SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Integrals
Exercise 7.9

1. $\int_1^1 (x+1)dx$

Ans. $\text{Let} I = \int_1^1 (x+1)dx$

$$\int (x+1)dx = \frac{x^2}{2} + x = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(1) - F(-1)$

$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)$

$= \frac{1}{2} + 1 - \frac{1}{2} + 1$

$= 2$

2. $\int_2^3 \frac{1}{x}dx$

Ans. $\text{Let } I = \int_2^3 \frac{1}{x}dx$

$$\int \frac{1}{x}dx = \log|x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(3) - F(2)$

$= \log|3| - \log|2| = \log\frac{3}{2}.$

3. $\int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)dx$

Ans. $\text{Let} I = \int_1^2 \left(4x^3 - 5x^2 + 6x + 9\right)dx$

$$\int \left(4x^3 - 5x^2 + 6x + 9\right)dx = 4\left(\frac{x^4}{4}\right) - 5\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + 9(x)$$

$$= x^4 - \frac{5x^3}{3} + 3x^2 + 9x = F(x)$$

By second fundamental theorem of calculus, we obtain

$I = F(2) - F(1)$

$I = \left\{ 2^4 - \frac{5.(2)^3}{3} + 3(2)^2 + 9(2) \right\} - \left\{ (1)^4 - \frac{5(1)^3}{3} + 3(1)^2 + 9(1) \right\}$

$= \left(16 - \frac{40}{3} + 12 + 18 \right) - \left(1 - \frac{5}{3} + 3 + 9 \right)$

$= 16 - \frac{40}{3} + 12 + 18 - 1 + \frac{5}{3} - 3 - 9$

$= 33 - \frac{35}{3}$

$= \frac{99 - 35}{3}$

$= \frac{64}{3}$

4. $\int_0^{\frac{x}{4}} \sin 2x dx$

Ans. $\text{Let} I = \int_0^{\frac{x}{4}} \sin 2x dx$

$\int \sin 2x dx = \left(\frac{-\cos 2x}{2} \right) = F(x)$

By second fundamental theorem of calculus, we obtain

$I = F\left(\frac{\pi}{4}\right) - F(0)$

$= -\frac{1}{2}\left[\cos 2\left(\frac{\pi}{4}\right) - \cos 0 \right]$

$= -\frac{1}{2}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0 \right]$

$= -\frac{1}{2}[0 - 1]$

$= \frac{1}{2}$

5. $\int_0^{\frac{x}{2}} \cos 2x dx$

Ans. $\text{Let} I = \int_0^{\frac{x}{2}} \cos 2x dx$

$$\int \cos 2x dx = \left(\frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{2}\right) - F(0)$$

$$= \frac{1}{2}\left[\sin 2\left(\frac{\pi}{2}\right) - \sin 0\right]$$

$$= \frac{1}{2}\left[\sin \pi - \sin 0\right]$$

$$= \frac{1}{2}\left[0 - 0\right] = 0$$

6. $\int_4^5 e^x dx$

Ans. $\text{Let} I = \int_4^5 e^x dx$

$$\int e^x dx = e^x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(5) - F(4)$$
$$= e^5 - e^4$$
$$= e^4(e-1)$$

7. $\int_0^{\frac{x}{4}} \tan x dx$

Ans. $\text{Let} I = \int_0^{\frac{x}{4}} \tan x dx$

$$\int \tan x dx = -\log\left|\cos x\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F(0)$$

$$= -\log\left|\cos\frac{\pi}{4}\right| + \log\left|\cos 0\right|$$

$$= -\log\left|\frac{1}{\sqrt{2}}\right| + \log|1|$$

$$= -\log(2)^{-\frac{1}{2}}$$

$$= \frac{1}{2}\log 2$$

8. $\int_{\frac{\pi}{6}}^{\frac{x}{4}} \operatorname{cosec} x dx$

Ans. $\text{Let} I = \int_{\frac{x}{6}}^{\frac{\pi}{4}} \operatorname{cosec} x dx$

$$\int \operatorname{cosec} x dx = \log|\operatorname{cosec} x - \cot x| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F\left(\frac{\pi}{4}\right) - F\left(\frac{\pi}{6}\right)$$

$$= \log\left|\operatorname{cosec}\frac{\pi}{4} - \cot\frac{\pi}{4}\right| - \log\left|\operatorname{cosec}\frac{\pi}{6} - \cot\frac{\pi}{6}\right|$$

$$= \log\left|\sqrt{2} - 1\right| - \log\left|2 - \sqrt{3}\right|$$

$$= \log\left(\frac{\sqrt{2} - 1}{2 - \sqrt{3}}\right)$$

9. $\int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

Ans. $\text{Let} I = \int_0^1 \frac{dx}{\sqrt{1 - x^2}}$

$$\int \frac{dx}{\sqrt{1 - x^2}} = \sin^{-1} x = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$
$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2}$$

