SpeedLabs

MATHS

CBSE 10th

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Pair of linear equations
Exercise-3.6

Q.1 Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$

$2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Sol:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows.

$\frac{p}{2}+\frac{q}{3}=2 \Rightarrow 3p+2q-12=0$ (1)

$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \Rightarrow 2p+3q-13=0$ (2)

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$\frac{p}{10}=\frac{1}{5}$ and $\frac{q}{15}=\frac{1}{5}$

$p=2$ and $q=3$

$\frac{1}{x}=2$ and $y=\frac{1}{3}$

$x=\frac{1}{2}$ and $y=\frac{1}{3}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$2p+3q=2$ (1)

$4p-9q=-1$ (2)

Multiplying equation (1) by 3, we obtain

$6p+9q=6$ (3)

Adding equation (2) and (3), we obtain

$10p=5$

$p=\frac{1}{2}$ (4)

Putting in equation (1), we obtain

$2\times\frac{1}{2}+3q=2$

$3q=1$

$q=\frac{1}{3}$

$p=\frac{1}{\sqrt{x}}=\frac{1}{2}$

$\sqrt{x}=2$

$x=4$

and $q = \frac{1}{\sqrt{y}} = \frac{1}{3}$

$\sqrt{x} = 3$

$y = 9$

hence, $x = 4, y = 9$

(iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} - 4y = 23$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$3p + 3y = 14 \Rightarrow 4p + 3y - 14 = 0$ (1)

$3p - 4y = 23 \Rightarrow 3p - 4y - 23 = 0$ (2)

By cross-multiplication, we obtain

$$\frac{p}{-69 - 56} = \frac{y}{-42 - (-92)} = \frac{1}{-16 - 9}$$

$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$

$\frac{p}{-125} = \frac{-1}{25}$ and $\frac{y}{50} = \frac{-1}{25}$

$p = 5$ and $y = -2$

$p = \frac{1}{x} = 5y = -24$

(iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$$\frac{6}{x - 1} - \frac{3}{y - 2} = 1$$

Putting $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$ in the given equation, we obtain

$5q + q = 2$ (1)

$6p - 3q = 1$ (2)

Multiplying equation (1) by 3, we obtain

$15p + 3q = 2$ (1)

Adding (2) an (3), we obtain

$21p = 7$

$p = \frac{1}{3}$

Putting this value in equation (1), we obtain

$5 \times \frac{1}{3} + q = 2$

$q = 2 - \frac{5}{3} = \frac{1}{3}$

$p = \frac{1}{x-1} = \frac{1}{3}$

$x - 1 = 3$

$x = 4$

$q = \frac{1}{y-2} = \frac{1}{3}$

$y - 2 = 3$

$y = 5$

$x = 4,$

$y = 5$

(v) $\frac{7x-2y}{xy} = 5$

$\frac{7}{y} - \frac{2}{x} = 5$ (1)

$\frac{8x+7y}{xy} = 15$

$\frac{8}{y} + \frac{7}{x} = 15$ (2)

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$ in the given equation, we obtain

$-2p + 7q = 5 \Rightarrow -2p + 7q = 0$ (3)

$7p + 8q = 15 \Rightarrow 7p + 8q - 15 = 0$ (4)

By cross-multiplication method, we obtain

$\frac{p}{-105-(-40)} = \frac{q}{-35-30} = \frac{1}{-16-49}$

$\frac{p}{-65} = \frac{q}{-65} = \frac{1}{-65}$

$p = 1$ and $q = 1$

$p = \frac{1}{x} = 1 \quad q = \frac{1}{y} = 1$

$x = 1 \quad y = 1$

(vi) $6x + 3y = 6xy$

$2x + 4y = 5xy$

$\frac{6}{y} + \frac{3}{x} = 6$ (1)

$2x + 4y = 5xy$

$\frac{2}{x} + \frac{4}{x} = 5$ (2)

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$ in the given equation, we obtain

$3p + 6q - 6 = 0$

$4p + 2q - 5 = 0$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$$

$$\frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$$

$p = 1$ and $q = \frac{1}{2}$

$p = \frac{1}{x} = 1 \quad q = \frac{1}{y} = 1$

$x = 1 \qquad y = 2$

(vii) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

Putting $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$ in the given equation, we obtain

