



SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Integrals

Exercise - 7.6

1. $x \sin x$

Ans. Let $I = \int x \sin x \, dx$

Taking x as first function and $\sin x$ as second function and integrating by parts, we obtain

$$I = x \sin x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin x \, dx \right\} dx$$

$$= x(-\cos x) - \int 1 \cdot (-\cos x) \, dx$$

$$= -x \cos x + \sin x + C$$

2. $x \sin 3x$

Ans. Let $I = \int x \sin 3x \, dx$

Taking x as first function and $\sin 3x$ as second function and integrating by parts, we obtain

$$I = x \int \sin 3x \, dx - \int \left\{ \left(\frac{d}{dx} x \right) \int \sin 3x \, dx \right\} dx$$

$$= x \left(\frac{-\cos 3x}{3} \right) - \int 1 \cdot \left(\frac{-\cos 3x}{3} \right) dx$$

$$= \frac{-x \cos 3x}{3} + \frac{1}{3} \int \cos 3x \, dx + C$$

$$= \frac{-x \cos 3x}{3} + \frac{1}{9} \sin 3x + C$$

3. $x^2 e^x$

Ans. Let $I = \int x^2 e^x \, dx$

Taking x^2 as first function and e^x as second function and integrating by parts, we obtain

$$I = x^2 e^x \, dx - \int \left\{ \left(\frac{d}{dx} x^2 \right) \int e^x \, dx \right\} dx$$

$$= x^2 e^x - \int 2x \cdot e^x \, dx = x^2 e^x - 2 \int x \cdot e^x \, dx$$

Again integrating by parts, we obtain

$$= x^2 e^x - 2x \cdot e^x - \left[x \cdot \int \left\{ \left(\frac{d}{dx} x \right) \int e^x \, dx \right\} dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - e^x \right]$$

$$= x^2 e^x - 2 x e^x + 2 e^x + C$$

$$= e^x (x^2 - 2x + 2) + C$$

4. $x \log x$

Ans. Let $I = \int x \log x dx$

Taking $\log x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log x \int x dx - \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx$$

$$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log x}{2} - \frac{x^2}{4} + C$$

5. $x \log 2x$

Ans. Let $I = \int x \log 2x dx$

Taking $\log 2x$ as first function and x as second function and integrating by parts, we obtain

$$I = \log 2x \int x dx - \left\{ \left(\frac{d}{dx} 2 \log x \right) \int x dx \right\} dx$$

$$= \log 2x \cdot \frac{x^2}{2} - \int \frac{2}{2x} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \int \frac{x}{2} dx$$

$$= \frac{x^2 \log 2x}{2} - \frac{x^2}{4} + C$$

6. $x^2 \log x$

Ans. Let $I = \int x^2 \log x \, dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \left(\frac{x^3}{3} \right) - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \int \frac{x^3}{3} dx$$

$$= \frac{x^3 \log x}{3} - \frac{x^3}{9} + C$$

7. $x \sin^{-1} x$

Ans. Let $I = \int x \sin^{-1} x \, dx$

Taking $\sin^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$I = \sin^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int x \, dx \right\} dx$$

$$= \sin^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} dx - \frac{1}{\sqrt{1-x^2}} \right\} dx$$

$$= \frac{x^2 \sin^{-1} x}{2} + \frac{1}{2} \int \left\{ \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - \sin^{-1} x \right\} + C$$

$$\begin{aligned}
 &= \frac{x^2 \sin^{-1} x}{2} + \frac{x}{4} \sqrt{1-x^2} + \frac{1}{4} \sin^{-1} x - \frac{1}{2} \sin^{-1} x + C \\
 &= \frac{1}{4} 2x^2 - 1 \sin^{-1} x + \frac{x}{4} \sqrt{1-x^2} + C
 \end{aligned}$$

8. $x \tan^{-1} x$

Ans. Let $I = \int x \sin^{-1} x \, dx$

Taking $\tan^{-1} x$ as first function and x as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= \tan^{-1} x \int x \, dx - \int \left\{ \left(\frac{d}{dx} \tan^{-1} x \right) \int x \, dx \right\} dx \\
 &= \tan^{-1} x \left(\frac{x^2}{2} \right) - \int \frac{1}{\sqrt{1-x^2}} \cdot \frac{x^2}{2} dx \\
 &= \frac{x^2 \tan^{-1} x}{2} + \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx \\
 &= \frac{x^2 \tan^{-1} x}{2} + \frac{1}{2} \int \left(\frac{x^2+1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right) dx \\
 &= \frac{x^2 \tan^{-1} x}{2} + \frac{1}{2} \int \left(1 - \frac{1}{1+x^2} \right) dx \\
 &= \frac{x^2 \tan^{-1} x}{2} + \frac{1}{2} (x - \tan^{-1} x) + C \\
 &= \frac{x^2}{2} \tan^{-1} x - \frac{x}{2} + \frac{1}{2} \tan^{-1} x + C
 \end{aligned}$$