10. $\int_0^1 \frac{dx}{1+x^2}$

Ans. $\text{Let} I = \int_0^1 \frac{dx}{1+x^2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \frac{\pi}{4}$$

11. $\int_2^3 \frac{dx}{x^2-1}$

Ans. $\text{Let} I = \int_2^3 \frac{dx}{x^2-1}$

$$\int \frac{dx}{x^2-1} = \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2}\left[\log\left|\frac{3-1}{3+1}\right| - \log\left|\frac{2-1}{2+1}\right|\right]$$

$$= \frac{1}{2}\left[\log\left|\frac{2}{4}\right| - \log\left|\frac{1}{3}\right|\right]$$

$$= \frac{1}{2}\left[\log\frac{1}{2} - \log\frac{1}{3}\right]$$

$$= \frac{1}{2}\left[\log\frac{3}{2}\right]$$

12. $\int_0^{\frac{x}{2}} \cos^2 x\,dx$

Ans. $\text{Let} I = \int_0^{\frac{x}{2}} \cos^2 x dx$

$$\int \cos^2 x dx = \int \left(\frac{1+\cos 2x}{2}\right) dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = \left[F\left(\frac{\pi}{2}\right) - F(0)\right]$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin \pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$$

$$= \frac{\pi}{4}$$

13. $\int_2^3 \frac{x dx}{x^2+1}$

Ans. $\text{Let} I = \int_2^3 \frac{x}{x^2+1} dx$

$$\int \frac{x}{x^2+1} dx = \frac{1}{2} \int \frac{2x}{x^2+1} dx = \frac{1}{2} \log\left(1+x^2\right) = F(x)$$

By second fundamental theorem of calculus, we obtain

$$I = F(3) - F(2)$$

$$= \frac{1}{2}\left[\log\left(1+(3)^2\right) - \log\left(1+(2)^2\right)\right]$$

$$= \frac{1}{2}\left[\log(10) - \log(5)\right]$$

$$= \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$$

14. $\int_0^1 \frac{2x+3}{5x^2+1} dx$

Ans. $\text{Let } I = \int_0^1 \frac{2x+3}{5x^2+1} dx$

$$\int_0^1 \frac{2x+3}{5x^2+1}dx = \frac{1}{5}\int \frac{5(2x+3)}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x+15}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5x^2+1}dx$$

$$= \frac{1}{5}\int \frac{10x}{5x^2+1}dx + 3\int \frac{1}{5\left(x^2+\frac{1}{5}\right)}dx$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{5}\cdot\frac{1}{\frac{1}{\sqrt{5}}}\tan^{-1}\frac{x}{\frac{1}{\sqrt{5}}}$$

$$= \frac{1}{5}\log\left(5x^2+1\right) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}x\right)$$

$$= F(X)$$

BY SECOND FUNDAMENTAL THOREM OF CALCULUS ,WE OBTAIN

$$I = F(1) - F(0)$$

$$= \left\{\frac{1}{5}\log(5+1) + \frac{3}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\right)\right\} - \left\{\frac{1}{5}\log(1) + \frac{3}{\sqrt{5}}\tan^{-1}(0)\right\}$$

$$= \frac{1}{5}\log 6 + \frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$$

15. $\int_0^4 xe^{x^2}dx$

Ans. Let $I = \int_0^4 xe^{x^2}dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

