

Board – CBSE

Class –12

Topic – Matrices

(i). Order, Addition, Multiplication and transpose of matrices:

LEVEL I

1. If a matrix has 5 elements, what are the possible orders it can have? [CBSE 2011]
2. Construct a 3×2 matrix whose elements are given by $a_{ij} = \frac{1}{2}|i - 3j|$
3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find $A - 2B$.
4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA .

LEVEL II

5. For the following matrices A and B , verify $(AB)^T = B^T A^T$.
where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = [-1 \quad 2 \quad 1]$
6. Give example of matrices A & B such that $AB = O$, but $BA \neq O$, where O is a zero matrix and A, B are both non zero matrices.
7. If B is skew symmetric matrix, write whether the matrix (ABA^T) is symmetric or skew symmetric.
8. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find a and b so that $A^2 + aI = bA$

LEVEL III

9. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$
10. Express the matrix A as the sum of a symmetric and a skew symmetric matrix, where:
$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$
11. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n-1)}{a-1} \\ 0 & 1 \end{bmatrix}$, $n \in \mathbb{N}$

(ii) Cofactors & Ad joint of a matrix

LEVEL I

12. Find the co-factor of a_{12} in $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

13. Find the adjoints of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

LEVEL II

Verify $A(\text{adj}A) = (\text{adj}A)A = |A|I$ if

14. $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

15. $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

(iii) Inverse of a Matrix & Applications

LEVEL I

16. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A

[CBSE 2011]

17. If A is square matrix satisfying $A^2 = I$, then what is the inverse of A ?

18. For what value of k , the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

LEVEL II

19. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 12I = 0$. Hence find A^{-1}

20. If A, B, C are three non-zero square matrices of same order, find the condition on A such that $AB = AC \Rightarrow B = C$.

21. Find the number of all possible matrices A of order 3×3 with each entry 0 or 1 and for which

$$A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \text{ has exactly two distinct solutions.}$$

LEVEL III

22. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations:

$$2x - 3y + 5z = 11, \quad 3x + 2y - 4z = -5, \quad x + y - 2z = -3$$

23. Using matrices, solve the following system of equations:

a. $x + 2y - 3z = -4$

$$2x + 3y + 2z = 2$$

$$3x - 3y - 4z = 11$$

[CBSE 2011]

b. $4x + 3y + 2z = 60$

$$x + 2y + 3z = 45$$

$$6x + 2y + 3z = 70.$$

[CBSE 2011]

24. Find the product AB , where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and use it to solve the

equations $x - y = 3$, $2x + 3y + 4z = 17$, $y + 2z = 7$

25. Using matrices, solve the following system of equations:

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$$

$$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$$