SpeedLabs

MATHS

12^{th}

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1. $\int_a^b x\,dx$

ANS. It is known that,

$$\int_a^b f(x)dx = (b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f(a+(n-1)h)\right], \text{where } h=\frac{b-a}{n}$$

$$\text{Here}, a=a, b=b, \text{and } f(x)=x$$

$$\therefore \int_a^b x\,dx = (b-a)\lim_{x\to\infty}\frac{1}{n}\left[a+(a+h)\ldots(a+2h)\ldots a+(n-1)h\right]$$

$$=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[\left(\underbrace{a+a+a+\ldots+a}_{\text{n times}}\right)+\left(h+2h+3h+\ldots+(n-1)h\right)\right]$$

$$=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[na+h\left(1+2+3+\ldots+(n-1)\right)\right]$$

$$=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[na+h\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[na+\frac{n(n-1)(h)}{2}\right]$$

$$=(b-a)\lim_{x\to\infty}\frac{n}{n}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{x\to\infty}\left[a+\frac{(n-1)h}{2}\right]$$

$$=(b-a)\lim_{x\to\infty}\left[a+\frac{(n-1)(b-a)}{2n}\right]$$

$$=(b-a)\lim_{x\to\infty}\left[a+\frac{\left(1-\frac{1}{n}\right)(b-a)}{2}\right]$$

$$=(b-a)\left[a+\frac{(b-a)}{2}\right]$$

$$=(b-a)\left[\frac{2a+b-a}{2}\right]$$

$$=\frac{(b-a)(b+a)}{2}$$

$$=\frac{1}{2}\left(b^2-a^2\right)$$

2. $\int_0^5 (x+1)dx$

Ans. Let $I=\int_0^5 (x+1)dx$

It is know that,

$$\int_0^5 f(x)dx=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a+h)\ldots f(a+(n-1)h)\right], \text{where } h=\frac{b-a}{n}$$

Here, $a=0, b=5,$ and $f(x)=(x+1)$

$$\Rightarrow h=\frac{5-0}{n}=\frac{5}{n}$$

$$\therefore \int_0^5 (x+1)dx=(5-0)\lim_{x\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+\ldots+f\left((n-1)\frac{5}{n}\right)\right]$$

$$=5\lim_{x\to\infty}\frac{1}{n}\left[1+\left(\frac{5}{n}+1\right)+\ldots\left\{1+\left(\frac{5(n-1)}{n}\right)\right\}\right]$$

$$=5\lim_{x\to\infty}\frac{1}{n}\left[\underbrace{(1+1+1\ldots1)}_{\text{n times}}+\left[\frac{5}{n}+2\cdot\frac{5}{n}+3\cdot\frac{5}{n}+\ldots(n-1)\frac{5}{n}\right]\right]$$

$$=5\lim_{x\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\{1+2+3\ldots(n-1)\}\right]$$

$$=5\lim_{x\to\infty}\frac{1}{n}\left[n+\frac{5}{n}\cdot\frac{(n-1)n}{2}\right]$$

$$=5\lim_{x\to\infty}\frac{1}{n}\left[n+\frac{5(n-1)}{2}\right]$$

$$=5\lim_{x\to\infty}\left[1+\frac{5}{2}\left(1-\frac{1}{n}\right)\right]$$

$$=5\left[1+\frac{5}{2}\right]$$

$$=5\left[\frac{7}{2}\right]$$

$$=\frac{35}{2}$$

3. $\int_2^3 x^2 dx$

Ans. It is know that,

$$\int_a^b f(x)dx=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+2h)\ldots f\{a+(n-1)h\}\right],\text{where } h=\frac{b-a}{n}$$

Here, $a=2, b=3,$ and $f(x)=x^2$

$$\Rightarrow h=\frac{3-2}{n}=\frac{1}{n}$$

$$\therefore \int_2^3 x^2 dx=(3-2)\lim_{x\to\infty}\frac{1}{n}\left[f(2)+f\left(2+\frac{1}{n}\right)+f\left(2+\frac{2}{n}\right)\ldots f\left\{2+(n-1)\frac{1}{n}\right\}\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[(2)^2+\left(2+\frac{1}{n}\right)^2+\ldots\left(2+\frac{(n-1)}{n}\right)^2\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[2^2+\left\{2^2+\left(\frac{1}{n}\right)^2+2\cdot2\cdot\frac{1}{n}\right\}+\ldots+\left\{(2)^2+\frac{(n-1)^2}{n^2}+2\cdot2\cdot\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[\left(2^2+\underset{\text{n times}}{\ldots}+2^2\right)+\left\{\left(\frac{1}{n}\right)^2+\left(\frac{2}{n}\right)^2\right\}+2\cdot 2\cdot\left\{\frac{1}{n}+\frac{2}{n}+\frac{3}{n}+\ldots+\frac{(n-1)}{n}\right\}\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{1^2+2^2+3^2\ldots+(n-1)^2\right\}+\frac{4}{n}\left\{1+2+\ldots+(n-1)\right\}\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[4n+\frac{1}{n^2}\left\{\frac{n(n-1)(2n-1)}{6}\right\}+\frac{4}{n}\left\{\frac{n(n-1)}{2}\right\}\right]$$

