SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

1. Evaluate the Given limit: $\lim_{x\to 3} x+3$

Ans. $\lim_{x\to 3} x+3 = 3+3 = 6$

2. Evaluate the Given limit: $\lim_{x\to \pi}\left(x-\frac{22}{7}\right)$

Ans. $\lim_{x\to \pi}\left(x-\frac{22}{7}\right) = \left(\pi-\frac{22}{7}\right)$

3. Evaluate the Given limit: $\lim_{x\to 1} \pi r^2$

Ans. $\lim_{x\to 1} \pi r^2 = \pi(1)^2 = \pi$

4. Evaluate the Given limit: $\lim_{x\to 4}\frac{4x+3}{x-2}$

Ans. $\lim_{x\to 4}\frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{16+3}{2} = \frac{19}{2}$

5. Evaluate the Given limit: $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1}$

Ans. $\lim_{x\to -1}\frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$

6. Evaluate the Given limit: $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Ans. $\lim_{x\to 0}\frac{(x+1)^5-1}{x}$

Put x + 1 = y so that $y\to 1$ as $x\to 0$.

$$\text{Accordingly}, \lim_{x\to 0}\frac{(x+1)^5-1}{x} = \lim_{y\to 1}\frac{y^5-1}{y-1}$$

$$= \lim_{y\to 1}\frac{y^5-1^5}{y-1}$$

$$= 5.1^{5-1}$$

$$= 5$$

$$\therefore \lim_{x\to 0}\frac{(X+5)^5-1}{x}=5$$

7. Evaluate the Given limit: $\lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}$

Ans. At x = 2, the value of the given rational function takes the form $\frac{0}{0}$

$$\therefore \lim_{x\to 2}\frac{3x^2-x-10}{x^2-4}=\lim_{x\to 2}\frac{(x-2)(3x+5)}{(x-2)(x+2)}$$

$$=\lim_{x\to 2}\frac{3x+5}{x+2}$$

$$=\frac{3(2)+5}{2+2}$$

$$=\frac{11}{4}$$

8. Evaluate the Given limit: $\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$

Ans. At x = 2, the value of the given rational function takes the form $\frac{0}{0}$

$$\therefore \lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}=\lim_{x\to 3}\frac{(x-3)(x^2+9)}{(x-3)(2x+1)}$$

$$=\lim_{x\to 2}\frac{(x+3)(x^2+9)}{2x+1}$$

$$=\frac{(3+3)+(x^2+9)}{2(3)+1}$$

$$=\frac{6\times 18}{7}$$

$$=\frac{108}{7}$$

9. Evaluate the Given limit: $\lim_{x\to 0}\frac{ax+b}{cx+1}$

Ans. $\lim_{x\to 0}\frac{ax+b}{cx+1}=\frac{a(0)+b}{c(0)+1}=b$

10. Evaluate the Given limit: $\lim_{z\to 1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

Ans. $\lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}$

At z = 1, the value of the given function takes the form $\frac{0}{0}$.

Put $z^{\frac{1}{6}}$ so that z →1 as x → 1

$$\text{Accordingly}, \lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{3}}-1}=\lim_{x\to1}\frac{x^5-1}{x-1}$$

$$=\lim_{y\to1}\frac{x^5-1^5}{x-1}$$

$$=2.1^{2-1} \qquad \left[\lim_{x\to a}\frac{x^n-a^n}{x-a}\right]=na^{n-1}$$

$$=2$$

$$\therefore \lim_{z\to1}\frac{z^{\frac{1}{3}}-1}{z^{\frac{1}{6}}-1}=2$$

11. Evaluate the Given limit: $\lim_{x\to1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq0$

Ans. $\lim_{x\to1}\frac{ax^2+bx+c}{cx^2+bx+a}=\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}$

$$=\frac{a+b+c}{a+b+c}$$

$$=1 \qquad [a+b+c\neq0]$$

12. Evaluate the Given limit: $\lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

Ans. $\lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

At x = –2, the value of the given function takes the form $\frac{0}{0}$

$$\text{Now}, \lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to-2}\frac{\left(\frac{2+x}{2x}\right)}{x+2}$$

$$= \lim_{x \to -2} \frac{1}{2x}$$

$$= \frac{1}{2(-2)} = \frac{-1}{4}$$

13. Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{bx}$

Ans. $\lim_{x \to 0} \frac{\sin ax}{bx}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