9. $(\sin^{-1} x)^2$

Let $I = \int (\sin^{-1} x)^2 \cdot 1 \, dx$

Taking $(\sin^{-1} x)^2$ as first function and 1 as second function and integrating by parts, we obtain

$$\begin{aligned}
 I &= (\sin^{-1} x) \int 1 \, dx - \int \left\{ \frac{d}{dx} (\sin^{-1} x)^2 \cdot \int 1 \cdot dx \right\} dx \\
 &= (\sin^{-1} x)^2 \cdot x - \int \frac{2 \sin^{-1} x}{\sqrt{1-x^2}} \cdot x \, dx
 \end{aligned}$$

$$\begin{aligned}
&= x(\sin^{-1} x)^2 + \int \sin^{-1} x \cdot \left(\frac{-2x}{\sqrt{1-x^2}} \right) dx \\
&= x(\sin^{-1} x)^2 + \left[\sin^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \sin^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} \right\} dx \right] \\
&= x(\sin^{-1} x)^2 + \left[\sin^{-1} x \cdot \sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\
&= x(\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - \int 2 dx \\
&= x(\sin^{-1} x)^2 + 2\sqrt{1-x^2} \sin^{-1} x - 2x + C
\end{aligned}$$

10. $\frac{x \cos^{-1} x}{\sqrt{1-x^2}}$

Ans. Let $I = \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$

$$I = \frac{-1}{2} \int \frac{-2x}{\sqrt{1-x^2}} \cdot \cos^{-1} x dx$$

Taking \cos^{-1} as first function and $\left(\frac{-2x}{\sqrt{1-x^2}} \right)$ as second function and integrating by parts, we obtain

$$\begin{aligned}
I &= \frac{-1}{2} \left[\cos^{-1} x \int \frac{-2x}{\sqrt{1-x^2}} dx - \int \left\{ \left(\frac{d}{dx} \cos^{-1} x \right) \int \frac{-2x}{\sqrt{1-x^2}} dx \right\} dx \right] \\
&= \frac{-1}{2} \left[\cos^{-1} x \cdot 2\sqrt{1-x^2} - \int \frac{-1}{\sqrt{1-x^2}} \cdot 2\sqrt{1-x^2} dx \right] \\
&= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + \int 2 dx \right] \\
&= \frac{-1}{2} \left[2\sqrt{1-x^2} \cos^{-1} x + 2x \right] + C \\
&= - \left[\sqrt{1-x^2} \cos^{-1} x + x \right] + C
\end{aligned}$$

11. $x \sec^2 x$

Ans. Let $I = \int x \sec^2 x dx$

Taking x as first function and $\sec^2 x$ as second function and integrating by parts, we obtain

$$I = x \int \sec^2 x dx - \int \left\{ \left\{ \frac{d}{dt} x \right\} \int \sec^2 x dx \right\} dx$$

$$x = \tan x - \int 1 \cdot \tan x dx$$

$$= x \tan x + \log |\cos x| + C$$

12. $\tan^{-1} x$

Ans. Let $I = 1 \cdot \tan^{-1} x dx$

Taking $\tan^{-1} x$ as first function and 1 as second function and integrating by parts, we obtain

$$I = \tan^{-1} x \int 1 dx - \int \left\{ \left(\frac{d}{dt} \tan^{-1} x \right) \int 1 \cdot dx \right\} dx$$

$$= \tan^{-1} x \cdot x - \int \frac{1}{1+x^2} \cdot x dx$$

$$= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$$

$$= x \tan^{-1} x - \frac{1}{2} \log |1+x^2| + C$$

$$= x \tan^{-1} x - \frac{1}{2} \log (1+x^2) + C$$

13. $x(\log x)^2$

Ans. $I = \int x(\log x)^2 dx$

Taking $(\log x)^2$ as first function and 1 as second function and integrating by parts, we obtain.

