SpeedLabs

MATHS

CBSE 11th

TEEVRA EDUTECH PVT. LTD.

1. If ${}^{n}C_8 = {}^{n}C_2$, find ${}^{n}C_2$.

Ans. It is known that ${}^{n}C_a = {}^{n}C_b \Rightarrow a = b \text{ or } n = a + b$

Therefore,

$${}^{n}C_8 = {}^{n}C_2 \Rightarrow n = 8 + 2 = 10$$

$$\therefore {}^{n}C_2 = {}^{10}C_2 \Rightarrow \frac{10!}{2!(10-2)!} = \frac{10!}{2!(10-2)!} = \frac{10 \times 9 \times 8!}{2 \times 1 \times 8!} = 45$$

2. Determine n if

(i) $\therefore {}^{2n}C_2 : {}^{n}C_3 = 12 : 1$ (ii) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$

Ans. (i) $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!}{n!} = \frac{12}{1}$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$$

$$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$$

$$\Rightarrow 2n - 1 = 3(n-2)$$

$$\Rightarrow 2n - 1 = 3n - 6$$

$$\Rightarrow 3n - 2n = -1 + 6$$

$$\Rightarrow n = 5$$

(ii) $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1}$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(2n-3)!}{n!} = 11$$

$$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 11$$

$$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$$

$$\Rightarrow \frac{4(2n-1)}{n-2} =$$

$$\Rightarrow 4(2n-1) = 11(n-2)$$

$$\Rightarrow 8n - 4 = 11n - 22$$

$$\Rightarrow 11n - 8n = -4 + 22$$

$$\Rightarrow 11n - 8n = -4 + 22$$

$$\Rightarrow 3n = 18$$

$$\Rightarrow n = 6$$

3. How many chords can be drawn through 21 points on a circle?

Ans. For drawing one chord on a circle, only 2 points are required. To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted. Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time. Thus, required number of chords =

$$^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$$

4. In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Ans A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls. 3 boys can be selected from 5 boys in $^5C_3$ ways. 3 girls can be selected from 4 girls in $^4C_3$ ways. Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$$= \frac{5 \times 4 \times 3}{3! \times 2} \times \frac{4 \times 3!}{3!}$$

$$= 10 \times 4 = 40$$

5. Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

Ans. There are a total of 6 red balls, 5 white balls, and 5 blue balls.9 balls have to be selected in such a way that each selection consists of 3 balls of each colour. Here, 3 balls can be selected from 6 red balls in ways ${}^{6}C_{3}$

3 balls can be selected from 5 white balls in ways ${}^{5}C_{3}$.

3 balls can be selected from 5 blue balls in ways ${}^{5}C_{3}$.

Thus, by multiplication principle, required number of ways of selecting 9 balls.

$$= {}^{6}C_{3} \times {}^{5}C_{3} \times {}^{5}C_{3} = \frac{6!}{3!\,2!} \times \frac{5!}{3!\,2!} \times \frac{5!}{3!\,2!}$$
$$= \frac{6 \times 5 \times 4 \times 3!}{3! \times 3 \times 2} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1} \times \frac{5 \times 4 \times 3!}{3! \times 2 \times 1}$$
$$= 20 \times 10 \times 10 = 2000$$

6. Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Ans. In a deck of 52 cards, there are 4 aces. A combination of 5 cards have to be made in which there is exactly one ace. Then, one ace can be selected in ${}^{4}C_{1}$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_{4}$ ways. Thus, by multiplication principle, required number of 5 card combinations.

$$= {}^{48}C_{4} \times {}^{4}C_{1} = \frac{48!}{4!\,44!} \times \frac{4!}{1!\,3!}$$
$$= \frac{48 \times 47 \times 46 \times 45}{4 \times 3 \times 2 \times 1} \times 4$$
$$= 778320$$

7. In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Ans. Out of 17 players, 5 players are bowlers. A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers. 4 bowlers can be selected in ${}^{5}C_{4}$ ways and the remaining 7 players can be selected out of the 12 players in ways ${}^{12}C_{7}$. Thus, by multiplication principle, required number of ways of selecting cricket team.

$$= {}^{5}C_{4} \times {}^{12}C_{7} = \frac{5!}{4!\,1!} \times \frac{12!}{7!\,5!} = 5 \times \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 3960$$

8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Ans There are 5 black and 6 red balls in the bag. 2 black balls can be selected out of 5 black balls ${}^{5}C_{2}$ in ways and 3 red balls can be selected out of 6 red balls ${}^{6}C_{3}$ in ways. Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls.

$$= {}^5C_2 \times {}^6C_3 = \frac{5!}{3!2!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200$$

9. In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Ans There are 9 courses available out of which, 2 specific courses are compulsory for every student. Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^7C_3$ ways.

Thus, required number of ways of choosing the programme.

$$= {}^7C_3 = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35$$