$$\text{Now,} \lim_{x \to 0} \frac{\sin ax}{bx} = \lim_{x \to 0} \frac{\sin ax}{ax} \times \frac{ax}{bx}$$

$$= \lim_{x \to 0} \left(\frac{\sin ax}{ax} \right) \times \left(\frac{a}{b} \right)$$

$$= \frac{a}{b} \lim_{ax \to 0} \left(\frac{\sin ax}{ax} \right) \quad [x \to 0 \Rightarrow ax \to 10]$$

$$= \frac{a}{b} \times 1 \quad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$$

$$= \frac{a}{b}$$

14. Evaluate the Given limit: $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

Ans. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

$$\text{Now,} \lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\left(\frac{\sin ax}{ax} \right) \times ax}{\left(\frac{\sin bx}{bx} \right) \times ax}$$

$$= \left(\frac{a}{b} \right) \times \frac{\lim_{x \to 0} \left(\frac{\sin ax}{ax} \right)}{\lim_{x \to 0} \left(\frac{\sin bx}{bx} \right)} \quad \left[\begin{array}{l} x \to 0 \Rightarrow ax \to 0 \\ \text{and } x \to 0 \Rightarrow bx \to 0 \end{array} \right]$$

$$= \left(\frac{a}{b} \right) \times \frac{1}{1} \quad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$$

$$=\frac{a}{b}$$

15. Evaluate the Given limit: $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

Ans. $\lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}$

It is seen that x → π ⇒ (π – x) → 0

$$\therefore \lim_{x\to\pi}\frac{\sin(\pi-x)}{\pi(\pi-x)}=\frac{1}{\pi}\lim_{(\pi-x)\to 0}\frac{\sin(\pi-x)}{(\pi-x)}$$

$$\frac{1}{\pi}\times 1 \quad \left[\lim_{y\to 0}\frac{\sin y}{y}=1\right]$$

$$=\frac{1}{\pi}$$

16. Evaluate the given limit: $\lim_{x\to 0}\frac{\cos x}{\pi-0}$

Ans. $\lim_{x\to 0}\frac{\cos x}{\pi-0}=\frac{\cos 0}{\pi-0}=\frac{1}{\pi}$

17. Evaluate the Given limit: $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

Ans. $\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now,

$$\lim_{x\to 0}\frac{\cos 2x-1}{\cos x-1}=\lim_{x\to 0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1} \qquad \left[\cos x=1-2\sin^2\frac{x}{2}\right]$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}=\lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$=4\frac{\lim_{x\to0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$=4\frac{\left(\lim_{x\to0}\frac{\sin x}{x}\right)^2}{\left(\lim_{\frac{x}{2}\to0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}\qquad\left[x\to0\Rightarrow\frac{x}{2}\to0\right]$$

$$=4\frac{1^2}{1^2}\qquad\left[\lim_{y\to0}\frac{\sin y}{y}=1\right]$$

$$=4$$

18. Evaluate the Given limit: $\lim_{x\to0}\frac{ax+x\cos x}{b\sin x}$

Ans. $\lim_{x\to0}\frac{ax+x\cos x}{b\sin x}$

At $x = 0$, the value of the given function takes the form $\frac{0}{0}$.

Now,

$$\lim_{x\to0}\frac{ax+x\cos x}{b\sin x}=\frac{1}{b}\lim_{x\to0}\frac{x(a+\cos x)}{\sin x}$$

$$=\frac{1}{b}\lim_{x\to0}\left(\frac{x}{\sin x}\right)\times\lim_{x\to0}(a+\cos x)$$

$$=\frac{1}{b}\times\frac{1}{\left(\lim_{x\to0}\frac{\sin x}{x}\right)}\times\lim_{x\to0}(a+\cos x)$$

$$=\frac{1}{b}\times(a+\cos 0)\qquad\left[\lim_{x\to0}\frac{\sin x}{x}=1\right]$$

19. Evaluate the Given limit: $\lim_{x\to0}x\sec x$

Ans. $\lim_{x\to0}x\sec x=\lim_{x\to0}\frac{x}{\cos x}=\frac{0}{\cos}=\frac{0}{1}=0$

