SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

Inverse Trigonometric Functions
Exercise - 2.1

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Ans. Let $\sin^{-1}\left(-\frac{1}{2}\right)=y.$ then $\sin y\ -\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right)=\frac{1}{2}.$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\left(\frac{\pi}{6}\right).$

2. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Ans. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y.$ Then, $\cos y=\ y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\ \frac{\pi}{6}=\frac{\sqrt{3}}{2}$

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

3. Find the principal value of $\text{cosec}^{-1}\ (2)$

Ans. Let $\text{cosec}-1\ (2)=y.$ Then, $\text{cosec}\ y=2=\text{cosec}\left(\frac{\pi}{6}\right).$

We know that the range of the principal value branch of cosec^{-1} is $\left[\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}.$

Therefore, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$

4. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Ans. Let $\tan^{-1}\left(-\sqrt{3}\right)=y.$ Then, $\tan y=-\ \sqrt{3}=-\tan\ \frac{\pi}{3}=\tan\ \left(-\frac{\pi}{3}\right).$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right)$ is $\sqrt{3}$.

Therefore, the principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{3}$.

5. Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Ans. Let $\cos^{-1}\left(-\frac{1}{2}\right)=y$ Then, $\cos y=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is

$[0,\pi]$ and $\cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

6. Find the principal value of $\tan^{-1}(-1)$

Ans. Let $\tan^{-1}(-1)=y$. Then, $\tan y=-1=-\tan\left(\frac{\pi}{4}\right)=\tan\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of $\tan^{-1}$ is

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right)=-1$.

Therefore, the principal value of $\tan^{-1}(-1)$ is - $\frac{\pi}{4}$.

7. Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Ans. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=y$. Then, see $y=\frac{2}{\sqrt{3}}=\sec\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\sec^{-1}$ is

$[0,\pi]-\left\{\frac{x}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

8. Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Ans. Let $\cot^{-1}\left(\sqrt{3}\right) = y$. Then, $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is $(0,\pi)$ and $\cos = \left(\frac{\pi}{6}\right) = \sqrt{3}$.

Therefore, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$

9. Find the principal value of $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

Ans. Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$ Then, $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0,\pi]$ and

$\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$

10. Find the principal value of $\text{coses}^{-1}\left(\sqrt{2}\right)$

Ans. Let $\text{cosec}^{-1}\left(-\sqrt{2}\right) = y$. then, $\text{cosec}\, y = -\sqrt{2} = -\text{cosec}\left(\frac{\pi}{4}\right) = \text{cosec}\left(-\frac{\pi}{4}\right)$.

We know that the range of the principal value branch of cosec^{-1} is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right] - \{0\}$ and $\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$.

Therefore, the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $\frac{\pi}{4}$.

11. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

Ans. Let $\tan^{-1}(1) = x$. Then, $\text{tenx} = 1 = \tan\frac{\pi}{4}$.

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let$^{-1}\frac{1}{2} = y$. Then, $\cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=\frac{2\pi}{3}$

$\text{Let}\sin^{-1}\left(-\frac{1}{2}\right)=z.\text{Then},\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(6\frac{\pi}{6}\right).$

$\text{Let}\sin^{-1}\left(-\frac{1}{2}\right)=z.\text{ Then, }\sin z=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)=\sin\left(6\frac{\pi}{6}\right).$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

$\frac{\pi}{4}+\frac{2\pi}{3}+\frac{\pi}{3}$

$\frac{3\pi+8\pi-2}{12}=\frac{2\pi}{3}=\frac{3\pi}{4}$

12. Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Ans. $\text{Let}\cos^{-1}\left(\frac{1}{2}\right)=x.\text{ Then, }\cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right).$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

$\text{Let}\sin^{-1}\left(\frac{1}{2}\right)=y.\text{ Then, }\sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right).$

$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}+\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

13. Find the value of if $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ **(B)** $\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$ **(D)** $\frac{\pi}{2} < y < \frac{\pi}{2}$

Ans. It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

14. Find the value of $\tan^{-1} \sqrt{3} - \sec^{-1}(-2)$ is equal to

(A) n (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{3}$

And. Let $\tan^{-1} \sqrt{3} = x$ Then, $\tan x = \sqrt{3} = \tan \frac{\pi}{3}$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1} \sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$. Then, $\sec y = -2 = -\sec\left(\frac{\pi}{3}\right) = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left\{\frac{\pi}{2}\right\}$

$\therefore \sec -1\left(\sqrt{3}\right) - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$