As $x \to 0, t \to 0$ and as $x \to 1, t \to 1,$

$$\therefore I = \frac{1}{2}\int_0^1 e^t dt$$

$$\frac{1}{2}\int e^t dt = \frac{1}{2}e^t = F(t)$$

By second fundamental theorem of calculus, we obtain

$$I = F(1) - F(0)$$

$$=\frac{1}{2}e-\frac{1}{2}e^0$$

$$=\frac{1}{2}(e-1)$$

16. $\int_0^1 \frac{5x^2}{x^2+4x+3}$

Ans. Let $I=\int_1^2 \frac{5x^2}{x^2+4x+3}dx$

Dividing $5x^2$by $x^2+4x+3,$ we obtain

$$I=\int_1^2\left\{5-\frac{20x+15}{x^2+4x+3}\right\}dx$$

$$=\int_1^2 5dx-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$$=[5x]_1^2-\int_1^2\frac{20x+15}{x^2+4x+3}dx$$

$$=I=5-I_1, \text{where } I=\int_1^2\frac{20x+15}{x^2+4x+3}dx \qquad ....(1)$$

Consider I, $=\int_1^2\frac{20x+15}{x^2+4x+8}dx$

Let $20x+15=a\frac{d}{dx}(x^2+4x+3)+B$

$$=2Ax+(4A+B)$$

Equating the coefficients of x and constant term, we obtain

A = 10 and B = -25

$$=10\int_1^2\frac{2x+4}{x^2+4x+3}dx-25\int_1^2\frac{dx}{x^2+4x+3}$$

Let $x^2+4x+3=t$

$$\Rightarrow(2x+4)dx=dt$$

$$\Rightarrow I_1=10\int\frac{dt}{t}-25\int\frac{dx}{(x+2)^2-1^2}$$

$$=10\log t-25\left[\frac{1}{2}\log\left(\frac{x+2-1}{x+2+1}\right)\right]$$

$$=\left[10\log\left(x^2+4x+3\right)\right]_1^2-25\left[\frac{1}{2}\log\left(\frac{x+1}{x+3}\right)\right]_1^2$$

$$=\left[10\log 15-10\log 8\right]-25\left[\frac{1}{2}\log\frac{3}{5}-\frac{1}{2}\log\frac{2}{4}\right]$$

$$=\left[10\log(5\times 3)-10\log(4\times 2)\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]$$

$$=\left[10\log 5+10\log 3-10\log 4-10\log 2\right]-\frac{25}{2}\left[\log 3-\log 5-\log 2+\log 4\right]$$

$$=\left[10+\frac{25}{2}\right]\log 5+\left[-10-\frac{25}{2}\right]\log 4+\left[10-\frac{25}{2}\right]\log 3+\left[-10+\frac{25}{2}\right]\log 2$$

$$=\frac{45}{2}\log 5-\frac{45}{2}\log 4-\frac{5}{2}\log 3+\frac{5}{2}\log 2$$

$$=\frac{45}{2}\log\frac{5}{4}-\frac{5}{2}\log\frac{3}{2}$$

Substituting the value of I_1 in (1) we obtain

$$I=5-\left[\frac{45}{2}\log\frac{5}{4}\log\frac{3}{2}\right]$$

$$=5-\frac{5}{2}\left[9\log\frac{5}{4}-\log\frac{3}{2}\right]$$

17. $\int_0^{\frac{x}{4}}\left(2\sec^2 x+x^3+2\right)dx$

Ans. Let $I=\int_0^{\frac{x}{4}}\left(2\sec^2 x+x^3+2\right)dx$

$$\int\left(2\sec^2 x+x^3+2\right)dx=2\tan x+\frac{x^4}{4}+2x=F(X)$$

By second fundamental theorem of calculus, we obtain

$$I=F\left(\frac{\pi}{4}\right)-F(0)$$

$$=\left\{\left(2\tan\frac{\pi}{4}+\frac{1}{4}\left(\frac{\pi}{4}\right)^4+2\left(\frac{\pi}{4}\right)\right)-\left(2\tan 0+0+0\right)\right\}$$

$$= 2\tan\frac{\pi}{4} + \frac{\pi^4}{4^5} + \frac{\pi}{2}$$

$$= 2 + \frac{\pi}{2} + \frac{\pi^4}{1024}$$

18. $\int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

Ans. Let $I = \int_0^{\pi}\left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right)dx$