20. Evaluate the Given limit: $\lim_{x\to0}\frac{\sin ax+bx}{ax+\sin bx}a,b,a+b\neq0$

Ans. At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now,

$$\lim_{x\to0}\frac{\sin ax+bx}{ax+\sin bx}$$

$$=\lim_{x\to0}\frac{\left(\frac{\sin ax}{ax}\right)ax+bx}{ax+bx\left(\frac{\sin bx}{bx}\right)}$$

$$=\frac{\left(\lim_{x\to0}\frac{\sin ax}{ax}\right)\times\lim_{x\to0}(ax)+\lim_{x\to0}bx}{\lim_{x\to0}ax+\lim_{x\to0}bx\left(\lim_{x\to0}\frac{\sin bx}{bx}\right)}\quad\left[\text{As } x\to0\Rightarrow ax\to0 \text{ and } bx\to0\right]$$

$$=\frac{\lim_{x\to0}(ax)+\lim_{x\to0}bx}{\lim_{x\to0}ax+\lim_{x\to0}bx}\qquad\left[\lim_{x\to0}\frac{\sin x}{x}=1\right]$$

$$=\frac{\lim_{x\to0}(ax+bx)}{\lim_{x\to0}(ax+bx)}$$

$$=\lim_{x\to0}(1)$$

$$=1$$

21. Evaluate the Given limit: $\lim_{x\to0}(\operatorname{cosec}x-\cot x)$

Ans. At x = 0, the value of the given function takes the form $\infty-\infty$.

Now,

$$\lim_{x\to0}(\operatorname{cosec}x-\cot x)$$

$$=\lim_{x\to0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$$

$$=\lim_{x\to0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$=\lim_{x\to0}\frac{\left(\frac{1-\cos x}{\sin x}\right)}{\frac{\sin x}{x}}$$

$$=\frac{\lim_{x\to0}\frac{1-\cos x}{x}}{\lim_{h\to0}\frac{\sin x}{x}}$$

$$= \frac{0}{1} \quad \left[\lim_{h \to 0} \frac{1-\cos x}{x} = 0 \text{ and } \lim_{h \to 0} \frac{\sin x}{x} = 1\right]$$

$$= 0$$

22. $\lim_{x \to \frac{x}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

Ans. $\lim_{x \to \frac{x}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

At $x = \frac{\pi}{2}$, the value of the given function takes the form $\frac{0}{0}$.

Now, put $x - \frac{\pi}{2} = y$ so that $x \to \frac{\pi}{2}, y \to 0$.

$$\lim_{x \to \frac{x}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}} = \lim_{y \to 0} \frac{\tan 2\left(y + \frac{\pi}{2}\right)}{y}$$

$$= \lim_{y \to 0} \frac{\tan(\pi + 2y)}{y}$$

$$= \lim_{y \to 0} \frac{\tan 2y}{y} \left[\tan(\pi + 2y)\right]$$

$$= \lim_{y \to 0} \frac{\sin 2y}{y \cos 2y}$$

$$= \lim_{y \to 0} \left(\frac{\sin 2y}{2y} \times \frac{2}{\cos 2y}\right)$$

$$= \left(\lim_{y \to 0} \frac{\sin 2y}{2y}\right) \times \lim_{y \to 0} \left(\frac{2}{\cos 2y}\right)$$

$$= 1 \times \frac{2}{\cos 0}$$

$$= 1 \times \frac{2}{1}$$

$$= 2$$

23. Find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} 2x+3, \\ 3(x+1), \end{cases}$

Ans. The given function is

$$f(x) = \begin{cases} 2x+3, & x \le 0 \\ 3(x+1), & x > 0 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} [2x+3] = 2(0)+3 = 3$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} 3(x+1) = 3(0+1) = 3$$

$$\therefore \lim_{x \to 0} f(x) = \lim_{x \to 0} f(x) = \lim_{x \to 0} f(x) = 3$$

$$\lim_{x \to 1} f(x) = \lim_{x \to 1} 3(x+1) = 3(1+1) = 6$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} 3(x+1) = 3(1+1) = 6$$

$$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = 6$$

24. Find $\lim_{h \to 1} f(x)$, where $f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ -x^2 - 1, & x > 1 \end{cases}$

Ans. The given function is

$$f(x) = \begin{cases} x^2 - 1, & x \le 1 \\ -x^2 - 1, & x > 1 \end{cases}$$

$$\lim_{h \to 1^-} f(x) = \lim_{h \to 1} \left[x^2 - 1 \right] = 1^2 - 1 = 1 - 1 = 0$$

$$\lim_{h \to 1^+} f(x) = \lim_{h \to 1} \left[-x^2 - 1 \right] = -1^2 - 1 = -1 - 1 = -2$$

It is observed that $\lim_{h \to 1^-} f(x) \ne \lim_{h \to 1^+} f(x)$.

