SpeedLabs

MATHS

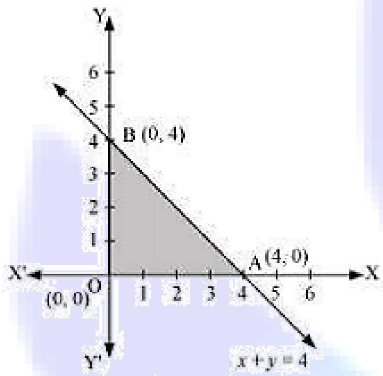
CBSE 12th

TEEVRA EDUTECH PVT. LTD.

1. Maximize Z = 3x + 4y

Subject to the constraints: $x+y\leq 4, x\geq 0, y\geq 0$

Ans. The feasible region determined by the constraints, $x+y\leq 4, x\geq 0, y\geq 0$ is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), and B (0, 4). The values of Z at these points are as follows.

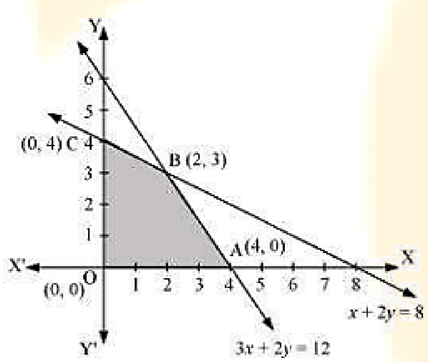
Corner point	Z = 3x + 4y	
O (0, 0)	0	
A (4, 0)	12	
B (0, 4)	16	→ Maximum

Therefore, the maximum value of Z is 16 at the point B (0, 4).

2. Minimize Z = –3x + 4y

subject to $x+2y\leq 8, 3x+2y\leq 12, x\geq 0, y\geq 0$

Ans. The feasible region determined by the system of constraints, $x+2y\leq 8, 3x+2y\leq 12, x\geq 0, y\geq 0$ and $y\geq 0$ is as follows.



The corner points of the feasible region are O (0, 0), A (4, 0), B (2, 3), and C (0, 4). The values of Z at these corner points are as follows.

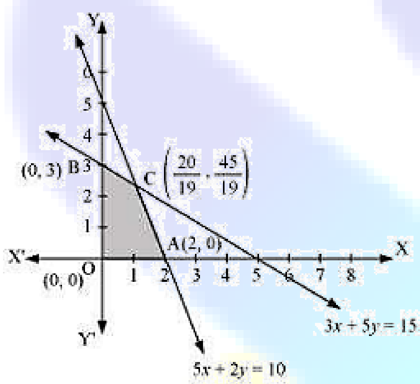
Corner point	$Z = -3x + 4y$	
0 (0, 0)	0	
A (4, 0)	–12	→ Minimum
B (2, 3)	6	
C (0, 4)	16	

Therefore, the minimum value of Z is -12 at the point (4, 0).

3. Maximize $Z = 5x + 3y$

subject to $3x+5y \le 15, 5x+2y \le 10, x \ge 0, y \ge 0$

Ans. The feasible region determined by the system of constraints, $3x+5y \le 15, 5x+2y \le 10, x \ge 0, y \ge 0$ and $y \ge 0$ are as follows.



The corner points of the feasible region are O (0, 0), A (2, 0), B (0, 3), and $c\left(\frac{20}{19}, \frac{45}{19}\right)$ The values of Z at these corner points are as follows.

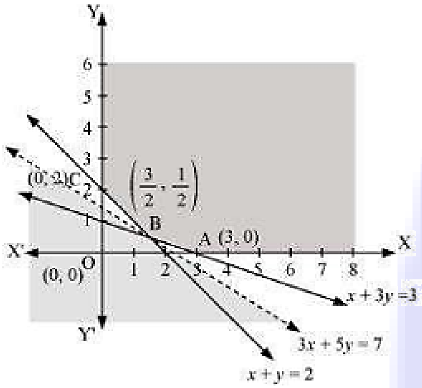
Corner point	$Z = 5x + 3y$	
0 (0, 0)	0	
A (2, 0)	10	
B (0, 3)	9	
$c\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{235}{19}$	→ Maximum

Therefore, the maximum value of Z is $\frac{235}{19}$ at the point $\left(\frac{20}{19}, \frac{45}{19}\right)$

4. Minimize $Z = 3x + 5y$

such that $x+3y \ge 3, x+y \ge 2, x, y \ge 0$

Ans. The feasible region determined by the system of constraints, $x+3y \ge 3, x+y \ge 2, \text{and} x, y \ge 0$ is as follows.



