^2+10 = (A+C)x^3(B+D)x^3 + (4A+3C)x + (4B+3D)$$

Equating the coefficients of x^3, x^2, x, and constant term, we obtain

A + C = 0

B + D = 4

4A + 3C = 0

4B + 3D = 10

On solving these equations, we obtain

A = 0, B = - 2 , C = 0, and D = 6

$$\therefore \frac{(4x^2+10)}{(x^2+3)(x^2+4)} = \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}$$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 - \left(\frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)} \right)$$

$$\Rightarrow \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx = \int \left\{1 + \frac{-2}{(x^2+3)} + \frac{6}{(x^2+4)}\right\}$$

$$= \int \left\{1 + \frac{2}{x^2(\sqrt{3})^2} - \frac{6}{x^2+2^2}\right\}$$

$$= x + 2\left(\frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}}\right) - 6\left(\frac{1}{2}\tan^{-1}\frac{x}{2}\right) + C$$

$$= x + \frac{2}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} - 3\tan^{-1}\frac{x}{2} + C$$

19. $\frac{2x}{(x^2+1)(x^2+3)}$

Ans. $\frac{2x}{(x^2+1)(x^2+3)}$

Let $x^2 = t \Rightarrow 2x\,d\,x = dt$

$$\therefore \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \frac{dt}{(t+1)(t+3)} \qquad ....(1)$$

$$\text{Let } \frac{1}{(t+1)(t+3)} + \frac{A}{(t+1)} + \frac{B}{(t+3)}$$

$$1 = A(t+3) + B(t+1) \qquad .... (2)$$

Substituting t = - 3 and t = - 1 in equation (1), we obtain

$$A = \frac{1}{2} \text{ and } B = -\frac{1}{2}$$

$$\therefore \frac{1}{(t+1)(t+3)} + \frac{A}{2(t+1)} + \frac{B}{2(t+3)}$$

$$\Rightarrow \int \frac{2x}{(x^2+1)(x^2+3)}dx = \int \left\{\frac{1}{2(t+1)} + \frac{1}{2(t+3)}\right\}dx$$

$$= \frac{1}{2}\log|(t+1)| - \frac{1}{2}\log|(t-3)| + C$$

$$= \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C$$

$$= \frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C$$

20. $\frac{1}{x\left(x^4-4\right)}$

Ans. $\frac{1}{x\left(x^4-4\right)}$

Multiplying numerator and denominator by x^3, we obtain

$$\frac{1}{x\left(x^4-4\right)} = \frac{x^3}{x^4\left(x^4-1\right)}$$

$$\therefore \int \frac{1}{x\left(x^4-4\right)}\,dx = \int \frac{x^3}{x^4\left(x^4-1\right)}\,dx$$

Let $x^4 = t \Rightarrow 4x^3\,dx = dt$

$$\therefore \int \frac{1}{x\left(x^4-1\right)}\,dx = \frac{1}{4}\int \frac{dt}{t\left(t-1\right)}\,dx$$

$$\text{Let } \frac{1}{t\left(t-1\right)} = \frac{A}{t} + \frac{B}{\left(t-1\right)}$$

$$1 = A\left(t-1\right) + Bt \qquad \ldots.(1)$$

Substituting t = 0 and 1 in (1), we obtain

A = - 1 and B = 1

$$\Rightarrow \frac{1}{t\left(t-1\right)} = \frac{-1}{t} + \frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{x\left(x^4-1\right)}\,dx = \frac{1}{4}\left\{\frac{-1}{t} + \frac{1}{t-1}\right\}dt$$

$$= \frac{1}{4}\log\left|\frac{t-1}{t}\right| + C$$

$$= \frac{1}{4}\log\left|\frac{x^4-1}{t}\right| + C$$

21. $\frac{1}{\left(e^x-1\right)}$ [Hint: Put $e^x = t$]

Ans. $\frac{1}{\left(e^x-1\right)}$

Let $e^x = t \Rightarrow e^x\,dx = dt$

$$\Rightarrow \int \frac{1}{e^x-1}\,dx = \int \frac{1}{t-1}\times\frac{dt}{t} = \int \frac{1}{t\left(t-1\right)}dt$$

$$\text{Let } \frac{1}{t\left(t-1\right)} = \frac{A}{t}+\frac{B}{t-1}$$

$$1 = A\left(t-1\right)+Bt$$

$$\therefore \frac{1}{t\left(t-1\right)} = \frac{-1}{t}+\frac{1}{t-1}$$

$$\Rightarrow \int \frac{1}{t\left(t-1\right)}\,dt = \log\left|\frac{t-1}{t}\right|+C$$

$$= \log\left|\frac{e^x-1}{e^x}\right|+C$$

22. $\int \frac{x\,dx}{\left(x-1\right)\left(x-2\right)}$ equals

A. $\log\left|\frac{\left(x-1\right)^2}{x-2}\right|+C$ B. $\log\left|\frac{\left(x-1\right)^2}{x-1}\right|+C$

C. $\log\left|\left(\frac{x-1}{x-2}\right)^2\right|+C$ D. $\log\left|\left(x-1\right)\left(x-2\right)\right|+C$

Ans. Let $\frac{x}{\left(x-1\right)\left(x-2\right)} = \frac{A}{\left(x-1\right)}+\frac{B}{\left(x-2\right)}$

$$x = A\left(x-2\right)+B\left(x-1\right) \qquad ....(1)$$

Substituting x = 1 and 2 in (1), we obtain

A = - 1 and B = 2

$$\therefore \frac{x}{\left(x-1\right)\left(x-2\right)} = \frac{1}{\left(x-1\right)}+\frac{2}{\left(x-2\right)}$$

$$\Rightarrow \int \frac{x}{\left(x-1\right)\left(x-2\right)}\,dx = \int \left\{\frac{1}{\left(x-1\right)}+\frac{2}{\left(x-2\right)}\right\}dx$$

$$= -\log|x-1| + 2\log|x-2| + C$$

$$= \log\left|\frac{(x-2)^2}{x-1}\right| + C$$

Hence, the correct Answer is B.

23. $\int \frac{dx}{x\left(x^2+1\right)}$ equals

A. $\log|x| - \frac{1}{2}\log\left(x^2+1\right) + C$

B. $\log|x| + \frac{1}{2}\log\left(x^2+1\right) + C$

C. $-\log|x| + \frac{1}{2}\log\left(x^2+1\right) + C$

D. $\frac{1}{2}\log|x| + \frac{1}{2}\log\left(x^2+1\right) + C$

Ans. Let $\frac{1}{x\left(x^2+1\right)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$$1 = A\left(x^2+1\right) + (Bx+C)x$$

Equating the coefficients of x^2, x , and constant term, we obtain

A + B = 0

C = 0

A = 1

On solving these equations, we obtain

A = 1, B = - 1 , and C = 0

$$\therefore \frac{1}{x\left(x^2+1\right)} = \frac{1}{x} + \frac{-x}{x^2+1}$$

$$\Rightarrow \int \frac{1}{x\left(x^2+1\right)}\,dx \int\left\{\frac{1}{x} + \frac{-x}{x^2+1}\right\}$$

$$= \log|x| - \frac{1}{2}\log\left|x^2+1\right| + C$$

Hence, the correct Answer is A.