SpeedLabs

MATHS

CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. $3\sin^{-1}x = \sin^{-1}(3x - 4x3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ prove.

Ans. To prove: $3\sin^{-1}x = \sin^{-1}(3x - 4x^3), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$ prove.

Let $x = \sin\theta$. Then, $\sin^{-1}x = \theta$.

We have,

R.H.S. $= \sin^{-1}(3x - 4x^3) = \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta$

$= 3\sin^{-1}x$

$=$ L.H.S

2. $3\cos^{-1}x = \cos-(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$

To prove: $3\cos^{-1}x = \cos-(4x^3 - 3x), x \in \left[\frac{1}{2}, 1\right]$

Let $x = \cos\theta$. Then, $\cos^{-1}x = \theta$

We have,

R.H.S. $= \cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$

$= 3\cos^{-1}x$

$=$ R.H.S

3. Prove $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Ans. To prove: prove $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

L.H.S. $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}} \quad \left[\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$=\tan^{-1}\frac{\frac{48\times 77}{11\times 24}}{\frac{11\times 24-14}{11\times 24}}$$

$$=\tan^{-1}\frac{48+77}{264-14}=\tan^{-1}\frac{125}{250}=\tan^{-1}\frac{1}{2}=\text{R.H.S.}$$

4. Prove: $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

Ans. To prove: $2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\frac{31}{17}$

L.H.S. $= 2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}$

$$= 2\tan^{-1}\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)}+\tan^{-1}\frac{1}{7} \quad \left[2\tan^{-1}x=\tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= 2\tan^{-1}\frac{1}{1-\left(\frac{1}{2}\right)}+\tan^{-1}\frac{1}{7}$$

$$= 2\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{1}{7}$$

$$= 2\tan^{-1}\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}} \quad \left[\tan^{-1}x=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{\left(\frac{28+3}{21}\right)}{\left(\frac{21-4}{21}\right)}$$

$$= \tan^{-1}\frac{31}{17}=\text{R.H.S.}$$

5. Write the function in the simplest form: $\tan^{-1}\frac{1+x^2-1}{x}, x \neq 0$

Ans. Write the function in the simplest form: $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$

$$\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$$\therefore \tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\frac{\sec\theta-1}{\tan\theta} = \tan^{-1}\left(\frac{1-\cos\theta}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$$

6. Write the function in the simplest form:

$$\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$$

Ans. $\tan^{-1}\frac{1}{\sqrt{x^2-1}}, |x| > 1$

Put $x = \operatorname{cosec}\theta \Rightarrow \theta = \operatorname{cosec}^{-1} x$

$$\therefore \tan^{-1}\frac{1}{\sqrt{x^2-1}} = \tan^{-1}\frac{1}{\sqrt{\operatorname{cosec}^2\theta-1}}$$

$$= \tan^{-1}\left(\frac{1}{\cos\theta}\right) = \tan^{-1}(\tan\theta)$$

$$= \theta = \operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1} x \qquad \left[\operatorname{cosec}^{-1}x + \sec^{-1} x = \frac{\pi}{2}\right]$$

7. Write the function in the simplest form:

$$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$$

Ans. $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x<\pi$

$\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$

$\tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)\tan^{-1}\left(\tan\frac{x}{2}\right)$

$= \frac{x}{2}$

8. Write the function in the simplest form:

$\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), 0<x<\pi$

Ans. $\tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

$= \tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right)$

$= \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$

$= \tan^{-1}(1)\tan^{-1}\left(\tan^{-1}\right)$ $\left[\tan^{-1}\frac{x-y}{1-xy} = \tan^{-1}x-\tan^{-1}y\right]$

$= \frac{\pi}{4}-x$

9. Write the function in the simplest form:

$\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x|<a$

Ans. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$

$$\text{Put} x = a\sin\theta \Rightarrow \frac{x}{a}\sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\therefore \tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} = \tan^{-1}\left(\frac{a\sin\theta}{a^2 - a^2\sin^2\theta}\right)$$

$$= \tan^{-1}\frac{a\sin\theta}{a\sqrt{1 - \sin^{-1}\theta}} = \tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$= \tan^{-1}(\tan\theta) = \theta = \sin^{-1}\frac{x}{a}$$

10. Write the function in the simplest form:

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right), a > 0; a > \frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$$