It can be seen that the feasible region is unbounded.

The corner points of the feasible region are A (3, 0), $B\left(\frac{3}{2},\frac{1}{2}\right)$ and C (0, 2) The values of Z at these corner points are as follows.

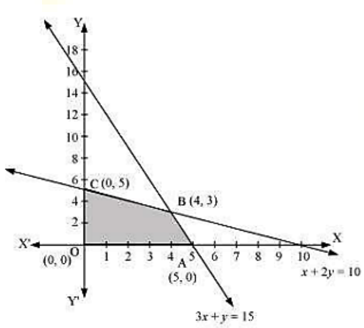
Corner point	Z = 3x + 5y	
A (3, 0)	9	
$B\left(\frac{3}{2},\frac{1}{2}\right)$	7	→ Smallest
C (0, 2)	10	

As the feasible region is unbounded, therefore, 7 may or may not be the minimum value of Z. For this, we draw the graph of the inequality, $3x + 5y < 7$, and check whether the resulting half plane has points in common with the feasible region or not. It can be seen that the feasible region has no common point with $3x + 5y < 7$ Therefore, the minimum value of Z is 7 at $\left(\frac{3}{2},\frac{1}{2}\right)$

5. Maximize $Z = 3x + 2y$

subject to $x+2y \le 10, 3x+y \le 15, x, y \ge 0$

Ans. The feasible region determined by the constraints, $x+2y \le 10, 3x+y \le 15, x, y \ge 0$ and $y \ge 0$ is as follows.



The corner points of the feasible region are A (5, 0), B (4, 3), and C (0, 5) The values of Z at these corner points are as follows.

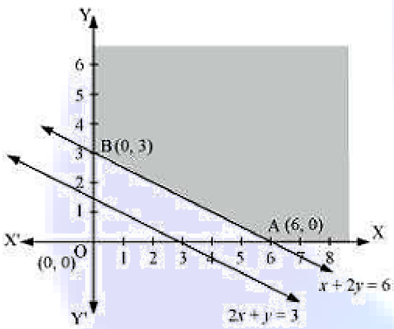
Corner point	Z = 3x + 5y	
A (5, 0)	15	
B (4, 3)	18	→ Maximum
C (0, 5)	10	

Therefore, the maximum value of Z is 18 at the point (4, 3).

6. Minimize Z = x + 2y

subject to $2x+y\geq 3, x+2y\geq 6, x, y\geq 0$

Ans. The feasible region determined by the constraints, $2x+y\geq 3, x+2y\geq 6, x, y\geq 0$ and $y\geq 0$ is as follows.



The corner points of the feasible region are A (6, 0) and B (0, 3). The values of Z at these corner points are as follows.

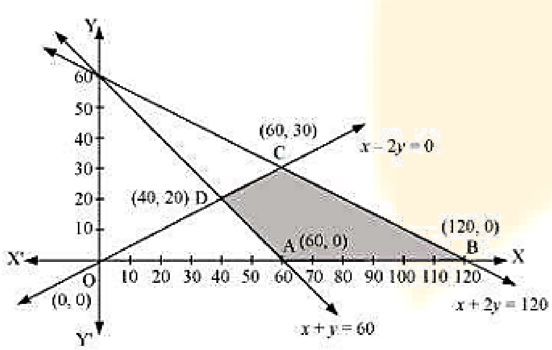
Corner point	Z = x + 2y
A (6, 0)	6
B (0, 3)	6

It can be seen that the value of Z at points A and B is same. If we take any other point such as (2, 2) on line x + 2y = 6, then Z = 6 Thus, the minimum value of Z occurs for more than 2 points. Therefore, the value of Z is minimum at every point on the line, x + 2y = 6

7. Minimize and Maximize Z = 5x + 10y

subject to $x+y\leq 120, x+y\geq 60, x-2y\geq 0, x, y\geq 0$

Ans. The feasible region determined by the constraints, $x+y\leq 120, x+y\geq 60, x-2y\geq 0, x, y\geq 0$ and $y\geq 0$ is as follows.