Hence, $\lim_{h \to 1} f(x)$ does not exist.

25. Evaluate $\lim_{x \to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x \ne 1 \\ 0, & x = 1 \end{cases}$

Ans. The given function is

$$f(x) = f(x) = \begin{cases} \frac{|x|}{x}, & x \ne 1 \\ 0, & x = 1 \end{cases}$$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0^-} \left[\frac{|x|}{x} \right]$$

$$= \lim_{x \to 0} \left(\frac{-x}{x} \right) \qquad [\text{When } x \text{ is negative}, |x| = -x]$$

$$= \lim_{x \to 0} (-1)$$

$$= -1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{x}{x}\right) \qquad \left[\text{When x is positive}, |x| = x\right]$$

$$= \lim_{x\to 0}(1)$$

$$= 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{x}{x}\right) \qquad \left[\text{When x is positive}, |x| = x\right]$$

$$= \lim_{x\to 0}(1)$$

$$= 1$$

It is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

26. Find $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Ans. The given function is

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0^-}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{x}{-x}\right) \qquad \left[\text{When x } x < 0, |x| = -x\right]$$

$$= \lim_{x\to 0}(-1)$$

$$= -1$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0^+}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{x}{x}\right) \qquad \left[\text{When } x > 0, |x| = x\right]$$

$$= \lim_{x\to 0}(1)$$

$$= 1$$

It is observed that $\lim_{x\to0^-} f(x) \neq \lim_{x\to0^+} f(x)$.

Hence, $\lim_{x\to0} f(x)$ does not exist.

27. Find $\lim_{x\to5} f(x)$, where $f(x) = |x| - 5$

Ans. The given function is $f(x) = |x| - 5$

$$\lim_{x\to5^-} f(x) = \lim_{x\to5^-}\left[|x|-5\right]$$
$$= \lim_{x\to5}(x-5) \qquad \left[\text{When } x>0, |x| = x\right]$$
$$= 5-5$$
$$= 0$$

$$\lim_{x\to5^+} f(x) = \lim_{x\to5^+}\left[|x|-5\right]$$
$$= \lim_{x\to5}(x-5) \qquad \left[\text{When } x>0, |x| = x\right]$$
$$= 5-5$$
$$= 0$$

$$\therefore \lim_{x\to5^-} f(x) = \lim_{x\to5^+} f(x) = 0$$

Hence, $\lim_{x\to5^-} f(x) = 0$

28. Suppose $f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax & x>1 \end{cases}$ and if $\lim_{x\to1} f(x) = f(1)$ what are possible values of a and b?

Ans. The given function is

$$f(x) = \begin{cases} a+bx, & x<1 \\ 4, & x=1 \\ b-ax & x>1 \end{cases}$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1}(b+ax) = b+a$$
$$\lim_{x\to1^+} f(x) = \lim_{x\to1}(b-ax) = b-a$$
$$f(1) = 4$$

It is given that $\lim_{x\to1} f(x) = f(1)$

$$\therefore \lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x) = \lim_{x\to1} f(x) = f(1)$$

$\Rightarrow a+b=4$ and $b-a=4$

On solving these two equation, we obtain $a=0$ and $b=4$

Thus, the respective possible values of a and b are 0 and 4.

29. Let $a_1, a_2, ...a_n$ be fixed real numbers and define a function

$$f(x) = (x - a_1)(x - a_2)(x - a_2).....(x - a_n)$$

What is $\lim_{x \to a_1} f(x)$? For some $a \neq a_1, a_2......, a_n$ compute $\lim_{x \to a} f(x)$.