Ans. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$

$$\text{Put } x = a\tan\theta \Rightarrow \frac{x}{a} = \tan\theta \Rightarrow \theta = \tan^{-1}\frac{x}{a}$$

$$\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = \tan^{-1}\left(\frac{3a^2.a\tan\theta - a^3\tan\theta}{a^3 - 3a.a^2\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3a^2\tan\theta - a^3\tan^3\theta}{a^3 - 3a^2\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{3a\theta - \tan^3\theta}{1 - \tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3\tan^{-1}\frac{x}{a}$$

11. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Ans. $\text{Let} \sin^{-1}\frac{1}{2} = x. \text{ Then, } \sin x = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right).$

$$\therefore \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left[2\cos\left(2\sin-1\frac{1}{2}\right)\right]=\tan^{-1}\left[2\cos\left(2\times\frac{\pi}{6}\right)\right]$$

$$=\sin^{-1}\left[2\cos\frac{\pi}{3}\right]=\tan^{-1}\left[2\times\frac{1}{2}\right]$$

$$=\tan^{-1}=\frac{\pi}{4}$$

12. Find the value of $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

Ans. $\cot\left(\tan^{-1}a+\cot^{-1}a\right)$

$$=\cot\left(\frac{\pi}{2}\right) \qquad \left[\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$$

$$=0$$

13. Find the value of $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right],|x|<1, y>0 \text{ and } xy<1$

Ans. Let $x = \tan\theta$. Then, $\theta = \tan^{-1} x$.

$$\therefore \sin^{-1}\frac{2x}{1+x^2}=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}\left(\sin 2\theta\right)=2\tan^{-1}x$$

Let $y = \tan\theta$. Then, $\theta = \tan^{-1} y$.

$$\therefore \cos^{-1}\frac{1-y^2}{1+y^2}=\cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)=\cos^{-1}\left(\cos 2\phi\right)=2\tan^{-1}y$$

$$\therefore \tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$=\tan\frac{1}{2}\left[2\tan^{-1}x+2\tan^{-1}y\right]$$

$$=\tan\left[\tan^{-1}x+\tan^{-1}y\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$=\frac{x+y}{1-xy}$$

14. if $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of x.

$$\sin\left(\sin-1\frac{1}{5}+\cos^{-1}x\right)=1$$

$$\Rightarrow \sin\left(\sin^{-1}\frac{1}{5}\right)\cos\left(\cos^{-1}x\right)+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(cis^{-1}x\right)=1$$

$$\left[\sin\left(A+B\right)=\sin A\cos B+\cos A\sin B\right]$$

$$\Rightarrow \frac{1}{5}\times x+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1$$

$$\Rightarrow \frac{x}{5}+\cos\left(\sin^{-1}\frac{1}{5}\right)\sin\left(\cos^{-1}x\right)=1 \text{............}(1)$$

$$\text{Now, let } \sin^{-1}\frac{1}{5}=y.$$

$$\text{Then, } \sin y\frac{1}{5}\Rightarrow \cos y=\sqrt{1-\left(\frac{1}{5}\right)^2}=\frac{2\sqrt{6}}{5}\Rightarrow y=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)$$

$$\therefore \sin^{-1}\frac{1}{5}=\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right) \text{.......................}(2)$$

$$\text{Let } \cos^{-1}x=z.$$

$$\text{Then, } \cos z=x\Rightarrow \sin z=\sqrt{1-x^2}\Rightarrow z=\sin^{-1}\left(\sqrt{1-x^2}\right).$$

$$\therefore \cos^{-1}x=\sin^{-1}\left(\sqrt{1-x^2}\right) \text{..................}(3)$$

From (1), (2), and (3) we have:

$$\frac{x}{5}+\cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right).\sin\left(\sin^{-1}\sqrt{1-x^2}\right)=1$$

$$\Rightarrow \frac{x}{5}+\frac{2\sqrt{6}}{5}\cdot\sqrt{1-x^2}=1$$

$$\Rightarrow x+2\sqrt{6}\sqrt{1-x^2}=5$$

$$\Rightarrow 2\sqrt{6}\sqrt{1-x^2}=5-x$$

On squaring both sides, we get :

$$(4)(6)(1-x^2)=25+x^2-10x$$

$$\Rightarrow 24-24x^2=25+x^2-10x$$

$$\Rightarrow 25x^2-10x+1=0$$

$$\Rightarrow (5x-1)^2=0$$

$$\Rightarrow (5x-1)=0$$

$$\Rightarrow x=\frac{1}{5}$$

Hence, the value of x is $\frac{1}{5}$.