$10p + 2q = 4 \Rightarrow 10p + 2q - 4 = 0 \qquad (1)$

$15p - 5q = -2 \Rightarrow 15p - 5q + 2 = 0 \qquad (2)$

Using cross-multiplication method, we obtain

$$\frac{p}{4-20} = \frac{q}{-60-(20)} = \frac{1}{-50-30}$$

$$\frac{p}{-16} = \frac{q}{-80} = \frac{1}{-80}$$

$$\frac{p}{-16} = \frac{1}{-80} \text{ and } \frac{q}{-80} = \frac{1}{-80}$$

$p = \frac{1}{5}$ and $q = 1$

$p = \frac{1}{x+y} = \frac{1}{5}$ and $q = \frac{1}{x-y} = 1$

$x + y = 5 \qquad (3)$

and $x - y = 1 \qquad (4)$

Adding equation (3) and (4), we obtain

$2x = 6$

$x = 3 \qquad (5)$

Substituting in equation (3), we obtain

$y = 2$

Hence, $x = 3, y = 2$

(viii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$$

Putting $\frac{1}{3x+y} = p$ and $\frac{1}{3x-y} = q$ in the given equation, we obtain

$$p + q = \frac{3}{4} \qquad (1)$$

$$\frac{p}{2} - \frac{q}{2} = \frac{-1}{8}$$

$$p - q = \frac{-1}{4} \qquad (2)$$

Adding (1) and (2), we obtain

$$2p = \frac{3}{4} - \frac{1}{4}$$

$$2p = \frac{1}{2}$$

$$p = \frac{1}{4}$$

Substituting in (2), we obtain

$$\frac{1}{4} - q = \frac{-1}{4}$$

$$q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$p = \frac{1}{3x+y} = \frac{1}{4}$$

$$3x + y = 4 \qquad (3)$$

$$q = \frac{1}{3x-y} = \frac{1}{2}$$

$$3x - y = 2 \qquad (4)$$

Adding equations (3) and (4), we obtain

$$6x = 6$$

$$x = 1 \qquad (5)$$

Substituting in (3), we obtain

$$3(1) + y = 4$$

$$y = 1$$

Hence, $x = 1, y = 1$

Q.2 Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Sol:

(i) Let the speed of Ritu in still water and the speed of stream be x km/h and y km/h respectively.

Speed of Ritu while rowing

Upstream $=(x-y)$ km/h

Downstream $=(x+y)$km/h

According to question,

$2(x+y)=20$

$x+y=10$ (1)

$2(x-y)=4$

$x-y=2$ (2)

Adding equation (1) and (2), we obtain

$2x=12 \Rightarrow x=6$

Putting this in equation (1), we obtain

$y=4$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii) Let the number of days taken by a woman and a man be x and y respectively.

Therefore, work done by a woman in 1 day $=\frac{1}{x}$

Work done by a man in 1 day $=\frac{1}{y}$

According to the question,

$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$

$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$

$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$

$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$ in the given equation, we obtain

$2p+5q=\frac{1}{4}$

$8p+20q=1$

$3p+6q=\frac{1}{3}$

$9p+18q=1$

By cross-multiplication, we obtain

$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$

$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$

$\frac{p}{-2}=\frac{-1}{36}$ and $\frac{q}{-1}=\frac{1}{-36}$

$p=\frac{1}{18}$ and $q=\frac{1}{36}$

$p=\frac{1}{x}=\frac{1}{18}$ and $q=\frac{1}{y}=\frac{1}{36}$

$x=18 \quad y=36$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be u km/h and v km/h respectively.

According to the given information,

$\frac{60}{u}+\frac{240}{v}=4$ (1)

$\frac{100}{u}+\frac{200}{v}=\frac{25}{6}$ (2)

Putting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in the given equation, we obtain

$60p+240q=4$ (3)

$100p+200q=\frac{25}{6}$

$600p+1200q=25$ (4)

Multiplying equation (3) by 10, we obtain

$600p + 240q = 40$ (5)

Subtracting equation (4) from (5), we obtain

$1200q = 15$

$q = \frac{15}{1200} = \frac{1}{80}$ (6)

Substituting in equation (3), we obtain

$60p + 3 = 4$

$60p = 1$

$p = \frac{1}{60}$

$p = \frac{1}{u} = \frac{1}{60}$ and $q = \frac{1}{v} = \frac{1}{80}$

$u = 60\text{km/h}$ and $v = 80\ \text{km/h}$

Hence, speed of train $= 60\ \text{km/h}$

Speed of bus $= 80\ \text{km/h}$