SpeedLabs

MATHS

CBSE 9th

TEEVRA EDUTECH PVT. LTD.

INTRODUCTION TO EUCLIDS GEOMETRY

Exercise- 5.1

Q 1. Which of the following statements are true and which are false? Give reasons for your answers.

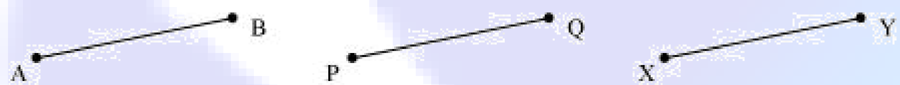
(i) Only one line can pass through a single point.

(ii) There are infinite number of lines which pass through two distinct points.

(iii) A terminated line can be produced indefinitely on both the sides.

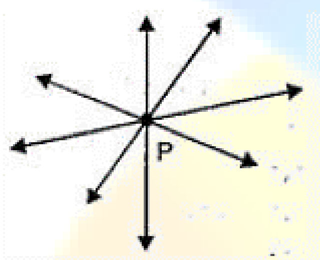
(iv) If two circles are equal, then their radii are equal.

(v) In the following figure, if AB = PQ and PQ = XY, then AB = XY.

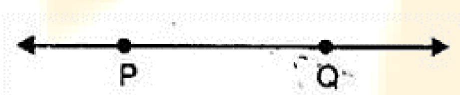


Ans - **(i)** False

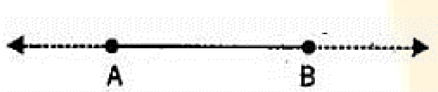
Correct statement: Infinite many lines can pass through a single point. This is self-evident and can be seen visually by the student given below:



(ii) False because the given statement contradicts the postulate I of the Euclid that assures that there is a unique line that passes through two distinct points. Through two points P and Q a unique line can be drawn.



(iii) True : Reason: We need to consider Euclid's Postulate 2: "A terminated line can be produced indefinitely."



(iv) True. If two circles are equal, then their centre and circumference will coincide and hence, the radii will also be equal.

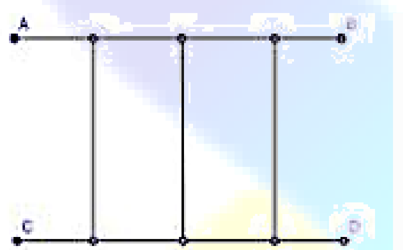
(v) True. It is given that AB and XY are two terminated lines and both are equal to a third line PQ. Euclid's first axiom states that things which are equal to the same thing are equal to one another. Therefore, the lines AB and XY will be equal to each other.

Q 2. Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(i) parallel lines **(ii)** perpendicular lines **(iii)**Line segment **(iv)** radius of a circle **(v)** square

Ans - **(i)** Parallel lines

Two lines are said to be parallel, when the perpendicular distance between these lines is always constant or we can say that the lines that never intersect each other are called as parallel lines. We need to define line first, in order to define parallel lines.

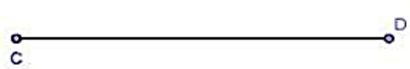


(ii) Perpendicular lines

Two lines are said to be perpendicular lines, when angle between these two lines is 90° We need to define line and angle, in order to define perpendicular lines.

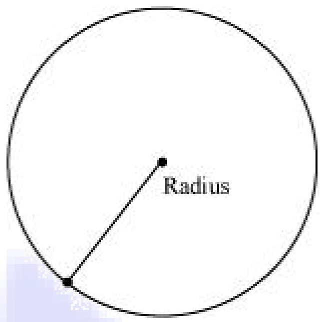
(iii) Line segment

A line of a fixed dimension between two given points is called as a line segment. We need to define line and point, in order to define a line segment.



(iv) Radius of circle

It is the distance between the centres of a circle to any point lying on the circle. To define the radius of a circle, we must know about point and circle.



(v) A square is a quadrilateral having all sides of equal length and all angles of same measure, i.e.,

To define square, we must know about quadrilateral, side, and angle.

Q 3. Consider the two 'postulates' given below:

(i) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(ii) There exists at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent? Do they follow from Euclid's postulates? Explain.

Ans - We are given with following two postulates

(i) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(ii) There exists at least three points that are not on the same line.

The undefined terms in the given postulates are point and line.

The two given postulates are consistent, as they do not refer to similar situations and they refer to two different situations.

We can also conclude that, it is impossible to derive at any conclusion or any statement that contradicts any well-known axiom and postulate.

The two given postulates do not follow from the postulates given by Euclid.

The two given postulates can be observed following from the axiom, "Given two distinct points, there is a unique line that passes through them".

Q 4. If a point C lies between two points A and B such that AC = BC, then prove that $AC = \frac{1}{2}AB$. Explain by drawing the figure.

Ans - We are given that a point C lies between two points B and C, such that AC = BC.

We need to prove that $AC = \frac{1}{2}AB$

Let us consider the given below figure.

We are given that AC = BC….

(i) An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

(ii) Let us add AC to both sides of equation (i).

AC + AC = BC + AC. An axiom of the Euclid says that "Things which coincide with one another are equal to one another." We can conclude that BC + AC coincide with AB, or

AB = BC + AC….(ii)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

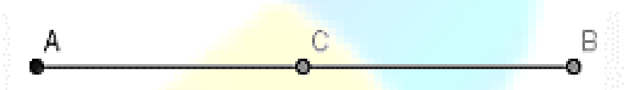
From equations (i) and (ii),

we can conclude that

AC + AC = AB, or2AC = AB.

An axiom of the Euclid says that "Things which are halves of the same things are equal to one another."

Therefore, we can conclude that.



Q 5. In the above question, point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Ans- Let there be two mid-points, C and D.

C is the mid-point of AB.

AC = CB

AC +AC = BC + AC (Equals are added on both sides) … (1)

Here, (BC + AC) coincides with AB. It is known that things which coincide with one another are equal to one another.

∴ BC + AC = AB … (2)

It is also known that things which are equal to the same thing are equal to one another. Therefore, from equations (1) and (2), we obtain

AC + AC = AB

⇒ 2AC = AB … (3)

Similarly, by taking D as the mid-point of AB, it can be proved that

2AD = AB … (4)

From equation (3) and (4), we obtain

2AC = 2AD (Things which are equal to the same thing are equal to one another.)

⇒ AC = AD (Things which are double of the same things are equal to one another.)

This is possible only when point C and D are representing a single point.

Hence, our assumption is wrong and there can be only one mid-point of a given line segment.

Q 6. In the following figure, if AC = BD, then prove that AB = CD.

Ans - We are given that AC = BD.

We need to prove that AB = CD in the figure given below. From the figure, we can conclude that

AC = AB + BC, and

BD = CD + BC.

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

AB + BC = CD + BC. (i)

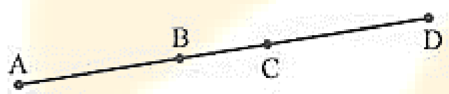
An axiom of the Euclid says that "when equals are subtracted from equals, the remainders are also equal."

We need to subtract BC from equation (i), to get

AB + BC – BC = CD + BC – BC

AB = CD.

Therefore, we can conclude that the desired result is proved.



Q 7. Why is axiom 5, in the list of Euclid's axioms, considered as a 'universal truth'? (Note that the question if not about fifth postulate)

Ans - We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.