The corner points of the feasible region are A (60, 0), B (120, 0), C (60, 30), and D (40, 20). The values of Z at these corner points are as follows.

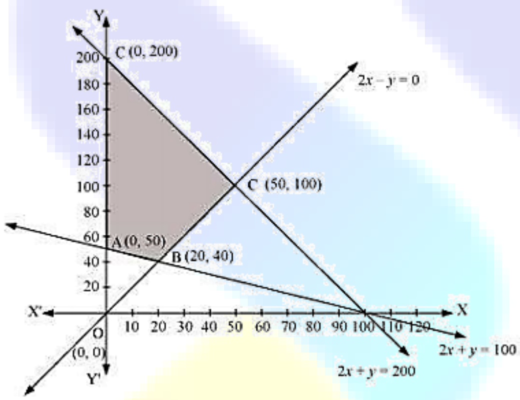
Corner point	$Z = 5x + 10y$	
A (60, 0)	300	→ Minimum
B (120, 0)	600	→ Minimum
C (60, 30)	600	→ Minimum
D (40, 20)	400	

The minimum value of Z is 300 at (60, 0) and the maximum value of Z is 600 at all the points on the line segment joining (120, 0) and (60, 30).

8. Minimize and Maximize $Z = x + 2y$

subject to $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200; x, y \geq 0$

Ans. The feasible region determined by the constraints, $x + 2y \geq 100, 2x - y \leq 0, 2x + y \leq 200; x, y \geq 0$ and $y \geq 0$ is as follows.



The corner points of the feasible region are A (0, 50), B (20, 40), C (50, 100), and D (0, 200). The values of Z at these corner points are as follows.

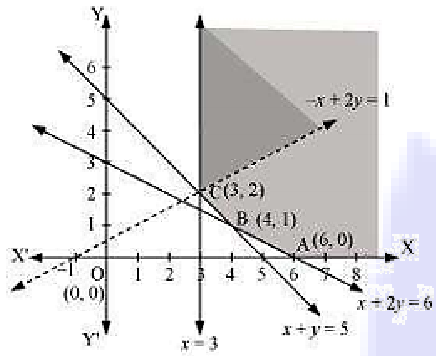
Corner point	$Z = x + 2y$	
A (0, 50)	100	→ Minimum
B (20, 40)	100	→ Minimum
C (50, 100)	250	
D (0, 200)	400	→ Minimum

The maximum value of Z is 400 at (0, 200) and the minimum value of Z is 100 at all the points on the line segment joining the points (0, 50) and (20, 40).

9. Maximize $Z = -x + 2y$, subject to the constraints:

$x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$

Ans. The feasible region determined by the constraints, $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0$ and $y \geq 0$ is as follows.



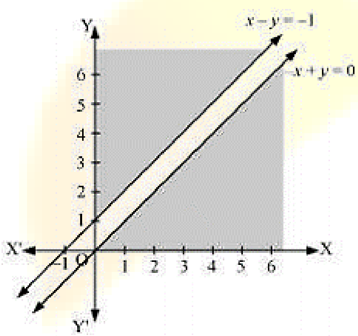
It can be seen that the feasible region is unbounded. The values of Z at corner points A (6, 0), B (4, 1), and C (3, 2) are as follows.

Corner point	$Z = -x + 2y$
A (6, 0)	$Z = -6$
B (4, 1)	$Z = -2$
C (3, 2)	$Z = 1$

As the feasible region is unbounded, therefore, $Z = 1$ may or may not be the maximum value. For this, we graph the inequality, $-x + 2y > 1$, and check whether the resulting half plane has points in common with the feasible region or not. The resulting feasible region has points in common with the feasible region. Therefore, $Z = 1$ is not the maximum value. Z has no maximum value.

10. Maximize $Z = x + y$, subject to $x - y \le -1, -x + y \le 0, x, y \ge 0$

Ans. The region determined by the constraints, $x - y \le -1, -x + y \le 0, x, y \ge 0$ is as follows.



There is no feasible region and thus, Z has no maximum value.