$$= -\int_0^{\pi}\left(\cos^2\frac{x}{2} - \sin^2\frac{x}{2}\right)dx$$

$$= -\int_0^{\pi}\cos x\,dx$$

$$= -\int_0^{\pi}\cos x\,dx = \sin x\ F(x)$$

By second fundamental theorem of calculus , we obtain

$$I = F(\pi) - F(0)$$

$$= \sin\pi - \sin 0$$

$$= 0$$

19. $\int_0^2 \frac{6x+3}{x^2+4}dx$

Ans. Let $I = \int_0^2 \frac{6x+3}{x^2+4}dx$

$$\int\frac{6x+3}{x^2+4}dx = 3\int\frac{2x+1}{x^2+4}dx$$

$$= 3\int\frac{2x}{x^2+4}dx + 3\int\frac{1}{x^2+4}dx$$

$$= 3\log\left(x^2+4\right) + \frac{3}{2}\tan^{-1}\frac{x}{2} = F(x)$$

By second fundamental theorem of calculus , we obtain

$$I = F(2) - F(0)$$

$$= \left\{3\log\left(2^2+4\right) + \frac{3}{2}\tan^{-1}\left(\frac{2}{2}\right)\right\} - \left\{3\log(0+4) + \frac{3}{2}\tan^{-1}\left(\frac{0}{2}\right)\right\}$$

$$= 3\log 8 + \frac{3}{2}\tan^{-1}1 - 3\log 4 - \frac{3}{2}\tan^{-1}0$$

$$= 3\log 8 + \frac{3}{2}\left(\frac{\pi}{4}\right) - 3\log 4 - 0$$

$$= 3\log\left(\frac{8}{4}\right) + \frac{3\pi}{8}$$

$$= 3\log 2 + \frac{3\pi}{8}$$

20. $\int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

Ans. Let $I = \int_0^1 \left(xe^x + \sin\frac{\pi x}{4}\right)dx$

$$\int\left(xe^x + \sin\frac{\pi x}{4}\right)dx = x\int e^x dx - \int\left\{\left(\frac{d}{dx}x\right)\int e^x dx\right\}dx + \left\{\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right\}$$

$$= xe^x - \int e^x dx - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= xe^x - e^x - \frac{4\pi}{\pi}\cos\frac{x}{4}$$

$$= F(x)$$

By second fundamental theorem of calculus , we obtain

$$I = F(1) - F(0)$$

$$= \left(1.e^1 - e^1 - \frac{4}{\pi}\cos\frac{\pi}{4}\right) - \left(0.e^0 - e^0 - \frac{4}{\pi}\cos 0\right)$$

$$= e - e - \frac{4}{\pi}\left(\frac{1}{\sqrt{2}}\right) + 1 + \frac{4}{\pi}$$

$$= 1 + \frac{4}{\pi} - \frac{2\sqrt{2}}{\pi}$$

21. $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$ equals

(A) $\frac{\pi}{3}$ (B) $\frac{2\pi}{3}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{12}$

Ans. $\int \frac{dx}{1+x^2} = \tan^{-1} x = F(x)$

By second fundamental theorem of calculus , we obtain

$$\int_1^{\sqrt{3}} \frac{dx}{1+x^2} = F\left(\sqrt{3}\right) - F(1)$$

$$= \tan^{-1}\sqrt{3} - \tan^{-1}1$$

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \frac{\pi}{12}$$

Hence the correct answer is D.

22. $\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2}$ equals

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{24}$ (D) $\frac{\pi}{4}$

Ans. $\int \frac{dx}{4+9x^2} = \int \frac{dx}{(2)^2 + (3x)^2}$

Put $3x = t \Rightarrow 3dx = dt$

$$\therefore \int \frac{dx}{(2)^2 + (3x)^2} = \frac{1}{3}\int \frac{dt}{(2)^2 + t^2}$$

$$= \frac{1}{3}\left[\frac{1}{2}\tan^{-1}\frac{t}{2}\right]$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x}{2}\right)$$

$$= F(x)$$

By second fundamental of theorem we obtain

$$\int_0^{\frac{2}{3}} \frac{dx}{4+9x^2} = F\left(\frac{2}{3}\right) - F(0)$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3}{2}\cdot\frac{2}{3}\right) - \frac{1}{6}\tan^{-1}0$$

$$= \frac{1}{6}\tan^{-1}1 - 0$$

$$= \frac{1}{6}\times\frac{\pi}{4}$$

$$= \frac{\pi}{24}$$

Hence, the correct answer is C