SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Integrals
Exercise - 7.7

1. $\sqrt{4-x^2}$

Ans. Let $I=\int\sqrt{4-x^2}dx=\int\sqrt{(2)^2-(x)^2}dx$

It is known that, $\int\sqrt{a^2-x^2}dx=\frac{x}{2}\int\sqrt{a^2-x^2}\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\therefore I=\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\frac{x}{2}+C$

$=\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+C$

2. $\sqrt{1-4x^2}$

Ans. Let $I=\int\sqrt{1-4x^2}dx=\sqrt{(1)^2-(2x)^2}dx$

Let $2x=t\Rightarrow 2dx=dt$

$\therefore I=\frac{1}{2}\int\sqrt{(1)^2-(t)^2}dt$

It is known that $\int\sqrt{a^2-x^2}dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$\Rightarrow I=\frac{1}{2}\left[\frac{t}{2}\sqrt{1-t^2}+\frac{1}{2}\sin^{-1}t\right]+C$

$=\frac{t}{4}\sqrt{1-t^2}+\frac{1}{4}\sin^{-1}t+C$

$=\frac{2x}{4}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

$=\frac{x}{2}\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}2x+C$

3. $\sqrt{x^2+4x+6}$

Ans. Let $I=\int\sqrt{x^2+4x+6}\,dx$

$=\int\sqrt{x^2+4x+4+2}\,dx$

$$=\int\sqrt{\left(x^2+4x+4\right)+2}\,dx$$

$$=\int\sqrt{(x+2)^2+\left(\sqrt{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2+a^2}dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\frac{2}{2}\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

$$=\frac{(x+2)}{2}\sqrt{x^2+4x+6}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+C$$

4. $\sqrt{x^2+4x+1}$

Ans. Let $I=\int\sqrt{x^2+4x+1}\,dx$'

$$=\int\sqrt{(x+4x+4)-3}\,dx$$

$$=\int\sqrt{(x+2)^2-\left(\sqrt{3}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x+1}-\frac{3}{2}\log\left|(x+2)+\sqrt{x^2+4x+1}\right|$$

5. $\sqrt{1+4x-x^2}$

Ans. Let $I=\int\sqrt{1-4x-x^2}dx$

$$=\int\sqrt{1-\left(x^2+4x+4-4\right)}\,dx$$

$$=\int\sqrt{1+4-(x+2)^2}\,dx$$

$$=\int\sqrt{\left(\sqrt{5}\right)^2-(x+2)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}\,dx=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{1-4x-x^2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+C$$

6. $\sqrt{x^2+4x-5}$

Ans. Let $I=\int\sqrt{x^2+4x-5}\,dx$

$$=\int\sqrt{\left(x^2+4x+4\right)-9}\,dx$$

$$=\int\sqrt{(x+2)^2-(3)^2}$$

$$=\int\sqrt{(x+2)^2-(3)^2}\,dx$$

It is known that, $\int\sqrt{x^2-a^2}dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{(x+2)}{2}\sqrt{x^2+4x-5}-\frac{9}{2}\log\left|(x+2)+\sqrt{x^2+4x-5}\right|+C$$

7. $\sqrt{1+3x-x^2}$

Ans. Let $I=\int\sqrt{1+3x-x^2}\,dx$

$$=\int\sqrt{1-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)}\,dx$$

$$=\int\sqrt{1-\left(1+\frac{9}{4}\right)-\left(x-\frac{3}{2}\right)^2}$$

$$=\int\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{a^2-x^2}=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+C$

$$\therefore I=\frac{x-\frac{3}{2}}{2}\sqrt{1+3x-x^2}+\frac{13}{4\times 2}\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{13}}{2}}\right)+C$$

$$=\frac{2x-3}{4}\sqrt{1+3x-x^2}+\frac{13}{8}\sin^{-1}\left(\frac{2x-3}{\sqrt{13}}\right)+C$$

8. $\sqrt{x^2+3x}$

Ans. Let $I=\int\sqrt{x^2+3x}$

$$=\int\sqrt{x^2+3x+\frac{9}{4}-\frac{9}{4}}\,dx$$

$$=\int\sqrt{\left(x+\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2}\,dx$$

It is known that, $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+C$

$$\therefore I=\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x}-\frac{\frac{9}{4}}{2}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

$$=\frac{(2x+3)}{2}\sqrt{x^2+3x}-\frac{9}{8}\log\left|\left(x+\frac{3}{2}\right)+\sqrt{x^2+3x}\right|+C$$

9. $\sqrt{1+\frac{x^2}{9}}$

Ans. Let $I=\int\sqrt{1+\frac{x^2}{9}}\,dx=\frac{1}{3}\int\sqrt{9+x^2}\,dx=\frac{1}{3}\int\sqrt{(3)^2+x^2}\,dx$

It is known that, $\int\sqrt{x^2+a^2}\,dx=\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore I=\frac{1}{3}\left[\frac{x}{2}\sqrt{x^2+9}+\frac{9}{2}\log\left|x+\sqrt{x^2+9}\right|\right]+C$$

$$=\frac{x}{6}\sqrt{x^2+9}+\frac{3}{2}\log\left|x+\sqrt{x^2+9}\right|+C$$

10. $\int\sqrt{1+x^2}\,dx$

Ans. It is known that, $\int\sqrt{a^2+x^2}\,dx=\frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{x^2+a^2}\right|+C$

$$\therefore\int\sqrt{1+x^2}\,dx=\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\log\left|x+\sqrt{1+x^2}\right|+C$$

11. $\int \sqrt{x^2 - 8x + 7}\, dx$

Ans. Let $I = \int \sqrt{x^2 - 8x + 7}\, dx$

$= \int \sqrt{\left(x^2 - 8x + 16\right) - 9}\, dx$

$= \int \sqrt{\left(x - 4\right)^2 - \left(3\right)^2}\, dx$

It is known that, $\int \sqrt{x^2 + a^2}\, dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C$

$\therefore I = \frac{(x-4)}{2}\sqrt{x^2 - 8x + 7} - \frac{9}{2}\log\left|\left(x - 4\right) + \sqrt{x^2 - 8x + 7}\right| + C$