$$=\lim_{x\to\infty}\frac{1}{n}\left[4n+\frac{n\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}+\frac{4n-4}{2}\right]$$

$$=\lim_{x\to\infty}\left[4+\frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)+2-\frac{2}{n}\right]$$

$$=4+\frac{2}{6}+2$$

$$=\frac{19}{3}$$

4. $\int_1^4\left(x^2-x\right)dx$

Ans. Let $I=\int_1^4\left(x^2-x\right)dx$

$$=\int_1^4 x^2dx-\int_1^4 xdx$$

Let $I=I_1-I_2$, where $I_1=\int_1^4 x^2dx$ and $I_2=\int_1^4 xdx$ ...(1)

It is know that,

$$\int_a^b f(x)dx=(b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+f(a+(n-1)h)\right], \text{where } h=\frac{b-a}{n}$$

For $I_1 = \int_1^4 x^2 dx$,

$a = 1, b = 4, \text{and } f(x) = x^2$

$\therefore h = \frac{4-1}{n} = \frac{3}{n}$

$$I_1 = \int_1^4 x^2 dx = (4-1)\lim_{x\to\infty}\frac{1}{n}\left[f(1) + f(1+h) + \ldots + f(1+(n-1)h)\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[1^2 + \left(1+\frac{3}{n}\right)^2 + \left(1+2\cdot\frac{3}{n}\right)^2 + \ldots\left(1+\frac{(n-1)3}{n}\right)^2\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[1^2 + \left\{1^2 + \left(\frac{3}{n}\right)^2\right\} + \ldots + \left\{1^2 + \left(\frac{(n-1)3}{n}\right)^2 + \frac{2\cdot(n-1)\cdot 3}{n}\right\}\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[\left(\underbrace{1^2 + \ldots + 1^2}_{\text{n times}}\right) + \left(\frac{3}{n}\right)^2\left\{1^2 + 2^2 + \ldots + (n-1)^2\right\} + 2\cdot\frac{3}{n}\left\{1 + 2 + \ldots + (n-1)\right\}\right]$$

$$= = 3\lim_{x\to\infty}\frac{1}{n}\left[n + \frac{9}{n^2}\left\{\frac{(n-1)(n)(2n-1)}{6}\right\} + \frac{6}{n}\left\{\frac{(n-1)(n)}{2}\right\}\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[n + \frac{9n}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) + \frac{6n-6}{2}\right]$$

$$= 3\lim_{x\to\infty}\left[1 + \frac{9}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right) + 3 - \frac{3}{n}\right]$$

$$= 3\lim_{x\to\infty}\left[1+3+3\right]$$

$= 3[7]$ ....(2)

$I_1 = 21$

For $I_2 = \int_1^4 x dx$,

$a = 1, b = 4, \text{and } f(x) = x$

$\Rightarrow h = \frac{4-1}{n} = \frac{3}{n}$

$$\therefore I_2 = (4-1)\lim_{x\to\infty}\frac{1}{n}\Big[f(1)+f(1+h)+\ldots f\big(a+(n-1)h\big)\Big]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\Big[1+(1+h)+\ldots+\big(1+(n-1)h\big)\Big]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[1+\left(1+\frac{3}{n}\right)+\ldots+\left\{1+(n-1)\frac{3}{n}\right\}\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[\left(\underset{\text{n time}}{1+1+\ldots+1}\right)+\frac{3}{n}\big(1+2+\ldots+(n-1)\big)\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[n+\frac{3}{n}\left\{\frac{(n-1)n}{2}\right\}\right]$$

$$= 3\lim_{x\to\infty}\frac{1}{n}\left[1+\frac{3}{2}\left(1-\frac{1}{n}\right)\right]$$

$$= 3\left[1+\frac{3}{2}\right]$$

$$= 3\left[\frac{5}{2}\right]$$

$$I_2 = \frac{15}{2} \qquad \text{.....(3)}$$

From equation (2) and (3), we obtain

$$I = I_1 + I_2 = 21 - \frac{15}{2} = \frac{27}{2}$$

5. $\int_1^1 e^x dx$

Ans. Let $I = \int_1^1 e^x dx$ ....(1)