$$I = (\log x)^2 \int x dx - \int \left[\left\{ \left(\frac{d}{dx} \log x \right)^2 \right\} \int x dx \right] dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[\int 2 \log x \cdot \frac{1}{x} \cdot \frac{x^2}{2} dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \int x \log x dx$$

Again integrating by parts, we obtain

$$\begin{aligned}
I &= \frac{x^2}{2}(\log x)^2 - \left[\log x \int x dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x dx \right\} dx \right] \\
&= \frac{x^2}{2}(\log x)^2 - \left[\frac{x^2}{2} - \log x - \int \frac{1}{x} \cdot \frac{x^2}{2} dx \right] \\
&= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx \\
&= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2} \log x + \frac{x^2}{4} + C
\end{aligned}$$

14. $(x^2 + 1) \log x$

Ans. Let $I = \int (x^2 + 1) \log x dx = \int x^2 \log x dx + \int \log x dx$

Let $I = I_1 + I_2 \dots \dots \dots (1)$

where, $I_1 = \int x^2 \log x dx$ and $I_2 = \int \log x dx$

$I_1 = \int x^2 \log x dx$

Taking $\log x$ as first function and x^2 as second function and integrating by parts, we obtain

$$I_1 = \log x \int x^2 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int x^2 dx \right\} dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{x^3}{3} \log x - \frac{1}{3} \left(\int x^2 dx \right)$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 \dots \dots \dots (2)$$

$I_2 = \int \log x dx$

Taking $\log x$ as first function and 1 as second function and integrating by parts, we obtain.

$$I_2 = \log x \int 1 dx - \int \left\{ \left(\frac{d}{dx} \log x \right) \int 1 dx \right\}$$

$$= \log x \cdot x - \int \frac{1}{x} \cdot x dx$$

$$= x \log x - \int 1 dx$$

$$= x \log x - x + C_2 \dots \dots (3)$$

Using equations (2) and (3) in (1), we obtain

$$I = \frac{x^3}{3} \log x - \frac{x^3}{9} + C_1 + x \log x - x + C_2$$

$$= \frac{x^3}{3} \log x - \frac{x^3}{9} + \log x - x + (C_1 + C_2)$$

$$= \left(\frac{x^3}{3} + x \right) \log x - \frac{x^3}{9} - x + C$$

15. $e^x (\sin x + \cos x)$

Ans. Let $I = \int e^x (\sin x + \cos x)$

Let $f(x) = \sin x$

A $f'(x) = \cos x$

A $I = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$I = e^x \sin x + C$

16. $\therefore I = e^x \sin x + C$

Ans. Let $I = \int \frac{xe^x}{(1+x)^2} dx = \int e^x \left\{ \frac{x}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1+x-1}{(1+x)^2} \right\} dx$

$= \int e^x \left\{ \frac{1}{1+x} - \frac{1}{(1+x)^2} \right\} dx$

Let $f(x) = \frac{1}{1+x}$ A $f'(x) = \frac{-1}{(1+x)^2}$

$\Rightarrow \int \frac{xe^x}{(1+x)^2} dx = \int e^x \{f(x) + f'(x)\} dx$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$$\therefore \int \frac{xe^x}{(1+x)^2} dx = \frac{e^x}{1+x} + C$$

17. $e^x \left(\frac{1 + \sin x}{1 + \cos x} \right)$

Ans. $e^x \left(\frac{1 + \sin x}{1 + \cos x} \right)$

$$= e^x \left(\frac{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$

$$= \frac{1}{2} e^x \cdot \left(\frac{\sin \frac{x}{2} + \cos \frac{x}{2}}{\cos \frac{x}{2}} \right)^2$$

$$= \frac{1}{2} e^2 \left[\tan \frac{x}{2} + 1 \right]^2$$

$$= \frac{1}{2} e^2 \left(1 + \tan \frac{x}{2} \right)^2$$

$$= \frac{1}{2} e^2 \left[1 + \tan^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$= \frac{1}{2} e^2 \left[\sec^2 \frac{x}{2} + 2 \tan \frac{x}{2} \right]$$

$$\frac{e^x (1 + \sin x) dx}{(1 + \cos x)} = e^x \left[\frac{1}{2} \sec^2 \frac{x}{2} + \tan \frac{x}{2} \right] \dots (1)$$

$$\text{Let } \tan \frac{x}{2} = f(x) \text{ then } f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

