SpeedLabs

MATHS

# CBSE 10th

TEEVRA EDUTECH PVT. LTD.

**Q.1** Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

(ii) $2x + y = 5$

$3x + 2y = 8$

(iii) $3x - 5y = 20$

$6x - 10 = 40$

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

**Sol:**

(i) $x - 3y - 3 = 0$

$3x - 9y - 2 = 0$

$$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) $2x + y = 5$

$3x + 2y = 8$

$$\frac{a_1}{a_2} = \frac{2}{3}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-8}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\frac{x}{-8 - (-10)} = \frac{y}{-15 + 16} = \frac{1}{4 - 3}$$

$\frac{x}{2} = \frac{y}{1} = 1$

$\frac{x}{2} = 1, \frac{y}{1} = 1$

$x = 2, y = 1$

(iii) $3x - 5y = 20$

$6x - 10 = 40$

$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) $x - 3y - 7 = 0$

$3x - 3y - 15 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \frac{c_1}{c_2} = \frac{-7}{15}$

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations. By cross-multiplication,

$\frac{x}{45 - (21)} = \frac{y}{-21 - (-15)} = \frac{1}{-3 - (-9)}$

$\frac{x}{24} = \frac{y}{-6} = \frac{1}{6}$

$\frac{x}{24} = \frac{1}{6}$ and $\frac{y}{-6} = \frac{1}{6}$

$x = 4$ and $y = -1$

**Q.2** (i) For which values of a and b will the following pair of linear equations have an infinite number of solutions?

$2x + 3y = 7$

$(a - b)x + (a + b)y = 3a + b - 2$

(ii) For which value of k will the following pair of linear equations have no solution?

$3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

**Sol:**

(i) $2x + 3y = 7$

$(a - b)x + (a + b)y - (3a + b - 2) = 0$

$$\frac{a_1}{a_2} = \frac{2}{a - b}, \frac{b_1}{b_2} = \frac{3}{a + b}, \frac{c_1}{c_2} = \frac{-7}{-(3a + b - 2)} = \frac{7}{(3a + b - 2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{2}{a - b} = \frac{7}{(3a + b - 2)}$$

$6a + 2b - 4 = 7a - 7b$

$a - 9b = -4$ (1)

$$\frac{2}{a - b} = \frac{3}{a + b}$$

$2a + 2b = 3a - 3b$

$a - 5b = 0$ (2)

Subtracting (1) from (2), we obtain

$4b = 4 \Rightarrow b = 1$

Substituting this in equation (2), we obtain

$a - 5 \times 1 = 0$

$a = 5$

Hence, $a = 5$ and $b = 1$ are the values for which the given equations give infinitely many solutions.

(ii) $3x + y = 1$

$(2k - 1)x + (k - 1)y = 2k + 1$

$$\frac{a_1}{a_2} = \frac{3}{2k - 1}, \frac{b_1}{b_2} = \frac{1}{k - 1}, \frac{c_1}{c_2} = \frac{-1}{-2k - 1} = \frac{1}{2k + 1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

$$\frac{3}{2k - 1} = \frac{1}{k - 1}$$

$3k - 3 = 2k - 1$

$k = 2$

Hence, for $k = 2$, the given equation has no solution.

**Q.3** Solve the following pair of linear equations by the substitution and cross multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

**Sol:**

$8x + 5y = 9$ (i)

$3x + 2y = 4$ (ii)

From equation (ii), we obtain

$x = \frac{4 - 2y}{3}$ (iii)

Substituting this value in equation (i), we obtain

$8\left(\frac{4 - 2y}{3}\right) + 5y = 9$

$32 - 16y + 15y = 27$

$-y = -5$

$y = 5$ (iv)

Substituting this value in equation (i), we obtain

$3x + 10 = 4$

$x = -2$

Hence, $x = -2, y = 5$

Again, by cross-multiplication method, we obtain

$8x + 5y = 9$

$3x + 2y = 4$

$\frac{x}{-20 - (-18)} = \frac{y}{-27 - (-32)} = \frac{1}{16 - 15}$

$\frac{x}{-2} = \frac{y}{5} = \frac{1}{1}$

$\frac{x}{-2} = 1$ and $\frac{y}{5} = 1$

$x = -2$ and $y = 5$

**Q.4** Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii) A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. Find the fraction.

(iii) Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv) Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v) The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

**Sol:**

(i) Let x be the fixed charge of the food and y be the charge for food per day.

According to the given information,

$x + 20y = 1000 \qquad (1)$

$x + 26y = 180 \qquad (2)$

Subtracting equation (1) from equation (2), we obtain

$6y = 180$

$y = 30$

Substituting this value in equation (1), we obtain

$x + 20 \times 30 = 1000$

$x + 1000 - 600$

$x = 400$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii) Let the fraction be $\frac{x}{y}$.

According to the given information,

$$\frac{x-1}{y} = \frac{1}{3} \Rightarrow 3x - y = 3 \qquad (1)$$

$$\frac{x}{y+8} = \frac{1}{4} \Rightarrow 4x - y = 8 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$x = 5 \qquad (3)$

Putting this value in equation (1), we obtain

$15 - y = 3$

$y = 12$

Hence, the fraction is $\frac{5}{12}$

(iii) Let the number of right answers and wrong answers be x and y respectively.

According to the given information,

$3x - y = 40$ (1)

$4x - 2y = 50$

$2x - y = 25$ (2)

Subtracting equation (2) from equation (1), we obtain

$x = 15$ (3)

Substituting this in equation (2), we obtain

$30 - y = 25$

$y = 5$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of 1st car and 2nd car be u km/h and v km/h.

Respective speed of both cars while they are travelling in same direction = $(u - v)$ km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = $(u + v)$ km/h

According to the given information,

$5(u - v) = 100$

$u - v = 20$ (1)

$1(u + v) = 100$ (2)

Adding both the equations, we obtain

$2u = 120$

$u = 6 =$ km/h (3)

Substituting this value in equation (2), we obtain

$v = 40$ km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be x unit and y unit respectively.

Area = xy

According to the question,

$(x - 5)(y + 3) = xy - 9$

$3x - 5y - 6 = 0$ (1)

$(x+3)(y+2) = xy + 67$

$2x + 3y - 61 = 0 \qquad (2)$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$x = 17, y = 9$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.