It is know that

$$\int_a^b f(x)dx = (b-a)\lim_{x\to\infty}\frac{1}{n}\Big[f(a)+f(a+h)\ldots f(a+(n-1)h\Big], \text{where } h = \frac{b-a}{n}$$

$$\text{Here}, a=-1, b=1, \text{and } f(x)=e^x$$

$$\therefore h=\frac{1+1}{n}=\frac{2}{n}$$

$$\therefore=(1+1)\lim_{x\to\infty}\frac{1}{n}\left[f(-1)+f\left(-1+\frac{2}{n}\right)+f\left(-1+2.\frac{2}{n}\right)+\ldots+f\left(-1+\frac{(n-1)2}{n}\right)\right]$$

$$=2\lim_{x\to\infty}\frac{1}{n}\left[e^{-1}+e^{\left(-1+\frac{2}{n}\right)}+e^{\left(-1+2\frac{2}{n}\right)}+\ldots e^{\left(-1+(n-1)\frac{2}{n}\right)}\right]$$

$$=2\lim_{x\to\infty}\frac{1}{n}\left[e^{-1}\left\{1+e^{\frac{2}{n}}+e^{\frac{4}{n}}e+e^{\frac{6}{n}}+e^{(n-1)\frac{2}{n}}\right\}\right]$$

$$=2\lim_{x\to\infty}\frac{1}{n}\frac{e^{-1}}{n}\left[\frac{e^{\frac{2n}{n}-1}}{e^{\frac{2}{n}-1}}\right]$$

$$=e^{-1}\times 2\lim_{x\to\infty}\frac{1}{n}\left[\frac{e^2-1}{e^{\frac{2}{n}-1}}\right]$$

$$=\frac{e^{-1}\times 2\left(e^2-1\right)}{\lim_{x\to\infty}\left(\frac{e^{\frac{2}{n}}-1}{\frac{2}{n}}\right)\times 2}$$

$$=e^{-1}\left[\frac{2\left(e^2-1\right)}{2}\right] \qquad \left[\lim_{x\to\infty}\left(\frac{e^h-1}{h}\right)=1\right]$$

$$=\frac{e^2-1}{e}$$

$$=\left(e-\frac{1}{e}\right)$$

6. $\int_0^4\left(x+e^{2x}\right)dx$

Ans. It is know that,

$$\int_0^4 (x)dx = (b-a)\lim_{x\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+\ldots+f\left(a+(n-1)h\right)\right],\text{where } h=\frac{b-a}{n}$$

Here , a=0, b=4, and $f(x) = x + e^{2x}$

$$\therefore h = \frac{4-0}{n} = \frac{4}{n}$$

$$\Rightarrow \int_0^4 \left(x+e^{2x}\right)dx = (4-0)\lim_{x\to\infty}\frac{1}{n}\left[f(0)+f(h)+f(2h)+\ldots+f\left((n-1)h\right)\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[\left(0+e^{0}\right)+\left(h+e^{2h}\right)+\left(2h+e^{2.2h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[1+\left(h+e^{2h}\right)+\left(2h+e^{4h}\right)+\ldots+\left\{(n-1)h+e^{2(n-1)h}\right\}\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[\left\{h+2h+3h+\ldots+(n-1)h\right\}+\left(1+e^{2h}+e^{4h}+\ldots+e^{2(n-1)h}\right)\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[h\left\{1+2+\ldots(n-1)\right\}+\left(\frac{e^{2hn}-1}{e^{2hn}-1}\right)\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[\frac{\left(h(n-1)n\right)}{2}+\left(\frac{e^{2hn}-1}{e^{2hn}-1}\right)\right]$$

$$= 4\lim_{x\to\infty}\frac{1}{n}\left[\frac{4}{n}\cdot\frac{(n-1)n}{2}+\left(\frac{e^{8}-1}{e^{\frac{8}{n}}-1}\right)\right]$$

$$= 4(2) + 4\lim_{x\to\infty}\frac{\left(e^{8}-1\right)}{\left(\frac{e^{\frac{8}{n}}-1}{\frac{8}{n}}\right)8}$$

$$= 8 + \frac{4\cdot\left(e^{8}-1\right)}{8} \qquad \left(\lim_{x\to\infty}\frac{e^{x}-1}{x}=1\right)$$

$$= 8 + \frac{e^8 - 1}{2}$$

$$= \frac{15 + e^8}{2}$$