From equation (1), we obtain

$$\int \frac{e^x(1+\sin x)}{(1+\cos x)} dx = e^x \tan \frac{x}{2} + C$$

18. $e^x \left(\frac{1}{x} - \frac{1}{x^2} \right)$

Ans. Let $I = \int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

Also let, $\frac{1}{x} = f(x)$ $f'(x) = \frac{-1}{x^2}$

It is known that, $\int e^x \{f(x) + Ff'(x)\} dx = e^x f(x) + C$

$\therefore I = \frac{e^x}{x} + C$

19. $\frac{(x-3)e^x}{(x-1)^3}$

Ans. $\int e^x \left\{ \frac{x-3}{(x-1)^3} \right\} dx = \int e^x \left\{ \frac{x-1-2}{(x-1)^3} \right\} dx$

$= \int e^x \left\{ \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right\} dx$

Let $f(x) = \frac{1}{(x-1)^2}$ $f'(x) = \frac{-2}{(x-1)^3}$

It is known that, $\int e^x \{f(x) + f'(x)\} dx = e^x f(x) + C$

$= \int e^x \left\{ \frac{(x-3)}{(x-1)^2} \right\} dx = \frac{e^x}{(x-1)^2} + C$

20. $e^{2x} \sin x$

Ans. Let, $I = \int e^{2x} \sin x dx \dots \dots (1)$

Integrating by parts, we obtain

$I = \sin x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \sin x \right) \int e^{2x} dx \right\} dx$

$$\Rightarrow I = \sin x \cdot \frac{e^{2x}}{2} - \int \cos x \cdot \frac{e^{2x}}{2} dx$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \int e^{2x} \cos x dx$$

Again integrating by parts, we obtain

$$I = \frac{e^{2x} \cdot \sin x}{2} - \frac{1}{2} \left[\cos x \int e^{2x} dx - \int \left\{ \left(\frac{d}{dx} \cos x \right) \int e^{2x} dx \right\} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\cos x \cdot \frac{e^{2x}}{2} - \int (-\sin x) \frac{e^{2x}}{2} dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{1}{2} \left[\frac{e^{2x} \cos x}{2} + \frac{1}{2} \int e^{2x} \sin x dx \right]$$

$$\Rightarrow I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} - \frac{1}{4} I \quad [\text{From(1)}]$$

$$\Rightarrow I = \frac{1}{4} I = \frac{e^{2x} \cdot \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow \frac{5}{4} I = \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4}$$

$$\Rightarrow I = \frac{4}{5} \left[\frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} \right] + C$$

$$\Rightarrow I = \frac{e^{2x}}{5} [2 \sin x - \cos x] + C$$

21. $\sin^{-1} \left(\frac{2x}{1+x^2} \right)$

Ans. Let $x = \tan \theta$ A $dx = \sec^2 \theta d\theta$

$$\therefore \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1} (\sin 2\theta)$$

$$A \int \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

$$2 \left[\theta \int \sec^2 \theta d\theta - \int \left\{ \left(\frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right\} d\theta \right]$$

$$\begin{aligned}
&= 2\left[\theta \cdot \tan\theta - \int \tan\theta d\theta\right] \\
&= 2\left[\theta \tan\theta + \log|\cos\theta|\right] + C \\
&= 2\left[x \tan^{-1}x + \log\left|\frac{1}{\sqrt{1+x^2}}\right|\right] + C \\
&= 2x \tan^{-1}x + \log(1+x^2)^{-\frac{1}{2}} + C \\
&= 2x \tan^{-1}x + 2\left[-\frac{1}{2}\log(1+x^2)\right] + C \\
&= 2x \tan^{-1}x - \log(1+x^2) + C
\end{aligned}$$

22. $\int x^2 e^{x^3} dx$

Ans. Let $I = \int x^2 e^{x^3} dx$

Also, let $x^3 = t$ A $3x^2 dx = dt$

$$\Rightarrow I = \frac{1}{3} \int e^x dt$$

$$= \frac{1}{3} (e^x) + C$$

$$= \frac{1}{3} e^{x^3} + C$$

23. $\int e^x \sec x (1 + \tan x) dx$

Ans. $\int e^x \sec x (1 + \tan x) dx$

Let, $I = \int e^x \sec x (1 + \tan x) dx = \int e^x (\sec x + \sec x \tan x) dx$

Also, let $\sec x = f(x)$ A $\sec x \tan x = f'(x)$

$$\therefore I = e^x \sec x + C$$

Hence, the correct Answer is B.