15. If $\tan-1\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then find the value of x.

Ans. $\tan-1\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

$$\Rightarrow \text{tam}^{-1}\left[\frac{\frac{x-1}{x-2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right]=\frac{\pi}{4}$$

$$\tan^{-1}\left[x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}\right]$$

$$\Rightarrow \tan^{-1}\frac{(x-1)(x+2)+(x+1)(x-2)}{(x+1)(x-2)-(x-1)(x+2)}=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{x^2+x-2+x^2-x-2}{x^2-4-x^2+1}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left[\frac{2x^2-4}{-3}\right]=\frac{\pi}{4}$$

$$\Rightarrow \tan\left[\tan^{-1}\frac{4-2x^2}{3}\right]=\frac{\pi}{4}$$

$$\Rightarrow \frac{4-2x^2}{3}=3$$

$$\Rightarrow 4-2x2=3$$

$$\Rightarrow 2x2=4-3=1$$

$$\Rightarrow x=\pm\frac{1}{\sqrt{2}}$$

Hence, the value of x is $\pm\frac{1}{\sqrt{2}}$

16. Find the values of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

Ans. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

We know that $\sin^{-1}(\sin x) = x$ if $\frac{\pi}{3}\in\left[\frac{-\pi}{2}.\frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

Here, $\frac{2\pi}{3}\in\left[\frac{-\pi}{2}.\frac{\pi}{2}\right]$

Now, $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ can be written as:

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{2\pi}{3}\right)\right]=\sin^{-1}\left(\sin\frac{\pi}{3}\right)\text{where}\frac{\pi}{3}\in\left[\frac{-\pi}{2}.\frac{\pi}{2}\right]$$

$$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$$

17. Find the values of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Ans. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

Here, $\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

Now, can be written as: $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$\tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[-\tan\left(\frac{-3\pi}{4}\right)\right] = \tan^{-1}\left[-\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right) = \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right] \text{where} -\frac{\pi}{4} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(-\frac{-\pi}{4}\right)\right] = \frac{-\pi}{4}$$

18. Find the values of $\tan\left(\sin - 1 + \frac{3}{5} + \cos^{-1}\frac{3}{2}\right)$

Ans. $\text{Let} \sin - 1\frac{3}{5} = x. \text{ then}, \sin x = \frac{3}{5} \Rightarrow \cos x = \sqrt{1-\sin^2 x} = \frac{4}{5} \Rightarrow \sec x = \frac{5}{4}.$

$$\therefore \tan x = \sqrt{\sec^2 x - 1} = \sqrt{\frac{25}{16} - 1} = \frac{3}{4}$$

$$\therefore x = \tan^{-1}\frac{3}{4}$$

$$\therefore \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{3}{4} \text{.......(i)}$$

$\text{Now}, \cot^{-1}\frac{3}{2} = \tan^{-1}\frac{2}{3} \text{.......(ii)}$ $\left[\tan^{-1}\frac{1}{x} = \cot^{-1} x\right]$

$\text{Hence } \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

$= \tan\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$ $\qquad$ [Using (i) and (ii)]

$= \tan\left(\tan^{-1}\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)$ $\qquad$ $\left[\tan-1x+\tan-1y=\tan-1\frac{x+y}{1-xy}\right]$

$\tan\left(\tan^{-1}\frac{9+8}{12-6}\right)$

$= \tan\left(\tan^{-1}\frac{17}{6}\right) = \frac{17}{6}$

19. Find the values of $\cos-1\left(\cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. We know that cos−1 (cos x) = x if $x \in [0,\pi]$, which is the principal value branch of $\cos^{-1}x$.

Here, $\frac{7\pi}{6} \notin x \in [0,\pi]$.

Now, $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ can be written as:

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{-7\pi}{6}\right) = \cos^{-1}\left[\cos\left(2\pi-\frac{7\pi}{6}\right)\right]$ $\qquad$ $[\cos(2\pi+x)=\cos x]$

$= \cos^{-1}\left[\cos\frac{5\pi}{6}\right] = \text{where}\ \frac{5\pi}{6} \in [0,\pi]$

$\therefore \left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\frac{5\pi}{6}\right) = \frac{5\pi}{6}$

The correct answer is B.

20. Find the values of $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Ans. Let $\sin^{-1}\left(\frac{-1}{2}\right) = x.\text{than}, \sin x = \frac{-1}{2} = -\sin\frac{\pi}{6} = \sin\left(\frac{-\pi}{6}\right).$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right].$

$$\therefore \sin^{-1}\left(\frac{-1}{2}\right) = \frac{-\pi}{6}$$

$$\therefore \sin\left(\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right) = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$