Ans. The given function is $f(x) = (x - a_1) = (x - a_2)......(x - a_n)$

$$\lim_{x \to a_1} f(x) = \lim_{x \to a_1} \left[(x - a_1)(x - a_2)....(x - a_n)\right]$$

$$= \left[\lim_{x \to a_1}(x - a_1)\right]\left[\lim_{x \to a_1}(x - a_2)\right]....\left[\lim_{x \to a_1}(x - a_n)\right]$$

$$= (a_1 - a_1)(a_1 - a_2)....(a_1 - a_n) = 0$$

$$\therefore \lim_{h \to a_1} f(x) = 0$$

$$\text{Now,} \lim_{x \to a} f(x) = \lim_{x \to a} \left[(x - a_1)(x - a_2)....(x - a_n)\right]$$

$$= \left[\lim_{x \to a}(x - a_1)\right]\left[\lim_{x \to a}(x - a_2)\right]....\left[\lim_{x \to a}(x - a_n)\right]$$

$$= (a - a_1)(a - a_2)....(a - a_n) = 0$$

$$\therefore \lim_{x \to a} f(x) = (a - a_1)(a - a_2)....(a - a_n)$$

30. If $f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$.

For what value (s) of a does $\lim_{x \to a} f(x)$ exists?

Ans. The given function is

$$f(x) = \begin{cases} |x| + 1, & x < 0 \\ 0, & x = 0 \\ |x| - 1, & x > 0 \end{cases}$$

When $a = 0$,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (|x| + 1)$$

$$= \lim_{x \to 0}(-x + 1) \quad [\text{If } x < 0, |x| = -x]$$

$$= -0 + 1$$

$$= 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (|x| + 1)$$

$$= \lim_{x \to 0}(x - 1) \quad [\text{If } x > 0, |x| = -x]$$

$= 0 - 1$

$= -1$

Here, it is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist.

When $a < 0$,

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (|x|+1)$$
$$= \lim_{x\to 0} (-x+1) \quad [\, x < a < 0 \Rightarrow |x| = -x \,]$$
$$= -a + 1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (|x|+1)$$
$$= \lim_{x\to 0} (-x+1) \quad [\, a < a < 0 \Rightarrow |x| = -x \,]$$
$$= -a + 1$$
$$= -a + 1$$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = -a + 1$

Thus, limit of $f(x)$ exists at $x = a$, where $a < 0$

When $a > 0$

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^-} (|x|-1)$$
$$= \lim_{x\to a} (x-1) \quad [\, a < x < a \Rightarrow |x| = x \,]$$
$$= a - 1$$

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^+} (|x|-1)$$
$$= \lim_{x\to a} (x-1) \quad [\, 0 < a < x \Rightarrow |x| = x \,]$$
$$= a - 1$$

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = a - 1$

Thus, limit of $f(x)$ exists at $x = a$, where $a > 0$

Thus, $\lim_{x\to a} f(x)$ exists for all $a \neq 0$

31. If the function f(x) satisfies $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$, evaluate

Ans. $\lim_{x\to 1} \frac{f(x)-2}{x^2-1} = \pi$

$$\Rightarrow \frac{\lim_{x\to 1}(f(x)-2)}{\lim_{x\to 1}(x^2-1)} = \pi$$

$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi \lim_{x\to 1}(x^2-1)$$

$$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi(1^2-1)$$

$$\Rightarrow \lim_{x\to 1}(f(x)-2) = 0$$

$$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = 0$$

$$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0$$

$$\therefore \lim_{x\to 1} f(x) = 2$$

32. If $f(x) = \begin{cases} mx^2+n, & x<0 \\ nx+m, & 0 \le x \le 1 \\ nx^3+m, & x>1 \end{cases}$ For what integers m and n does $\lim_{x\to 0} f(x)$ and exist $\lim_{x\to 1} f(x)$?

Ans. The given function is

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0}(mx^2+n)$$
$$= m(0)^2 + n$$
$$= n$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0}(nx+n)$$
$$= m(0)+n$$
$$= m.$$

Thus, $\lim_{x\to 0} f(x)$ exists if m = n.

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(nx+m)$$
$$= m(0)+m$$
$$= m+n$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(nx^3+m)$$
$$= m(1)^3+m$$
$$= m+n$$

$$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x)$$

Thus, $\lim_{x\to 1} f(x)$ exists for any integral value of m and n.