

1 marks

1. Find the value of $\cos \left\{ \cos^{-1} \left(\frac{-\sqrt{3}}{2} \right) + \frac{\pi}{6} \right\}$
2. Find the value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$
3. Solve for x: $\sin \left[\sin^{-1} \frac{1}{5} + \cos^{-1} x \right] = 1$
4. Write the simplest form: $\tan^{-1} \left(\frac{\cos x}{1 - \sin x} \right)$
5. Considering the principle solutions. Find the number of solution of $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$
6. Find the principle value of $\sin^{-1} \left(\frac{-\sqrt{3}}{2} \right) + \cos^{-1} \left(\frac{-\sqrt{3}}{2} \right)$
7. Find the value of x if $\operatorname{cosec}^{-1} x + 2 \cot^{-1} 7 + \cos^{-1} \left(\frac{3}{4} \right)$
8. If $\cos^{-1} = \tan^{-1} x$. Show that $\sin(\cos^{-1} x) = x^2$
9. If $x > 0$ and $\sin^{-1} \left(\frac{5}{x} \right) + \sin^{-1} \left(\frac{12}{x} \right) = \frac{\pi}{2}$. Then find the value of x.
10. Prove that $\cos \left\{ 2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right\} + x = 0$

4 marks /6 marks

11. Prove that $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} = \frac{\pi}{4}$
12. if $x, y, z \in [-1, 1]$ such that $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$. Find the value of $x^{2006} + y^{2007} + z^{2008} - \frac{9}{x^{2006} + y^{2007} + z^{2008}}$
13. $\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$. Prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$
14. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$. prove that $x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz$
15. Prove that: $\sin 2[\cot^{-1}\{\cos(\tan^{-1} x)\}] = \sqrt{\frac{x^2+1}{x^2+1}}$
16. In any triangle ABC. If $A = \tan^{-1} 2$ and $B = \tan^{-1} 3$. Prove that $c = \frac{\pi}{4}$
17. If $x = \operatorname{cosec}[\tan^{-1}\{\cos(\cot^{-1}(\sec(\sin^{-1} a)))\}]$ and $y = \sec[\cot^{-1}(\operatorname{cosec}(\cos^{-1} a))]$ where $a \in [0, 1]$
Find the relationship between x and y in terms of a
18. Prove that: $\cot^{-1} \left[\frac{ab+1}{a-b} \right] + \cot^{-1} \left[\frac{bc+1}{b-c} \right] + \cot^{-1} \left[\frac{ca+1}{c-a} \right] = 0$

19. Solve for x: $\sin^{-1} \frac{2\alpha}{1+\alpha^2} + \sin^{-1} \frac{2\beta}{1+\beta^2} = 2 \tan^{-1} x$

20. Prove: $\cos^{-1} x - \cos^{-1} y = \cos^{-1} [xy + \sqrt{1-x^2} \cdot \sqrt{1-y^2}]$

21. If $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \pi$. Prove that $a + b + c = abc$

22. Prove that $\tan^{-1} \frac{yz}{xr} + \tan^{-1} \frac{zx}{yr} + \tan^{-1} \frac{xy}{zr} = \frac{\pi}{2}$ where $x^2 + y^2 + z^2 = r^2$

23. Solve for x: $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} 3$

24. Solve : $\sin[2 \cos^{-1}\{\cot(2 \tan^{-1} x)\}] = 0$

25. If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta$. prove that $9x^2 - 12xycos\theta + 4y^2 = 36 \sin^2 \theta$

Answer

1. -1

2. π

3. $\frac{1}{5}$

4. $\frac{\pi}{4} + \frac{x}{2}$

5. 2

6. $\frac{\pi}{2}$

7. $x = \text{cosec}^{-1} \frac{125}{117}$

8. $x = 13$

12. Zero: $x = 1, y = 1, z = 1$

17. $x^2 = y^2 = 3 - a^2$

19. $x = \frac{\alpha+\beta}{1-\alpha\beta}$

23. $x = -1$

24. $\pm 1, -1 \pm \sqrt{2}, 1 \pm \sqrt{2}$